Studies in the Structure of the American Economy

Leontief, Wassily W.

STUDIES IN THE STRUCTURE OF THE AMERICAN ECONOMY

THEORETICAL AND EMPIRICAL EXPLORATIONS IN INPUT-OUTPUT ANALYSIS

BY

WASSILY LEONTIEF

AND

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PREFACE

In this volume the Harvard Economic Research Project on the Structure of the American Economy presents some of the results of the first three years of its activity. The over-all objectives of the project and the methods of analytical procedure to be followed were described in the original statement of its research program, issued in March 1948, as follows:

The program of the Harvard Economic Research Project represents an attempt to lay an elaborate and well-integrated foundation for the empirical study of long-run problems. The ultimate aim of the program can be conceived as a 'tableau économique,' i.e., an internally consistent quantitative picture, for, say, two decades hence, which would show, in some detail, the outputs of various goods, and the inputs of resources, services, and the stock of capital required to produce them, on the basis of certain anticipations for such factors as the population, the magnitude of the economic activities of the government and the method of financing them, and so on. But this is an ultimate aim, perhaps only to be approached. The proximate aim is rather the study of the structural interrelations in the economy today and in the recent past; the forces making for changes in this structure, and the effect of changes in these forces in creating a new structure. The terms 'structure' and 'structural interrelation' are here used to denote the complex web of relations through which any given unit of resources is assimilated to the satisfaction of some economic want. It includes descriptions of the industrial structure, in which the distribution of outputs and the sources of the inputs of each industry are traced out: of the capital structure, i.e., the relations of stocks of investment goods to the output of each kind of product; and of the structure of consumption in relation to the distribution of income and other characteristics of consuming units.

Such a structural approach by its very nature cannot be purely descriptive or even strictly 'behavioristic.' A national economy in operation is a process of simultaneous adjustment in which a very large number of apparently separate but actually interdependent flows of production, distribution, consumption and investment are constantly affecting each other and ultimately are determined by a set of basic characteristics of the system. A valid explanation of the past, not to mention prognostication of the future develop-

ments must satisfy a test of internal consistency which it cannot possibly pass without having at the outset been formulated in strictly analytical, i.e., theoretical terms.

Because of its scope the research program will have to be carried out over a number of years and by many hands. The program will consist of a number of studies carried out by individuals or groups, designed so as to contribute in a cumulative fashion to the common goal described above. To secure this end:

(a) The theoretical conceptions underlying all individual studies will be developed with a view toward mutual consistency. Accordingly,

(b) all definitions and classifications used in the separate studies (in particular, the basic industrial and commodity classifications) will be identical or at least directly translatable into each other;

(c) the basic outlines of the individual studies will be designed and their findings formulated so as to make them fit each other within the framework of the over-all analytical scheme underlying the project as a whole.

(d) A systematic effort will be made to get away as much as possible from the use of grossly aggregative entities. All statistical studies will be conducted and the final quantitative findings formulated in terms of rather detailed industry, commodity and income classifications.

Of necessity the methods used in the individual projects will vary widely. There is, however, a common element in the methods to be used in all of the investigations. The procedure in every case will be to make hypotheses which can be understood and interpreted in terms of the behavior of individual households, or firms, and then to test them by means of any available data. Since much of the behavior of firms and households is sociologically motivated, it will be necessary to use data from these fields as well as more narrowly economic data in testing our hypotheses.

The twelve chapters and three separate appendices published here give a fairly comprehensive picture of the progress of this exploratory venture up to now.

No attempt has been made in this exposition to straighten out the naturally uneven line of advance of our inquiries. In some directions the road appeared to be relatively smooth. Difficult and occasionally treacherous terrain made the progress along the other paths slow and tortuous. Risks were taken—without risks there can be no true research—but also reasonable precautions. The channels of conceptual or, better to say, theoretical communication between the individual investigators were kept clear all the time and the heavy cargo of systematically organized factual information was brought up, if not to the most advanced points of exploration, then at least to within a short distance of them. On the other

hand, care was taken not to accumulate—even in the appendices—impressive but essentially useless dumps of statistical data.

Although I have carried on the basic research in American economic structure for some years, the Harvard Economic Research Project as such did not come into being until 1948. A four-year grant from the Rockefeller Foundation made possible the formation of a research organization which could push the research in this field farther and more effectively than I had previously been able to do with very limited research assistance. Part of our work, since 1949, has also been financed under a government contract by the United States Air Forces. I acknowledge with gratitude the financial support received from these agencies. Without their assistance this volume would not have been possible.

The United States Department of Labor was the first government agency to take active interest in the 'input-output' approach to the study of the American Economy and the continual co-operative relationship with its Bureau of Labor Statistics has benefited our work most decisively.

The authors of the various studies contained in this book are all present or former members of the senior research staff of the Project. Specific acknowledgment to all those who have worked in many capacities upon the research cannot be made in this limited space. Particular contributions of individuals are noted in various chapters.

Special acknowledgment should be made to Dr. Elizabeth W. Gilboy, Associate Director of the Project, who has served as editor of this volume and guided the development of the chapters from their earliest stages; to Robert Solow, Assistant Professor at the Massachusetts Institute of Technology, for his part in the basic empirical research on the capital structure; to A. Benjamin Handler, now Associate Professor at the University of Michigan, who supervised the later stages of the same work.

Others who should be mentioned are: Robert L. Allen, Mrs. Judith Balderston, Otto Bird, Mrs. Nancy Bromberger, Mrs. Carol Cameron, William Capron, Mrs. Sara Clark, Miss Bernadette Drolette, Mrs. Fay Greenwald, Mrs. Ruth Kahn, Miss Lora Katz, Robert Kavesh, Miss Elaine Kazanowski, Mrs. Mary Kazanowski, John Lansing, Irwin Leff, Richard Levitan, Mrs. Dolores McJilton, Leon Moses, Miss M. Janice Murphy, Mrs. Margaret Oliver, Myer Rashish, Richard Rosenthal, Ira Scott, Jr., Mrs. Martha Shoesmith, Burton Singer, Mrs. Edith Soodak, Miss Raya Spiegel, Carl Stevens, Tun Thin, and Miss Ruth Winer.

Mrs. Jean Allen is responsible for the excellent draftsmanship of most of the charts. The maps for Chapter 5 were drawn by Edward Schmitz. Walter C. Eberhard also drafted additional charts for Chapter 5. The index was prepared by James Henderson.

Certain discrepancies of minor magnitude can be found in comparing the figures in some of the tables. Some of these may be errors, despite

careful checking, but it is more likely that they are due to rounding and particularly to the continuing nature of the research. During the process of preparing the manuscript, and its publication, additional data were found and, where possible, included in the volume. It was not possible at this stage to revise all the original material for complete consistency with the new.

In solving the difficult technical problems involved in the production of this volume, the Oxford University Press, New York, showed exceptional forbearance toward what another less understanding publisher would have considered to be our unreasonably exacting demands.

In order to facilitate cross references, the mathematical equations are designated by the number of the chapter in which they appear as well as by continuous equation numbers within each chapter. The chapter numbers are placed first. For example, equation (3, 8) signifies equation 8 of Chapter 3; (10, 4) equation 4 of Chapter 10, et cetera. Equations in the mathematical notes following the various chapters are numbered similarly, with the addition of n to indicate their appearance in the note; (3n, 12) signifies equation 12 in the note to Chapter 3.

Acknowledgments to publishers and others who have kindly allowed us to use their material are given on a separate page.

WASSILY LEONTIEF

Cambridge, Massachusetts
July 1952

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Chapter 1

INTRODUCTION

Wassily Leontief

I. Economic Theory and Factual Analysis

In principle the nature of the co-operative relationship between theory and direct observation is clear and simple: The theory states what factual data are to be secured and how they are to be used within the framework of a particular analytical scheme; the observations provide the data.

Economic theory, like any other, distinguishes between data and the unknowns, between those aspects of an observed situation which are to be explained and those which are considered as 'given' and serve—within the framework of the particular theoretical structure—as the basis of the proposed explanation. The same element which within one theoretical context is treated as an unknown might be treated as a datum in another and vice versa. Thus, for example, the conventional 'partial equilibrium' theory explains the output of an industry and the price at which it is sold in terms of the 'given' market supply and demand functions; a more general analysis derives the shapes of these curves themselves from the given cost functions of the individual sellers and the demand functions of the buyers. Proceeding further in the same direction, it can be shown how the former can be explained on the basis of given engineering-production functions while the latter are derived from the 'preference varieties' of the individual households. Each one of these explanatory schemes would obviously require different sets of empirical data—the market supply and demand curves in the first case, the individual cost and demand functions in the second, and the technical production and psychological (or sociological) preference functions in the third.

As long as appropriate basic data are introduced into a theoretical argument in the purely symbolic form of hypothetical assumptions, the actual availability of corresponding information does not matter. If it comes up at all as a problem, it does so only to the extent that the theory is required to be operational in the most general, or rather conjunctive, sense of that word. The reference to a mountain on the other side of the moon is in this sense perfectly operational since one can easily imagine taking a photograph of it from an interplanetary rocket.

If, on the other hand, empirical implementation is considered as much a part of an economic argument as the consistent development of its logical consequences, the relationship between theoretical and factual analysis appears in a different light. How would contemporary economics fare if each of the more important theoretical schemes were paired off with the factual data required for their empirical implementation and vice versa? All too many theories would be found short of factual content and all too many assumed facts scattered beyond the pale of any relevant theoretical structure.

II. Economics as an Empirical Science

The engine of economic theory has reached, in the last twenty years, a high degree of internal perfection and has been turning over with much sound and fury. If the advance of economics as an empirical science is still rather slow and uncertain, the lack of sustained contact between the wheels of theory and the hard facts of reality is mainly to blame.

For many years the empirical basis of economic analysis was limited to a few data of common experience and the theorists directed their main efforts toward the construction of ever larger systems of as many possible conclusions as could logically be supported on the narrow base of a few factual premises. The ascetic attitude toward the use of direct empirical evidence has in the long run affected the nature of traditional economic theory to such an extent that by the time the flow of facts and figures began to increase, the imposing theoretical machine proved to be constitutionally unable to make use of it. Nor do many of its attendants seem to be much concerned over the resulting situation. Without marked change in pace and direction they proceed as before, adding here one assumption, removing there another; deriving here a new, and disproving there an old, conclusion.

Backed up against the all too narrow gate of analytical economics, the ever-increasing stream of statistical and other factual information found itself a separate outlet. I do not have in mind so much the applied and descriptive studies which for generations have constituted and always will make up the bulk of economic literature, but rather the particular type of analysis which concerns itself with the systematic description of various important phases of the economic process in quantitative statistical terms. Culminating in national income statistics, it covers production, consumption, prices, employment, and many other measurable aspects of economic activity.

This movement—connected closely with the development of governmental statistics—deserves great credit for having collected, organized, classified, and often even originated a great amount of invaluable em-

pirical data which otherwise would not have been presented in any intelligible form. Developed occasionally in ignorance and often in defiance of abstract theoretical tradition, this empirical descriptive school evolved a terminology and a set of methodological procedures all its own. These are related to the conceptual framework of pure theory only through their common origin in the language and practices of everyday economic intercourse. Aggregation and averaging are the two main statistical devices used in descriptive quantitative economics for simplified and generalized presentation of basic empirical data.

The persistent cleavage between a preponderantly deductive type of analysis, on the one side, and radical empiricism, on the other, has been widely recognized as a principal fault of economic science. Much original and imaginative work toward the eventual widening of the empirical basis of analytical economics was done by the modern school of statistical econometricians. In one important respect, however, this new school seems to be following in the footsteps of the old, deductive approach. It also has apparently accepted the paucity of direct empirical information as a natural, albeit regrettable, condition which economists will have to accept for years to come. In the same spirit in which pure economic theory aims at deriving the greatest number of possible conclusions from a few fundamental assumptions, the modern statistical school with utmost ingenuity explores various methods of drawing the largest possible set of statistical inferences from a small and strictly limited number of direct observations.

Both are fascinating endeavors worthy of the intensive application of the best scientific minds, but I submit that neither the elaborate deductive, theoretical models nor the most refined techniques of statistical inference can contribute much to the accomplishment of the principal task confronting contemporary economics: the task of widening radically and effectively its empirical basis. The problem we are facing is fundamentally not of a tactical nature; it is not simply a question of a better performance in one or another direction of a small adjustment in the line-up in a limited sector. Its solution would require redefinition of our general strategic objectives. It involves the engagement of considerable new forces and, in case such are not forthcoming, a large-scale redeployment of the available resources.

III. Statistical Inference

Both theoretical formulation and factual description must be reoriented if they are to be brought closer to each other. Much of contemporary abstract analysis is couched in aggregative terms. At worst, this robs it of any operational meaning; at best, it separates artificially the essen-

tially analytical task of defining the aggregates in terms of the directly observed 'real' variables from the rest of the theoretical argument and shifts it onto the shoulders of the empirical investigator, who often is even unaware of its true import.

Given a fixed set of primary empirical data and two alternative formulations of essentially the same theoretical problem—one of which utilizes these data directly while the other approaches them through intermediately defined aggregative variables—the former can generally be expected to make a more efficient use of the available material than the latter. It is one of the great accomplishments of economic theory that it has developed a remarkably well-organized framework for the analytical manipulation of systems involving a very large number of distinct but interrelated elements. One of the paradoxical aspects of its present situation is that, instead of guiding empirical research in the direction of differentiated sets of primary data, the theory itself seems to be reverting to grossly aggregative formulation. The reason for this—beside the purely didactic requirements of superficial simplicity—is possibly methodological.

The empirical components which are to be included in the theoretically prefabricated analytical schemes can be arrived at either through direct observation or through the use of various more-or-less intricate methods of indirect statistical inference. The production function of some particular industry can, for example, be obtained directly through collection of the relevant technical, engineering information or it can be determined indirectly by way of a rather intricate interpretation of the supply-demand reactions of the industry.

In the latter case the relations between the data and the unknown variables become actually opposite to those which exist in the formal explanatory scheme. Instead of considering the technical production function as known and deriving from it the unknown price and output reaction of the industry, the statistician, in the process of indirect inference, takes the price and output reactions as 'given' and derives from them the unknown shape of the underlying production function.

To avoid a possible misunderstanding of the following remarks, let it be stated at once that the fact that the same production function might subsequently be used to explain the price behavior of the industry cannot serve as a basis for a fundamental objection to indirect statistical inference. The use of invisible but indirectly described entities constitutes one of the most useful devices of scientific inquiry. On the other hand, there can be little doubt that direct observation offers the undisputable advantages of operational simplicity and, because of that, greater reliability.

Undue and exclusive emphasis on indirect statistical inference in some of the more recent empirical studies has at times put on this useful, nay indispensable, tool of empirical analysis a burden which it actually cannot

possibly sustain. Statistical rather than purely theoretical difficulties have forced much of the current empirical research into the narrow channel of unduly aggregative models. And even then, in their desperate search for sufficiently large 'samples,' the proponents of indirect statistical inference have found themselves driven toward the treacherous shoals of time-series analysis. There they face the fatal choice between strongly auto-correlated short series and series covering a longer span of years, which expose the investigator to the even more fundamental danger of assuming invariance in relationships which actually do change and even lose their identity over time.

Once economic theory is reformulated in such a way as to be able to absorb large amounts of detailed factual data, even the present supply of primary empirical information—disaggregated and sorted out in accordance with more concrete theoretical specifications—will go much further than it does now in putting economic analysis on a realistic basis. Next, or rather at the same time, must come a systematic and equally disciplined theoretical exploitation of new sources of factual information.

The distinction between direct observation and indirect inference is obviously a question of degree. In some instances simple unsophisticated observation will lead us far; in others—as in the problem of entrepreneurial and even consumer expectations—an indirect hypothetical approach can hardly be avoided.

As long as the analytical economist is satisfied with 'assumptions' rather than actual observations (direct and indirect alike), he is free to roam over the field of his inquiry more or less at random. The feeling that a given problem could certainly be solved if only the necessary factual information were at hand might easily induce him to pass it by in favor of another, where the possibilities of getting direct insight into the underlying factual relationships are more remote. This tendency to by-pass the easy empirical problems in favor of the more difficult (because more speculative) theoretical issues seems to be partly responsible for the lack of any clear-cut line of cumulative advance in our science. A less adventurous and more conservative approach would consider actual empirical implementation of the general theoretical argument as much a part of the solution of a given economic problem as its abstract formulation. The investigator will have discovered from hard experience that a point of strong resistance should be attacked only after all adjoining less difficult terrain has been already fully occupied.

IV. The Input-Output Approach

Both the simplified version of the classical general equilibrium theory—referred to as the input-output approach—which constitutes the theo-

retical framework of the present volume, and the first phase of its actual application to the study of the American economy, have already been described in considerable detail elsewhere.[1]

The systematic connection between the diverse and specialized inquiries presented in the following chapters can best be understood if they are approached against the background of a short recapitulation of these past results.

Considered from the point of view of the input-output scheme any national economy can be described as a system of mutually interrelated industries or—if one prefers a more abstract term—interdependent economic activities. The interrelation actually consists in the more or less steady streams of goods and services which directly or indirectly link all the sectors of the economy to each other. These flows can be observed and described in quantitative terms. A fold-in table in *The Interindustry Relations Study for 1947* represents, for example, the input-output structure of the American economy for the year 1947.[2] A part of this table is reproduced in Table 1 for illustrative purposes.

The whole system has been subdivided into 50 sectors comprising agriculture, various extractive and manufacturing industries, electric public utilities, three kinds of transportation, trade and other types of service industries. Foreign countries are treated as a separate industry. Households and government, the latter comprising all public institutions not engaged in regular productive activities, constitute the two large non-industrial sectors of the system.

The headings at the top and the left-hand side of the table indicate that each row and each corresponding column (except the very last one) is identified with a particular sector of the economy. The figures entered along any one horizontal row show how the annual 1947 output of the particular industry has been distributed among all the other industries, the households, and the government. This total output itself is shown at the end of the row, i.e. in the last right-hand column.

If examined column-wise, the same figures show the 1947 input structure of the economy; each vertical column contains the amounts of the products of all the different branches of the economy which have been absorbed by its own particular sector.

All figures in this table are shown in dollars. They might as well have been given in physical units appropriate for the description of the outputs of the individual sectors of the economy—tons of coal, bushels of

[1] *The Structure of American Economy*, 1919-1939, 2nd edition, revised, Oxford University Press, New York, 1951.

[2] By W. Duane Evans and Marvin Hoffenberg, *Review of Economics and Statistics*, May 1952. Three similar tables for 1919, 1929, and 1939 are included in the appendix to *The Structure of American Economy*.

wheat, ton miles of transportation, man-hours of work, and so on. As a matter of fact, the dollar figures entered in each particular row can be interpreted in this sense provided one defines the physical units in which they have been measured as 'the amount (i.e. number of tons, yards, ton miles, or hours) of the particular product purchasable for one dollar at the prevailing 1947 prices.' Only the 'total inputs' do not lend themselves

TABLE 1

Quantitative Input-Output Relations in the United States, 1947[1]
(millions of dollars)

Industry Purchasing

	Industry Producing	Agriculture and Fisheries	Food and Kindred Products	Nonferrous Metals	Iron and Steel	Motors and Generators	Motor Vehicles		Total Output
		(1)	(2)	(3)	(4)	(5)	(6)		(7)
(1)	1 Agriculture and fisheries	10,856	15,048	11	---	---	---	...	44,263
(2)	2 Food and kindred products	2,378	4,910	*	3	---	---	...	37,636
(3)	3 Nonferrous metals	---	---	2,599	324	366	176	...	6,387
(4)	4 Iron and steel	6	2	33	3,982	118	196	...	12,338
(5)	5 Motors and generators	---	---	---	---	317	---	...	1,095
(6)	6 Motor vehicles	111	3	*	*	---	4,401	...	14,265
	.	.	.	.	.	.	.	...	.
	.	.	.	.	.	.	.	...	.
	.	.	.	.	.	.	.	...	.
	Total	44,263	37,636	6,387	12,338	1,095	14,263	...	769,248

[1] This table reproduces a portion of the 50-industry input-output table for 1947, which is included in full in "The Interindustry Relations Study for 1947," *Review of Economics and Statistics*, May 1952.

*Less than $500,000.

to this kind of physical interpretation: tons of coal, yards of cloth, and man-hours of labor cannot be added for any useful purpose. Thus so long as the argument is conducted in physical terms—as it indeed will be in the following two or three pages—the bottom row of figures must be entirely neglected.

To explain the particular configuration of the flows of goods and services shown in the input-output table, one must turn to the analysis of the basic structural characteristics of the individual sectors of the economy.

Turning from description to explanation one can discern two sets of basic conditions which must be satisfied by any consistent set of inter-industrial commodity and service flows.

First, there are the balance requirements: the combined inputs of each commodity or service must equal its total output.

Second, there are the structural characteristics of all the individual sectors of the economy. These imply the existence of definite relationships between the quantities of all the outputs absorbed by any one particular industry and the level of its total output. Had the 1947 iron and steel output been larger than the $12,338 million worth (in 1947 prices) actually produced in that year, the amounts of coal, ore, power, labor, and of all other inputs required for the production of steel would, for obvious technical reasons, have also been greater than the actual 1947 figures entered in the appropriate column of our table. A lower steel output would, on the other hand, imply smaller inputs. Similar direct dependence must obviously exist between the level of output and the input requirements of every other productive sector of the national economy. In economic theory these structural input-output relationships are often referred to as the 'production functions' of the respective industries.

The description of the production function of an industry becomes particularly simple if the amount of each cost factor absorbed per unit of finished product is technologically fixed. A doubling of the output would imply in this case a doubling of all inputs. To compute the input requirements of an industry for a prescribed output, one would have only to know its 'input coefficient,' that is, the constant quantities of each of the various inputs absorbed per unit of its final product.

With a given set of input coefficients describing the internal structures of all the productive sectors of the economy and a known 'bill of final demand' a complete input-output table of the economy can be reconstructed from the bottom up through solution of a system of simultaneous linear equations.

The 'final demand' is meant here to comprise that part of the total production of each commodity which is allocated outside the industrial sectors of the economy and is consumed, for example, by households and

government. Defined in more appropriate negative terms the 'final demand' encompasses all demand which is not accounted for through the input requirements computed from the given set of input coefficients. If and when households are treated as an industry which absorbs finished consumers' goods as its inputs and turns out labor services as its output, the demand for consumers' goods should not be included in the 'final bill of goods'; it would be determined, i.e. explained, indirectly and simultaneously with the inputs of all other industrial sectors. This implies, of course, the knowledge of the shape of the empirical 'consumption function' of the households, i.e. of the structural relationship between the output of labor (employment) on the one hand and the corresponding inputs of various consumers' goods absorbed by the households on the other.

V. Application of Input-Output

The general theoretical scheme outlined above is designed to absorb and to exploit for analytical purposes a large amount of detailed factual information. It establishes a direct connection between the investigation of the separate factors of the economy on the one hand and the analysis of the national economy as a whole in terms of their mutual interrelaships on the other.

In its first stages the work in the field of interindustry relationships concerned itself mainly with the exploration of the formal properties and the numerical characteristics of the originally compiled input-output tables. These *Studies* carry the analysis beyond the confines of these initial figures. Elaboration of the conceptual framework keeps step with the exploration and utilization of new empirical material. Some of these studies are primarily theoretical and methodological. Others are empirical in the sense that they deal in detail with factual data. All are designed to be fitted into the over-all picture of the American economy. This implies not only a common theoretical background but also strict conformity to definite empirical standards.

The following chapters do not comprise a logical sequence in the sense that one picks up where the other leaves off. Even if they made a complete whole, which they certainly do not, the simple linear succession of verbal expositions cannot possibly reflect the much more intricate relationship between the separate component parts of our analytical system.

Three leading themes will be discerned as dominating the entire volume. One is the problem of stocks as contrasted with and related to flows of goods and services; the second is the question of structural change; and the third, the problem of technological, psychological, and other noneconomic bases of economic relationships.

The exclusion of stocks from the original input-output scheme limits its applicability as a general equilibrium theory to short-run analysis. Additions to stocks have to be treated in this case as components of the exogenously determined outside demand, i.e. as parts of the final bill of goods. The alternative device (resorted to in the first three parts of *The Structure of American Economy*) of assuming that a constant proportion of the flow of inputs absorbed by an industry is allocated to investment lacks theoretical as well as practical foundation, except in the very special and essentially uninteresting case of an evenly progressing economy.

The explicit introduction of stocks of commodities along with their flows in the basic input-output scheme is intended primarily to supply the conceptual framework for empirical analysis of the investment process. Traditional capital theory even more than other parts of abstract economic analysis has suffered from unduly aggregative formulations. In the present analysis the capital structure of the national economy is visualized as consisting of physically discernible stocks of specific commodities. The input-output ratios of the simple flow theory described above are supplemented by corresponding stock-flow relationships. These explicitly take account of the fundamental technological fact that the production of a flow of specific output requires not only the availability of corresponding flows of current inputs but also the existence of previously accumulated stocks of equipment, buildings, inventories, intermediate products, etc. With the introduction of stock-flow relationships the system, of necessity, acquires dynamic character; the present rates of output become theoretically—as they certainly are actually—dependent upon the accumulation of the past input (investment) flows.

A substantial portion of this volume comprising Chapter 6, as well as Appendix 1, is devoted to a detailed quantitative description of the capital structure of the American economy in the year 1939. This statistical inquiry, combined with the previously derived flow figures, has been specifically designed to supply the factual basis for a mathematical study of the dynamic properties of the system. An exhaustive analytical exploitation of the large sets of empirical capital coefficients thus obtained involves extensive computations which will not be completed for some time to come.

The dynamic properties of the economic system as derived from the stock-flow relationships account for only one aspect of economic change, that which can be explained in terms of invariant structural constants. The other, more deep-seated causes of development are to be found in the variation of the basic structural relationships themselves—that is, in modification of changes in consumers' tastes—and changes in the structure of

productive processes. To be sure, the distinction between structural variations on the one hand and formal dynamics on the other is only a relative one, since the separation of data from variables is methodologically determined. One can easily visualize a system more comprehensive in scope than the conventional general equilibrium theory. Technical input-output relationships and consumers' tastes, as well as the other data of the present system, would appear in it as the unknowns to be explained in terms of some other more fundamental relationships. Considered from such a higher point of view the approach to the problem of technical change as presented in this volume could be best defined as 'comparative statics.' Without attempting to explain the observed variation in the technical constants the discussion is limited to the study of the effects which such changes have on the variables of the system. The discussion of the substitution of new input combinations for the old, however, points the way toward a more fundamental explanation of structural change.

In the analysis of technological progress pure theory without factual implementation is even more helpless than in the other fields of economic inquiry. Chapter 2 describes the variation of the technological basis of the American economy between the years 1919, 1929, and 1939. These are the three dates for which more or less comparable sets of input-output figures are available. Attention is centered here on the effects which changes in the technical structure of the individual industries have had on the one hand and the labor requirements and, consequently, labor productivity computed for the economy as a whole on the other. A more detailed analysis of the process of technical change in two particular industries is presented in Chapters 7 and 11. In both instances the emphasis is put on the stock-flow relationships, i.e. on the capital structure.

Detailed and systematic examination of the technological structure of individual industries such as will be found in Chapter 10 leads beyond the limits of conventional economic analysis. While the methods of indirect statistical inference (discussed in the first part of these introductory observations) were designed to approach the basic structural relationships of the economic system from above, through observation of the dependent variables such as prices and total outputs, the empirical studies which led to construction of the flow charts and the compilation of stock-flow ratios presented in this volume approach the structural constant directly through observation of the actual stocks and flows. The third alternative which logically offers itself is that of deriving the relevant structural relationships 'from below.' In the study of the industrial sectors of the economy this means a detailed analysis of the technical background of productive processes, a reconstruction of technical coefficients from engineering data.

In principle at least, it has long been recognized that the ultimate determinants of the structural relationships which govern the operation of the economic system are to be sought outside the narrowly conceived domain of economic science. Notwithstanding their often expressed desire to co-operate with the adjoining disciplines, economists have more often than not developed their own brand of psychology, their special versions of sociology and their particular 'laws' of technology. If, with respect to the two other fields, such self-sufficiency might possibly be justified by lack of non-controversial, directly usable material, this certainly is not the case in relation to technology. As soon as the economist abandons grossly aggregative formulations he will find in engineering data a promising and accessible source of direct empirical information on the input-output structure of the individual industries. Part IV deals with explorations in this field.

The derivation of 'economic production functions' from engineering information is discussed from a general methodological point of view in Chapter 8. In Chapters 10 and 11 the production functions of the cotton textile and the air-transport industry are actually obtained in this way. Although laborious and costly, such an approach offers a useful, and in the long run indispensable, method of estimating relevant structural constants. First-hand information of this kind will be particularly helpful in the study of technical change. It also can contribute much to the solution of the difficult problem of industrial classification.

An adequately designed classification of industries constitutes the cornerstone of effective input-output analysis. It connects theory with facts by defining the variables in terms of which the observed economic system is to be described. In prevalent statistical practice industries are distinguished in terms of the finished commodities which they produce. Since most of the data used up to now in empirical input-output analysis were obtained from conventional statistical sources, the same 'classification by product' dominates the definition of our basic variables. A closer examination of actual manufacturing procedures raises the question whether the structural characteristics of various industries might better be accounted for if certain common basic processes—such as 'power generation,' 'machining,' etc.—with their own distinctive input-output patterns were split off from the conventionally defined industries and treated as separate productive activities. Such separation of technologically homogeneous processes will prove to be particularly useful in the study of structural change. It is well known that many of the most important technical innovations affect directly and nearly simultaneously a great number of apparently different industries because, behind the variety of products,

there often is concealed a similarity, or rather identity, of the basic productive processes. The problems of process analysis constitute the principal focus of Chapter 9 dealing with questions of industrial classification and aggregation. It points out the attractive features but also brings out the less obvious limitation of the process concept as applied to input-output analysis.

No other field of economic inquiry can suffer so much from theoretical over-simplification as the study of household behavior. The structure of consumers' tastes is less articulated than that of the more or less rationally organized productive processes. It cannot be easily approached 'from below' via economic psychology or quantitative sociology for the simple reason that neither of these disciplines does yet actually exist. Thus the analysis of the input-output structure of the households has to fall back on the methods of indirect statistical inference. The indirectness of such an approach can, however, be considerably reduced through the formulation of an explicit theoretical hypothesis based on careful consideration of relevant qualitative information, i.e. far-reaching stratification of the available qualitative data.

As soon as the total consumers' expenditures are described in terms of separate demands for various specific types of commodities and services, the substitution between them raises questions of foremost analytical importance. The empirical section of Chapter 12 is concerned with the problems of consumers' behavior and is devoted to a statistical analysis of the derivation of the relative price elasticities of three principal categories of consumers' goods. This chapter represents essentially the first exploratory sally into the field which will be given major attention in the research program of the Harvard Economic Research Project in the coming years.

The road toward disaggregation necessarily leads into the problem of regional breakdown of national totals and national averages. The analysis of the regional structure of the American economy as described in Part II represents the first admittedly crude attempt to approach the problem of spatial distribution of economic activities within the framework of empirical input-output analysis. The theoretical formulation underlying this analysis is expressly designed to describe the location of all the various branches of economic activity in its relation to their internal structure, on the one hand, and the balance requirements of the system as a whole, on the other. The regional balances of trade are accordingly explained in terms of the interregional distribution of industries and households. Applied to the actual regional distribution of all industries in the year 1939, this approach yields a detailed and consistent estimate of the regional balances of trade for the 48 separate states—an estimate which might be of some interest to the student of industrial location quite independent of its use in this particular investigation.

The immediate purpose of Chapter 4 is the determination of the differential regional impact of any given change in the 'bill of final demand.' In its formal aspect it represents a simple extension of the previously described derivation of the total industrial and employment impact of given variations in final demand (see Chapter 4). The second section (Chapter 5) presents the empirical results and contains a number of critical observations on these results as well as an outline of further research to be undertaken along the same general lines of analysis.

Chapter 2

STRUCTURAL CHANGE

Wassily Leontief

I. Economic Change

Within the framework of an explicitly formulated theoretical system, economic change can be explained either as structural change or as a dynamic process. In the first case, the variation of the dependent variables is simply related to the underlying changes in some of the basic data; in the second, the law of change itself is considered as given, i.e. built into the structure of the explanatory scheme. The law of change might, of course, be changing over time; this is the case of structural variation in a dynamic system.

In considering these distinctions, it is important to remember that they refer to differences in theories, that is, to different methods of describing and explaining the observed facts rather than to some intrinsic properties of the observed reality itself. Alternative theories, instead of being mutually exclusive, might furthermore be hierarchically related to each other. A dynamic theory could, for example, treat the data of the less general, static theory as its variables and thus, taking up where the latter leaves off, reduce what in the first instance appeared to be a structural change to a dynamic law. Such generalization of a theoretical approach would necessarily have to be accompanied by a corresponding widening, or rather deepening, of its empirical basis.

In this chapter the relations between the different sectors of the American economy in the years 1919, 1929, and 1939 are compared with each other and their differences interpreted in terms of static input-output analysis. In the next chapter a more general, dynamic version of the input-output theory is developed. As stated above, such theoretical generalization necessarily requires for its empirical implementation a large amount of additional factual information. Such information is made available through the study of the capital structure of the American economy in the year 1939 as presented in Part III of this volume. The third and last stage of the inquiry—the determination of the specific dynamic properties of the system through insertion of actual figures into the general formulae—has not yet been completed at the time of the present publication.

To define the concept of structural change within the framework of static input-output analysis, it is necessary to recapitulate its simple theoretical outlines in concise quantitative terms.

Let X_i represent the annual rate of total output (measured in appropriate physical units) of industry i; x_{ik}, the amount of the product of industry i absorbed annually by industry k; and y_i, the amount of the same product i made available for 'outside use,' i.e. for consumption apart from that of any one of the m industries explicitly included in the system under consideration. The over-all input-output balance of the whole national economy comprising m separate industries can be described in terms of m linear equations:

$$X_i - \sum_{k=1}^{m} x_{ik} = y_i \qquad i = 1, 2 \cdots, m \quad (2, 1)$$

The input-output structure of any particular industry is described by a set of 'technical coefficients,' a_{ik}, each of which states the amount of each particular input absorbed by that industry per unit of its own output. Thus the commodity flows included in the balance equations are subject to the following set of structural relationships:

$$x_{ik} = a_{ik}X_k \qquad \begin{matrix} i = 1, 2, \cdots, m \\ k = 1, 2, \cdots, m \end{matrix} \quad (2, 2)$$

Substituting (2, 2) in (2, 1) we have

$$X_i - \sum_{k=1}^{m} a_{ik}X_k = y_i \qquad i = 1, 2, \cdots, m \quad (2, 3)$$

This is a system of m linear equations in m unknowns. It can be solved for all the X_i's in terms of the given final demands, $y_1, y_2, \cdots, y_m$

$$X_i = \sum_{k=1}^{m} A_{ik}y_k \qquad i = 1, 2, \cdots, m \quad (2, 4)$$

That means that by inserting any given 'outside bill of goods,' $y^o{}_1, y^o{}_2, \cdots, y^o{}_m$, in the right-hand side of each equation one can determine the corresponding rate of output, X_i, of commodity i.

Each of the constants, A_{ik}, is in general a function of all the a's, i.e. it depends on the input coefficients of all the m industries.

The rectangular matrix,

$$a \equiv \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ & \cdot & & \cdot \\ & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix} \tag{2, 5}$$

each column of which comprises the input coefficients of one particular industry, can with good reason be referred to as the structural flow matrix or simply the flow structure of the corresponding economic systems.[1]

Economic systems with identical sets of input coefficients can be said to be structurally identical, and systems with unlike technical matrices structurally different. Structural change, in other words, is a change in the structural matrix of the system.

According to this terminology an increase or decrease in the output, X_i, of any industry can be caused by a change in the given bill of goods, a change in the structure of the system, or by a combination of both.

The usefulness of the static input-output approach in the study of an actual economy is conditioned by the relative invariance of its structural characteristics. The introduction of 'open systems'—with the bill of final demand considered as being dependent on some 'outside forces'—serves the purpose of separating the more stable from the less stable aspects of interindustrial relationships; the input coefficients included

[1] In conventional matrix notation, if A is defined as

$$A \equiv \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mm} \end{bmatrix} \tag{2, 6}$$

then $A = [I - a]^{-1}$—where I is the identity matrix

$$\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

—(2, 7) which implies that $A_{ik} = \dfrac{\Delta_{ki}}{|I - a|}$ where $|I - a|$ represents the determinant of matrix $[I - a]$ and Δ_{ki} the algebraic complement of element containing a_{ki} in that determinant

in the structural matrix describing the former, the unaccounted-for determinants of the given bill of goods represented by the latter.[2]

For analysis of long-run developments such as are associated with the notion of technical progress, exhaustion of natural resources, or variation in consumers' tastes, the study of structural change becomes of foremost importance. A study of this kind must answer two questions: 'How has the structure of the particular economic system actually changed?' and 'How did this change affect the outputs of the separate industries, the prices of different commodities; in short, how did it affect the magnitudes of the dependent variables of the given system?' An inquiry into the causes of the structural change itself is pointedly omitted from this study. It is omitted because such an inquiry could not possibly be pursued within the conceptual framework of the theoretical system described above: the 'structural data,' the change of which is being explained, cease to be constants and become 'unknowns' determined within the framework of some other, necessarily more general, system of theoretical reference.

The first of the two questions mentioned above is strictly factual; the second, although factual in its ultimate intent, raises some general methodological problems. Let these be taken up first.

II. Economic Structure over Time

Comparing the structure of an economic system in two stages of its historical development sufficiently removed from each other, one might easily find them to be as unlike as a butterfly and a caterpillar. Not only the relations between the separate sectors of the economy, between the various commodities and services, will have changed, but more than that, the commodities and services found in the two stages might turn out to be entirely dissimilar. A quantitative comparison, a measurement of the difference between the two stages, would in such a case be out of the question. This does not mean, however, that the relation between such entirely dissimilar stages in the historical development of a particular economic system cannot be analyzed and explained. Such analysis and explanation is possible because of the fundamental continuity of the process of change itself. However different the goods and services observed at the opposite ends of a long chain of economic transformation, its successive links are necessarily intermeshed. 'New' commodities would not have been introduced at all if they could not be, and actually had not

[2] An indirect empirical test of this particular type of stability assumption, as compared with invariances assumed in conventional aggregative extrapolations based on changes in gross national product, has been described in *The Structure of American Economy, 1919-1939*, 2nd ed., New York, 1951, p. 216. It definitely has turned out in favor of the former.

been, produced from the 'old' commodities and services, on the one hand, and if they were not put to some 'old' uses, replacing as inputs some of the previously produced goods, on the other. How many potential new products are confined to the disembodied existence of the blueprint stage because no practical way has yet been found to produce them by known methods from actually available inputs or because they cannot be directly or indirectly used in turning out some already produced and consumed commodities as services?

The formal structure of the chain relationship between the successive stages of a changing economic system can be easily described in terms of the already familiar input-output model.[3] Without entering into a detailed discussion of this procedure which actually has already been used for the solution of so-called linear programming problems, it is sufficient to state here that in the description of historical input-output relationships all commodities and services have to be distinguished not only by their physical identities but also according to the time of their production or consumption. Thus X_1 might, for example, represent 'the total steel production of 1900'; X_2, 'the total steel production of 1901'; X_5, 'the total number of locomotives produced in 1901', and a_{15} would then be a technical coefficient showing the amount of 'steel produced in 1900' absorbed per unit of 'locomotives produced in 1901.'

The final bill of goods would accordingly comprise the amounts of separate commodities consumed outside of the observed system in different years and, in particular, those outputs of the end year of the given chain which were carried over into the future.

With such a bill of goods and the complete set of historical structural coefficients, it becomes possible to 'derive' the outputs and consumption of all commodities over the whole period of time under consideration.

Each one of the commodities produced and consumed, as well as all the different structural relationships realized in the course of that development, would find its appropriate place in such an input-output system. The problem of change, i.e. the problem of the interrelation between the different variables within the historical sequence of time, is treated within such a generalized input-output scheme exactly in the same way as the relationship between the individual commodities and services is dealt with in the conventional input-output scheme. The difference between the structure of the economic system at its various stages of development becomes theoretically analogous to the difference between the structural characteristics of the various interconnected sectors of a given national economy.

[3] See, for example, *The Programming of Interdependent Activities* by George B Danzig in *Activity Analysis of Production and Allocation*, edited by Tjalling C. Koopmans, New York, 1951.

The operation of 'splicing time series' turns out in this formulation to be logically identical with that of 'aggregating commodities,' that is, combining distinct industries in conventional input-output analysis. All theoretical propositions and practical rules derived for the latter must obviously also apply to historical input-output relationships and consequently be valid as well for the former.

The same theoretical and practical arguments which can be raised in favor of the more diversified conceptual scheme of the input-output approach as against the broadly aggregative formulations of conventional national income analysis must be advanced in criticism of various attempts to depict the quantitative aspects of economic development in terms of artificially constructed long-run time series. A differentiated step-by-step description which would reflect the essential continuity of the economic process without assuming a non-existing qualitative uniformity will eventually offer a methodologically more satisfactory alternative.

III. The American Economy, 1919-39

The study of long historical sequences obviously requires a wealth of detailed factual information which we do not have at our disposal at the present time. What we do have is a relatively short three-link chain—the description of the input-output structure of the American economy in the years 1919, 1929, and 1939. With a few notable exceptions the identification of the same commodity groups in all the three positions does not present, in this case, an insurmountable difficulty.

In order to be made directly comparable the three original input-output tables on which the following empirical analysis [4] is based first had to be adjusted for discrepancies in industrial classification and then reduced to common physical units by appropriate deflation.

The first step consisted mainly in the aggregation of overlapping subgroups. It necessarily resulted in the sacrifice of much detail, particularly in the 1939 figures which were originally collected on the basis of an industrial classification much more refined than that used for the compilation of the 1919 and 1929 data. Thus, after the subsequent omission of a few sectors such as foreign countries and industries, not elsewhere classified, whose structural characteristics were not distinct enough to warrant this type of analysis, the comparative study was conducted in terms of 13 productive sectors of the economy, labor, and of course the final demand representing the open end of the system. The significant exclusion of construction industries from the subsequent analysis is due to the fact that a predominant proportion of the inputs originating in

[4] The statistical work was originally carried out by Bernadette Drolette; later, by Anne Grosse. The computations were finally checked by Nancy Bromberger.

that industry is made on capital account rather than current cost account; thus it depends upon the rate of expansion of the consuming industries rather than on their intrinsic distinct structural characteristics. To explain such investment inputs one has to resort to an explicitly formulated dynamic theory of a kind described, for example, in the next chapter. Within the framework of a static input-output analysis, all investment demand has to be included in the final bill of goods determined from the outside. There can be little doubt that similar investment items have not been properly eliminated from some of the other categories of inputs. Being relatively small as compared with the corresponding flows of the current cost type, they have distorted the resulting empirical picture only to a relatively limited degree.

In preparation for the second operation separate 1919 and 1929 price indices were compiled for each group of goods and services comprised in the output of an individual sector of the economy, 1939 prices being used as a base. Each row of the 1919 and 1929 input-output table was then divided by the appropriate index. The resulting tables, 1, 2, and 3, show the input-output structure of the American economy for the years 1919, 1929, and 1939 in comparable physical units. For each of the 13 different outputs these are defined as 'the amount of that particular kind of goods or services purchasable for one dollar at their 1939 price.'

No such roundabout procedure was used to obtain the labor inputs shown in the bottom row of each table These are actual employment figures given in man-hour units per year. In shifting from one industry to another, the workers typically change their work hours per year accordingly, so that a given total number of persons employed will supply a different amount of labor measured in man-hours as its industrial distribution changes.

The figures shown in italics are the technical input coefficients, the *a*'s of the structural matrix (2, 5). To obtain them, all inputs in the first column were divided by the same total output figure entered on the right-hand side of the first row; the inputs in the second column were divided by the corresponding total output from the second row, and so on.

The technical coefficients thus obtained are averages not only because each one of them refers to whole groups of industries with more or less different cost structures, but also in the sense that these ratios reflect whole series of techniques simultaneously employed in each individual line of production—from the oldest one still in use to the newest just barely introduced in the most modern producing units.

The first type of heterogeneity can be reduced through the use of a more detailed industrial classification. The second will eventually have to be resolved in terms of an explicitly formulated analysis of diffusion of technological change such as is described, for example, in Chapter 3 below.

TABLE 1

Input-Output Relations in the Economic System of the United States, 1919

	Ind. No.	Industry	1	2	3	4	5	6	7	8	9	10	11	12	16	All Other	Total Output (less imports)
			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
(1)	1	Agriculture and food	–	0	0	0	0	0	0	0	0	226.9	45.0	1242.7	0	8646.0	8962.9
				0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*.15617*	*.01291*	*.25649*	*0*		
(2)	2	Ferrous metals	15.3	–	69.4	1005.3	35.9	0	0	0	0	0	0	0	17.5	881.8	2010.7
			.00171		*.06405*	*.11841*	*.06100*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*.00326*		
(3)	3	Automobiles	0	0	–	0	0	0	0	0	0	0	0	0	0	1084.6	1083.9
			0	*0*		*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*		
(4)	4	Metal fabricating	533.0	19.0	246.4	–	15.2	6.1	38.7	40.2	191.8	8.3	91.7	114.5	473.8	6681.6	8489.9
			.05917	*.00914*	*.22732*		*.02583*	*.00533*	*.05412*	*.01494*	*.08043*	*.00571*	*.02631*	*.02363*	*.08827*		
(5)	5	Nonferrous metals	9.3	34.5	24.2	164.9	–	0	0	0	1.0	21.1	0	2.1	0	446.9	588.5
			.00104	*.01716*	*.02234*	*.01942*		*0*	*0*	*0*	*.00042*	*.01452*	*0*	*.00043*	*0*		
(6)	6	Nonmetallic minerals	39.0	21.1	11.4	17.9	2.4	–	2.4	0	10.6	44.6	7.3	.8	12.2	1122.5	1143.7
			.00435	*.01049*	*.01052*	*.00211*	*.00408*		*.00336*	*0*	*.00444*	*.03070*	*.00209*	*.00017*	*.00227*		
(7)	7	Petroleum and natural gas	9.9	13.2	3.7	13.2	7.5	9.9	–	21.9	8.7	5.0	4.1	2.1	31.0	601.7	715.1
			.00110	*.00676*	*.00341*	*.00155*	*.01274*	*.00866*		*.00814*	*.00365*	*.00341*	*.00118*	*.00043*	*.00578*		
(8)	8	Coal, coke and manufactured gas	73.0	379.6	9.3	116.3	45.3	94.7	32.9	–	128.6	61.7	57.6	52.5	453.7	1193.5	2690.5
			.00814	*.18879*	*.00858*	*.01370*	*.07698*	*.08280*	*.04601*		*.05392*	*.04247*	*.01653*	*.01084*	*.08452*		
(9)	9	Electric utilities	44.6	17.6	12.9	51.6	22.3	16.4	2.3	17.6	–	9.4	39.9	65.7	0	2084.5	2384.8
			.00498	*.00875*	*.01191*	*.00608*	*.03789*	*.01434*	*.00322*	*.00654*		*.00647*	*.01145*	*.01356*	*0*		
(10)	10	Chemicals	302.0	1.5	5.0	0	0	0	6.4	0	0	–	15.3	59.9	0	1185.1	1452.9
			.03369	*.00075*	*.00461*	*0*	*0*	*0*	*.00895*	*0*	*0*		*.00439*	*.01236*	*0*		
(11)	11	Lumber, paper, printing and publishing	130.2	0	31.3	78.3	0	0	0	0	0	34.2	–	17.8	21.4	3313.9	3485.5
			.01453	*0*	*.02889*	*.00922*	*0*	*0*	*0*	*0*	*0*	*.02354*		*.00367*	*.0[illegible]99*		
(12)	12	Textiles and leather	71.0	0	15.2	0	0	0	0	0	0	4.5	46.2	–	0	4903.1	4845.1
			.00792	*0*	*.01403*	*0*	*0*	*0*	*0*	*0*	*0*	*.00310*	*.01325*		*0*		
(13)	16	Transportation	897.8	192.0	83.0	157.6	34.4	178.6	125.1	728.7	0	8.6	321.9	0	–	2640.0	5367.8
			.10017	*.09549*	*.00766*	*.01856*	*.05845*	*.15615*	*.17494*	*.27084*	*0*	*.00592*	*.09235*	*0*			
(14)		Households (labor inputs)	2760.6	132.7	94.9	707.9	70.9	112.3	19.8	139.8	228.0	85.0	423.9	549.0	650.0		
			.30800	*.06600*	*.08759*	*.08338*	*.12056*	*.09823*	*.02765*	*.05197*	*.09561*	*.05851*	*.12162*	*.11331*	*.12110*		

Upper figure in each box represents commodity flow in 1919, including transportation costs but not trade margins. All inputs except employment are measured in millions of dollars, deflated to 1939 prices. Employment is measured in millions of man-hours.

Lower figures (in italics) are coefficients.

TABLE 2

Input-Output Relations in the Economic System of the United States, 1929

	Ind. No.	Industry	1 (1)	2 (2)	3 (3)	4 (4)	5 (5)	6 (6)	7 (7)	8 (8)	9 (9)	10 (10)	11 (11)	12 (12)	16 (13)	All Other (14)	Total Output (less imports) (15)
(1)	1	Agriculture and food	–	0	0	0	0	0	0	0	0	261.4	118.5	1095.5	0	11326.6	11281.1
				0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*.08708*	*.01949*	*.16190*	*0*		
(2)	2	Ferrous metals	15.7	–	391.2	1610.9	68.0	0	0	0	0	0	10.5	0	45.0	3422.9	3285.7
			.00139		*.14570*	*.14841*	*.06721*	*0*	*0*	*0*	*0*	*0*	*.00173*	*0*	*.00757*		
(3)	3	Automobiles	0	0	–	0	0	0	0	0	0	0	0	0	0	8134.4	2684.9
			0	*0*		*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*		
(4)	4	Metal fabricating	588.2	28.9	518.2	–	43.9	48.6	113.0	68.2	356.7	12.1	151.3	155.9	649.9	8147.5	10854.3
			.05214	*.00880*	*.17874*		*.04339*	*.02311*	*.04910*	*.02139*	*.09354*	*.00403*	*.02488*	*.02304*	*.10926*		
(5)	5	Nonferrous metals	11.0	76.6	78.7	405.1	–	0	0	0	21.4	44.2	4.8	.7	0	578.3	1011.7
			.00098	*.02331*	*.02931*	*.03732*		*0*	*0*	*0*	*.00561*	*.01472*	*.00079*	*.00010*	*0*		
(6)	6	Nonmetallic minerals	69.9	29.4	61.8	31.4	0	–	0	0	16.2	87.1	16.2	1.0	28.4	1933.2	2103.4
			.00620	*.00895*	*.02302*	*.00289*	*0*		*0*	*0*	*.00425*	*.02901*	*.00266*	*.00015*	*.00477*		
(7)	7	Petroleum and natural	36.7	38.1	4.7	25.4	13.4	28.2	–	29.6	26.8	11.3	7.1	6.4	67.0	2108.0	2301.3
		gas	*.00325*	*.01160*	*.00181*	*.00234*	*.01325*	*.01341*		*.00928*	*.00703*	*.00376*	*.00117*	*.00095*	*.01126*		
(8)	8	Coal, coke and	66.5	301.6	14.3	97.1	28.6	90.0	16.4	–	161.6	86.9	68.5	53.2	510.2	1701.3	3188.0
		manufactured gas	*.00589*	*.09179*	*.00533*	*.00895*	*.02827*	*.04279*	*.00713*		*.04238*	*.02895*	*.01126*	*.00786*	*.08578*		
(9)	9	Electric utilities	44.6	33.9	17.9	80.4	26.8	47.3	6.2	38.4	–	36.6	55.4	65.2	5.4	3355.3	3813.4
			.00395	*.01032*	*.00667*	*.00741*	*.02649*	*.02249*	*.00269*	*.01205*		*.01219*	*.00911*	*.00964*	*.00091*		
(10)	10	Chemicals	513.0	.9	31.6	40.0	0	12.1	16.7	0	0	–	76.2	204.5	0	2365.3	3001.9
			.04547	*.00027*	*.01177*	*.00369*	*0*	*.00575*	*.00726*	*0*	*0*		*.01253*	*.03023*	*0*		
(11)	11	Lumber, paper, print-	87.5	0	52.9	113.9	0	.9	0	0	0	10.9	–	14.6	23.7	6156.7	6081.0
		ing and publishing	*.00776*	*0*	*.01971*	*.01049*	*0*	*.00043*	*0*	*0*	*0*	*.00363*		*.00216*	*.00398*		
(12)	12	Textiles and leather	40.0	0	24.3	4.3	0	0	0	0	0	21.7	62.6	–	0	7137.4	6765.1
			.00355	*0*	*.00906*	*.00040*	*0*	*0*	*0*	*0*	*0*	*.00723*	*.01029*		*0*		
(13)	16	Transportation	893.4	283.4	168.1	88.4	30.3	242.6	338.8	1063.3	0	95.3	338.8	10.4	–	2405.6	5948.0
			.07919	*.08625*	*.06260*	*.00814*	*.02995*	*.11534*	*.14722*	*.33353*	*0*	*.03175*	*.05571*	*.00154*			
(14)		Households (labor	2682.8	137.5	116.3	551.3	69.6	112.6	67.8	122.4	312.4	88.3	448.0	544.5	629.4		
		inputs)	*.23781*	*.04184*	*.04312*	*.05079*	*.06881*	*.05354*	*.02948*	*.03838*	*.08193*	*.02943*	*.07368*	*.08049*	*.10581*		

Upper figure in each box represents commodity flow in 1929 including transportation costs but not trade margins. All inputs except employment are measured in millions of dollars, deflated to 1939 prices. Employment is measured in millions of man-hours

Lower figures (in italics) are coefficients.

TABLE 3

Input-Output Relations in the Economic System of the United States, 1939

	Ind. No.	Industry	1 (1)	2 (2)	3 (3)	4 (4)	5 (5)	6 (6)	7 (7)	8 (8)	9 (9)	10 (10)	11 (11)	12 (12)	16 (13)	All Other (14)	Total Output (less imports) (15)
(1)	1	Agriculture and food	–	0	0	0	0	0	0	0	0	216	190	717	0	16418	16517
				0	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*.07855*	*.03418*	*.12271*	*0*		
(2)	2	Ferrous metals	19	–	363	949	42	0	0	0	0	0	62	0	71	1098	2552
			.00115		*.14075*	*.10822*	*.03286*	*0*	*0*	*0*	*0*	*0*	*.01115*	*0*	*.01862*		
(3)	3	Automobiles	112	0	–	0	0	0	0	0	1	0	0	0	4	2464	2479
			.00678	*0*		*0*	*0*	*0*	*0*	*0*	*.00025*	*0*	*0*	*0*	*.00105*		
(4)	4	Metal fabricating	691	42	448	–	20	23	127	30	171	45	146	108	260	6712	8769
			.04184	*.01646*	*.17371*		*.01565*	*.01262*	*.04420*	*.01145*	*.04298*	*.01428*	*.02626*	*.01848*	*.06819*		
(5)	5	Nonferrous metals	29	109	93	271	–	4	1	1	54	88	5	2	0	1000	1278
			.00176	*.04271*	*.03606*	*.03090*		*.00219*	*.00035*	*.00048*	*.01357*	*.02793*	*.00090*	*.00034*	*0*		
(6)	6	Nonmetallic minerals	108	34	60	33	0	–	1	10	0	105	12	0	0	1480	1823
			.00654	*.01332*	*.02326*	*.00376*	*0*		*.00035*	*.00482*	*0*	*.03332*	*.00216*	*0*	*0*		
(7)	7	Petroleum and natural gas	192	50	5	21	8	38	–	27	50	24	12	10	66	2448	2873
			.01162	*.01959*	*.00194*	*.00239*	*.00626*	*.02084*		*.01301*	*.01257*	*.11762*	*.00216*	*.00171*	*.01731*		
(8)	8	Coal, coke, and manufactured gas	47	202	11	44	16	47	9	–	158	92	53	26	289	1094	2083
			.00285	*.07915*	*.00427*	*.00502*	*.01252*	*.02578*	*.00313*		*.03971*	*.02920*	*.00953*	*.00445*	*.07579*		
(9)	9	Electric utilities	129	42	17	83	36	45	11	34	–	36	76	73	14	3384	3979
			.00781	*.01646*	*.00659*	*.00947*	*.02817*	*.02468*	*.00383*	*.01638*		*.01142*	*.01367*	*.01249*	*.00367*		
(10)	10	Chemicals	415	36	31	110	3	17	13	0	0	–	109	326	15	2269	3151
			.02513	*.01411*	*.01202*	*.01254*	*.00235*	*.00933*	*.00452*	*0*	*0*		*.01961*	*.05579*	*.00395*		
(11)	11	Lumber, paper, printing, and publishing	120	0	17	48	0	26	2	0	1	11	–	15	5	5586	5559
			.00727	*0*	*.00639*	*.00547*	*0*	*.01426*	*.00070*	*0*	*.00025*	*.00349*		*.00257*	*.00131*		
(12)	12	Textiles and leather	95	0	63	13	1	2	0	0	0	11	78	–	0	5850	5843
			.00575	*0*	*.02443*	*.00148*	*.00078*	*.00110*	*0*	*0*	*0*	*.00349*	*.01403*		*0*		
(13)	16	Transportation	807	219	88	151	38	229	249	716	0	202	304	9	–	801	3813
			.04886	*.08582*	*.03412*	*.01722*	*.02973*	*.12562*	*.08667*	*.34489*	*0*	*.06411*	*.05469*	*.00154*			
(14)		Households (labor inputs)	3169	96	85	381	55	81	48	74	202	79	338	452	446		
			.19155	*.03735*	*.03431*	*.04329*	*.04121*	*.04422*	*.01652*	*.03530*	*.05066*	*.02173*	*.06069*	*.07688*	*.11686*		

Upper figure in each box represents commodity flow in 1939, including transportation but not trade margins. All inputs except employment are measured in millions of dollars. Labor inputs are measured in millions of man-hours.

Lower figures (in italics) are coefficients.

A very serious limitation on the scope of the quantitative study presented below is imposed by the fact that it reflects only the variations in the flow structure of the economy while neglecting the changes in its capital structure. A stock-flow matrix of the American economy, describing the capital requirements of the individual industries, is available for 1939 but not for any other year.

The large amounts of statistically unallocated outputs to be found in the 1919, 1929, and even the 1939 input-output tables reflect the defectiveness of the factual information on which the following quantitative analysis is based. This inherent weakness of the basic information must obviously affect also the numerical results presented below.

IV. Changes in Technical Coefficients

If the 1919, 1929, and 1939 sets of technical coefficients were identical, one would have said that the structure of all the sectors of the American economy included in this matrix remained invariant over these two decades. Actually the coefficients did change and this change itself, and in particular its effect on the total outputs of the individual industries and employment (labor input) in each of them, is the subject of the following analysis.

The simplest index of the change of any one particular coefficient between two points of time is the difference between its original and its final magnitude. For comparison of changes in two or more different coefficients, this index would be of little use since the absolute magnitude of any one of them depends upon the physical units in which the output and the input quantities are described. The labor input per ton of steel becomes larger if one describes it in labor hours rather than in labor days. This difficulty is eliminated if the differences are expressed in relative (percentage) terms, i.e. if they are divided by the original or the final value of the coefficients Since it often happens that either one or the other of these two values is zero or near zero, many of the relative changes would turn out to be infinite (a finite difference divided by zero). To avoid this inconvenience the differences can, for purposes of quantitative comparison, be related to the mean of the original and the final value of the coefficients.

If a_{ik} and a'_{ik} are the two magnitudes of a particular input coefficient which are to be compared, their difference is $a_{ik} - a'_{ik}$; their mean is $\frac{a_{ik} + a'_{ik}}{2}$, and the index of the relative change, $\bar{a}_{ik}$, is $\frac{2(a_{ik} - a'_{ik})}{(a_{ik} + a'_{ik})}$. The description of the structural change of the economic system, if presented in terms of an unweighted distribution of such indices—each related to

one particular input coefficient—would have neglected the fact that some of the input ratios belong to large, while the other to comparatively small, industries. This consideration gains in importance if it is remembered that even a most detailed industrial classification involves a certain degree of aggregation and with it a more or less arbitrary determination of the size of the individual industries. For example, by splitting any one industry in two nearly identical parts technologically, one could double the frequency with which the corresponding change indices would be represented in the combined size distribution of all structural changes occurring in the given economic system. To eliminate this source of possible distortion the individual indices should be weighted. The total value (price times quantity) of the corresponding input items can appropriately serve as a measure of the relative importance of the respective individual changes, if considered from the point of view of the system as a whole.

In the distributions shown on Charts 1, 2, and 3 each individual change index, $\bar{a}_{ik}$, is entered with the weight $\frac{x_{ik} + x'_{ik}}{2}$. This is an average of the first- and second-year value (expressed in 1939 prices) of the input item corresponding to the particular index. Chart 1 shows the weighted distributions of all technical changes for the years 1919-29 and 1929-39.

A negative change, i.e. a reduction in the input requirements per unit of output, can be loosely described as an increase in productivity. Since both distributions are nearly normal,[a] their respective means can serve as convenient statistical measures of the magnitude of the over-all change. From 1919 to 1929 the input coefficients of all the cost elements in all industries were on the average reduced by 14 per cent. In the following 1929-39 decade the average reduction amounted to only 11 per cent. Described in these terms as well as in others, as we shall see later, the rate of technical progress was slower in the period of the great depression than during the preceding years of the great boom.

The examination of the individual changes is being left to the reader. Their explanation in terms of particular detailed information obtained from the technological annals of the individual industries lies, as has already been stated, beyond the scope of the present study.

A complete description, not to say explanation, of structural change would, as has been mentioned before, require information not only on

[a] The statistical characteristics of the distribution shown in Charts 1, 2, and 3 are as follows:

	1919-29 DISTRIBUTION	1929-39 DISTRIBUTION
Mean	−.14	−.11
Standard deviation	.35	.35
Quartile coefficient of skewness	.17	.03

CHART 1
WEIGHTED DISTRIBUTION OF RELATIVE CHANGES IN TECHNICAL INPUT COEFFICIENTS

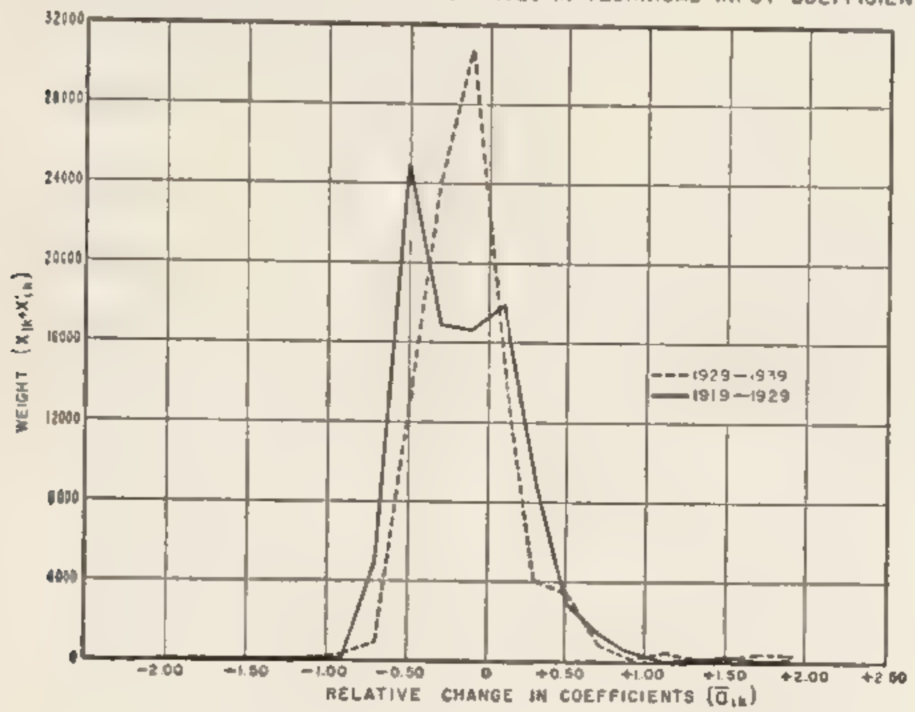


CHART 2
WEIGHTED DISTRIBUTION OF RELATIVE CHANGES IN TECHNICAL INPUT COEFFICIENTS
(EMPLOYMENT COEFFICIENTS OMITTED)

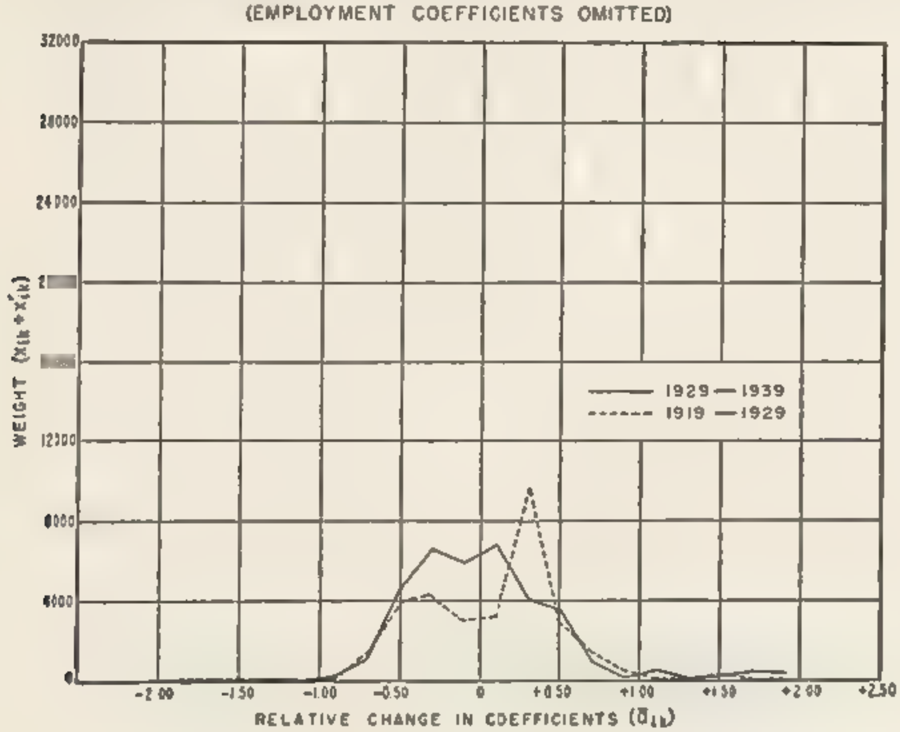


Table 4

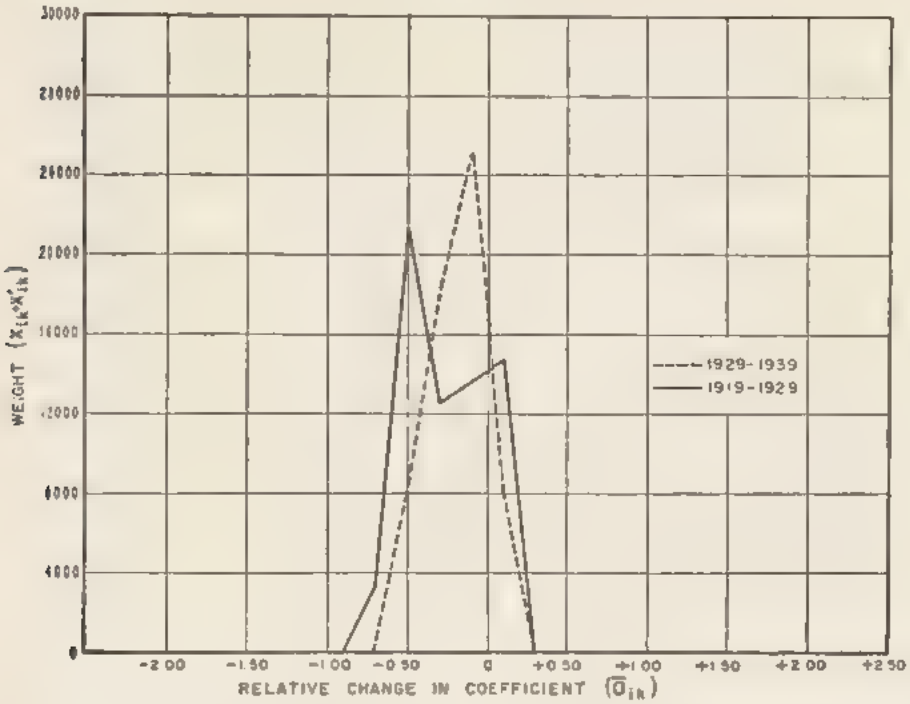
Relative Changes in Technical Input Coefficients
1929-39 and 1919-29

Relative Changes in Technical Coefficients $\bar{a}_{ik}$	Weights $(x_{ik} + x'_{ik})$ of Changes in: All Coefficients[1] 1929-39	All Coefficients[1] 1919-29	All Coefficients except Labor 1929-39	All Coefficients except Labor 1919-29	Labor Coefficients 1929-39	Labor Coefficients 1919-29
	(1)	(2)	(3)	(4)	(5)	(6)
−2.10 to −2.00	45.6	4.8	45.6	4.8	---	---
−2.00 to −1.90	---	---	---	---	---	---
−1.90 to −1.80	---	---	---	---	---	---
−1.80 to −1.70	---	---	---	---	---	---
−1.70 to −1.60	---	---	---	---	---	---
−1.60 to −1.50	---	---	---	---	---	---
−1.50 to −1.40	---	94.4	---	94.4	---	---
−1.40 to −1.30	69.9	---	69.9	---	---	---
−1.30 to −1.20	---	2.8	---	2.8	---	---
−1.20 to −1.10	---	---	---	---	---	---
−1.10 to −1.00	28.7	---	28.7	---	---	---
−1.00 to − .90	320.0	76.3	320.0	76.3	---	---
− .90 to − .80	---	---	---	---	---	---
− .80 to − .70	619.1	357.0	619.1	357.0	---	---
− .70 to − .60	418.1	4588.9	418.1	1156.9	---	3432.0
− .60 to − .50	3039.7	3196.5	1807.7	551.5	1232.0	2645.0
− .50 to − .40	9295.0	21650.2	2777.0	3275.2	6518.0	18375.0
− .40 to − .30	15348.4	9227.7	2683.4	1225.7	12665.0	8002.0
− .30 to − .20	8710.0	7662.1	3861.0	3155.1	4849.0	4507.0
− .20 to − .10	20476.1	15733.3	2375.1	2121.3	18101.0	13612.0
− .10 to 0	10444.1	948.0	3552.1	948.0	6892.0	---
0 to + .10	11487.9	3201.2	3290.9	2265.2	8197.0	936.0
+ .10 to + .20	3630.1	14771.7	3630.1	975.7	---	13796.0
+ .20 to + .30	3616.0	7547.4	3616.0	7547.4	---	---
+ .30 to + .40	521.9	2089.9	521.9	2089.9	---	---
+ .40 to + .50	755.8	2829.6	755.8	2829.6	---	---
+ .50 to + .60	2748.1	166.4	2748.1	166.4	---	---
+ .60 to + .70	709.3	811.3	709.3	811.3	---	---
+ .70 to + .80	326.7	671.6	326.7	671.6	---	---
+ .80 to + .90	191.4	301.0	191.4	301.0	---	---
+ .90 to +1.00	---	138.1	---	138.1	---	---
+1.00 to +1.10	152.7	---	152.7	---	---	---
+1.10 to +1.20	303.1	---	303.1	---	---	---
+1.20 to +1.30	105.0	54.7	105.0	54.7	---	---
+1.30 to +1.40	---	103.9	---	103.9	---	---
+1.40 to +1.50	259.8	68.1	259.8	68.1	---	---
+1.50 to +1.60	---	251.1	---	251.1	---	---
+1.60 to +1.70	---	---	---	---	---	---
+1.70 to +1.80	411.5	22.4	411.5	22.4	---	---
+1.80 to +1.90	26.9	---	26.9	---	---	---
+1.90 to +2.00	361.9	90.1	361.9	90.1	---	---

[1] The coefficients included in these distributions cover inputs into industries 1-12, 15, and 16 from industries 1-12, 16 and households (labor). Coefficients of inputs into these industries from industry 15 (construction) are omitted, since, in general, they describe purchases for investment purposes rather than for current account.

CHART 3

WEIGHTED DISTRIBUTION OF RELATIVE CHANGES IN LABOR INPUT COEFFICIENTS



the flows of inputs but also the *stocks* of machinery, equipment, and other durable or at least storable factors involved in various productive processes in the different stages of their technical development.

In Charts 2 and 3 the changes in the labor-input coefficients are separated from the variation in all the other input ratios.[6] The prevalence of 'direct labor-saving' changes in 1919-29 as against the relative predominance of saving in non-labor inputs during the 1929-39 decade is clear.

Segregated by individual industries the same changes are shown in Chart 4. Each bar depicts the variation in the output structure of one particular industry as represented by a weighted average of the change-indices of all its separate input coefficients. Since total value figures are again used as weights, these averages might be interpreted as indices of 'real cost' reductions in the particular industries.

V. Technological Change

Although on the average the input ratios in most industries decrease with time, we find that in nearly each individual industry some of them

[6] Means of the distribution of changes in input coefficients are as follows:

	1919 29 DISTRIBUTION	1929-39 DISTRIBUTION
All coefficients	−.14	−.11
All non-labor coefficients	.6	.3
Labor coefficients only	−.36	−.16

TABLE 5

Weighted Column Means of Relative Changes in Technical Input Coefficients, 1929-39 and 1919-29

	Ind. No.	Industry	Mean Relative Change in Input Coefficients $\frac{\Sigma \bar{a}(x'_{ik} + x_{ik})}{(x'_{ik} + x_{ik})}$ 1929-39 (1)	1919-29 (2)
(1)	1	Agriculture and food	-.2077	-.1877
(2)	2	Ferrous metals	+.0124	-.3935
(3)	3	Automobiles	-.2637	-.1927
(4)	4	Metal fabricating	-.1615	-.2921
(5)	5	Nonferrous metals	-.4657	-.4686
(6)	6	Nonmetallic minerals	-.0947	-.4392
(7)	7	Petroleum and natural gas	-.4884	-.0721
(8)	8	Coal, coke, and manufactured gas	-.0163	-.0579
(9)	9	Electric public utilities	-.4404	-.1091
(10)	10	Chemicals	+.0058	-.4632
(11)	11	Lumber, paper, printing, and publishing	-.0780	-.4210
(12)	12	Textiles and leather	-.0585	-.3207
(13)	15	Construction	-.0633	+.1095
(14)	16	Transportation	+.0413	-.0752
		Mean[1]	-.1266[1]	-.1533[1]

The coefficients represent averages of all coefficients for inputs into industries 1-12, 15, and 16, including labor inputs but excluding inputs from industry 15 (construction).

are increasing while the others become smaller. This observation points directly to the question of factor substitution in its relation to technical change. In its general aspects this problem has been thoroughly and definitely explored by the traditional theory of production. More recently it also was reformulated in somewhat different and, for the present purposes, more convenient terms of linear programming with alternative factor combinations. The following general remarks are thus strictly limited in scope. They are intended only to prepare the necessary conceptual background for the empirical analysis at hand.

The substitution of a lower for a higher input ratio does not require any elaborate explanation whenever it can be interpreted as an independent technical change, a reduction in any one or more coefficients, with the rest of the structural matrix remaining the same, will always result in a more efficient utilization of resources. It makes it possible to produce any given bill of goods with smaller total outputs (and since the bill of goods is fixed, also smaller total inputs) of all commodities and services.

CHART 4

It is the observed increase in various input ratios which calls for a special explanation. Occasionally increased input requirements might be caused by changing external circumstances of production such, for example, as exhaustion of natural resources, change in climate, etc. In most instances, however, the increase in the technical coefficient of certain kinds of inputs is associated with a reduction of the input ratios of some other commodities or services absorbed by the same industry. The adoption of a new method of production involves a simultaneous change in all its input ratios and the reduction in some of them could not be realized without corresponding increases in the other. In short, a whole new column of coefficients—which represents within the structural matrix of the whole economy the technical characteristics of the particular industry under consideration—is being substituted for the old one. Other columns might or might not be changed at the same time, but the subsequent

analysis is based on the assumption that the individual industries are structurally independent of each other in the sense that the technical *possibility* of substituting a new set of coefficients in any one column of a given structural matrix is in no way conditioned by the changes which might take place in any other column of the same matrix This assumption should, of course, be interpreted as an empirical requirement which has to be satisfied by the industrial classification in terms of which the structural changes are actually analyzed.

What can be said in general about the combined effects of two independent technological changes in relation to the separate effects of each one of them? Described in terms of the changes in total outputs required to produce some fixed bill of final demand, these effects can be shown (see the mathematical note at the end of the chapter) to satisfy the following relationship:

$$(X_l - X'''_l) = (X_l - X'_l)\frac{X'''_i}{X'_i} + (X_l - X''_l)\frac{X'''_k}{X''_k}$$

The subscripts i and k refer to the outputs of the two industries which experience the structural changes. Subscript l indicates the output of any industry included in the system under consideration: X_l is the original level of its output; X'_l shows the output of industry l as affected by the technical change in industry i (with the structure of industry k remaining unchanged), and X''_l reflects the effect of a similarly isolated structural change in industry k, while X'''_l represents the output of industry l as it would have been if both structural changes occurred simultaneously. Furthermore X'_i is the output of industry i as it would have been in the case in which its own technical structure had changed but the structure of industry k remained the same, while X'_k similarly reflects the effect of a change in industry i on its own output with no change in industry k; similarly X''_i and X''_k reflect the effects of technical changes in industry k on the outputs of industries i and k respectively. Finally X'''_i and X'''_k describe the outputs of these industries resulting from the combination of both structural changes.

If this relation is applied to a system in which both structural changes have in fact occurred simultaneously X_l, X'''_l, X'''_k, and X'''_i must obviously be positive,[7] while X'_l, X''_l, X'_i, and X''_k can conceivably be negative, since they refer to virtual not actually observed situations. This

[7] Actually it is possible to visualize an economic system with such a set of technical coefficients that the solution of the corresponding input-output equations would indicate negative outputs for any positive bill of final demand. This implies that positive outputs could be achieved in this system only if at least some elements of the bill of goods were negative In other words, the productivity of such an economy is so low that it can operate only if subsidized from the outside But in this case too the actually observed outputs would be positive.

means that the separate effects of the two structural changes, i.e. $(X_i - X'_i)$ and $(X_i - X''_i)$, might have been negative while their combined effect, $(X_i - X'''_i)$, still could be positive and vice versa.

If, however—as actually happens in the case of the structural changes of the American economy analyzed below—all the virtual as well as actual outputs are positive, the combined effect, $(X_i - X'''_i)$, can be positive only if at least one of the separate effects, i.e. either $(X_i - X'_i)$ or $(X_i - X''_i)$, is also positive. This, however, still leaves the possibility for the positive combined effect to be smaller or larger than either one of the separate effects.

In a trivial case in which the technical progress in each industry had consisted in reductions in some technical coefficients with increases in none, the separate as well as the combined effects of such changes would result in a reduction of all total outputs associated with any given bill of goods. Since $\frac{X'''_i}{X'_i}$ and $\frac{X'''_k}{X'_k}$ would then be necessarily positive but smaller than 1, the combined effect of any two positive changes would be larger than any one of the two separate effects but smaller than their sum:

$$(X_i - X'''_i) > (X_i - X'_i)$$

and

$$(X_i - X'''_i) > (X_i - X''_i)$$

but

$$(X_i - X'''_i) < (X_i - X'_i) + (X_i - X''_i)$$

VI. The Effects of Structural Change

Tables 6 and 7 present the results of a series of computations designed to show the separate and the combined effects of the structural changes which have actually taken place in the various productive sectors of the American economy in the years 1919-29 and 1929-39.

The same approach was used to derive both sets of figures. Thus the general description of the terms in which the 1919-29 comparison is presented applies also to the 1929-39 results.

In Table 7 the 1939 bill of final demand (as described in Column 14 of Table 3) is used as the basis of all subsequent comparisons. Column 1 shows the actual output of all the 16 productive sectors of the economy as well as the total actual employment (Row 14) in these sectors in the year 1939. Column 8 shows what the outputs and the employment would have been if the same 1939 bill of goods were produced with the 1929 rather than the 1939 technical relationships prevailing in all the industries. These figures are computed through the solution of 14 simultaneous input-output equations of the type described by set (2, 3) on page 18

TABLE 6

Effects on Industrial Outputs and Labor Requirements of Substituting 1919 Input Coefficients in the 1929 Matrix[1]

	Ind. No.	Industries with 1919 Input Structure (all other industries have 1929 input structure)	Outputs Required to Produce 1929 Bill of Final Demand								
			None (actual 1929 output)	Agriculture and Food (1)	Heavy Industry (2, 3, 4 10)	Nonferrous Metals and Nonmetallic Minerals (5, 6)	Fuel and Power (7, 8, 9)	Light Industry (11, 12)	Transportation (16)	All	Actual 1919 Outputs
			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1)	1	Agriculture and food	11281.1	12925.6	13153.3	12926.5	12928.0	13568.1	12927.6	13772.1	8962.9
(2)	2	Ferrous metals	3285.7	3427.4	2851.5	3389.1	3388.4	3406.3	3354.2	2807.8	2010.7
(3)	3	Automobiles	2684.9	2689.6	2689.6	2689.6	2689.6	2689.6	2689.6	2689.6	1083.9
(4)	4	Metal fabricating	10854.3	11138.2	11143.5	10968.2	10935.6	11095.1	10872.6	11100.8	8489.9
(5)	10	Chemicals	3001.9	3209.7	3311.2	3345.5	3362.8	3205.6	3358.2	2984.9	1452.9
(6)	5	Nonferrous metals	1011.7	1241.0	992.0	1234.7	1213.9	1236.0	1230.6	964.5	588.5
(7)	6	Nonmetallic minerals	2193.4	2273.2	2262.3	2304.5	2307.2	2297.2	2282.8	2225.2	1143.7
(8)	7	Petroleum and natural gas	2301.3	2397.6	2395.1	2413.4	2404.3	2424.3	2386.3	2306.5	715.1
(9)	8	Coal, coke, and manufactured gas	3188.0	3329.3	3598.1	3441.7	3401.1	3360.5	3259.3	3981.3	2690.5
(10)	9	Electric utilities	3813.4	3861.9	3816.2	3844.4	3830.2	3894.2	3839.4	3845.6	2384.8
(11)	11	Lumber, paper, printing, and publishing	6081.0	6568.6	6558.4	6478.4	6477.9	6496.8	6477.7	6661.2	3485.5
(12)	12	Textiles and leather	6765.1	7359.3	7299.5	7302.8	7302.9	7323.6	7302.9	7381.6	4845.1
(13)	16	Transportation	5948.0	6486.5	6176.2	6382.5	6091.7	6503.5	6182.4	6788.0	5367.8
(14)		Computed labor requirements	8156.2[2]	7385.9	7109.6	6621.8	6513.2	7179.1	6512.8	9204.7	5975.0
(15)		Labor requirements computed as above but with 1929 labor coefficients throughout	5882.9	6478.6	6462.1	6455.0	6419.0	6627.3	6418.3	6685.2	4454.8[3]
(16)		Excess of computed over actual 1929 labor inputs	+2273.3	+1503.0	+1216.7	+ 738.9	+ 630.3	+1296.2	+ 629.9	+3321.6	+ 92.1
(17)		Excess over actual 1929 labor inputs of requirements computed as above but with 1929 coefficients throughout	0	+ 595.7	+ 579.2	+ 572.1	+ 536.1	+ 744.4	+ 535.4	+ 802.3	−1428.1

[1]Outputs are measured in millions of dollars, deflated to 1939 prices. Labor requirements are in millions of man-hours.

[2]Labor requirements with 1929 input structure but 1919 labor coefficients.

[3]Labor required to produce the 1919 bill of goods with 1919 input structure but 1929 labor coefficients.

TABLE 7

Effects on Industrial Outputs and Labor Requirements of Substituting 1929 Input Coefficients in the 1939 Matrix[1]

Ind. No.	Industries with 1929 Input Structure (all other industries have 1939 input structure)	None (actual 1939 output)	Agriculture and Food (1)	Heavy Industry (2, 3, 4, 10)	Nonferrous Metals and Nonmetallic Minerals (5, 6)	Fuel and Power (7, 8, 9)	Light Industry (11, 12)	Transportation (16)	All	Actual 1929 Outputs
		Outputs Required to Produce 1939 Bill of Final Demand								
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1)	1 Agriculture and food	16542	16535.9	16563.6	16515.1	16517.8	16648.1	16516.5	16710.5	11281.1
(2)	2 Ferrous metals	2582	2570.9	2909.9	2601.6	2580.1	2501.5	2526.5	2948.0	3285.7
(3)	3 Automobiles	2579	2467.6	2579.2	2578.9	2578.1	2579.9	2574.9	2462.0	2684.9
(4)	4 Metal fabricating	8811	8956.5	8730.4	8824.1	9008.9	8787.4	8925.5	9403.7	10854.3
(5)	10 Chemicals	3183	3489.0	3037.3	3142.3	3162.9	2964.5	3137.9	3174.3	3001.9
(6)	5 Nonferrous metals	1326	1276.4	1230.7	1277.5	1253.2	1268.8	1281.1	1202.5	1011.7
(7)	6 Nonmetallic minerals	1823	1827.5	1790.6	1823.7	1830.4	1820.8	1841.1	1819.4	2103.4
(8)	7 Petroleum and natural gas	2902	2748.1	2842.9	2870.0	2847.5	2861.7	2850.6	2649.2	2301.3
(9)	8 Coal, coke, and manufactured gas	2083	2177.4	2168.2	2131.1	2114.0	2094.6	2113.9	2425.6	3188.0
(10)	9 Electric utilities	3979	3923.4	3949.9	3974.6	3970.0	3935.0	3970.5	3824.1	3813.4
(11)	11 Lumber, paper, printing and publishing	5572	5569.3	5636.2	5534.0	5557.7	5557.0	5570.3	5633.4	6081.0
(12)	12 Textiles and leather	5375	5805.5	5896.1	5839.6	5843.3	5822.2	5843.2	5746.2	6765.1
(13)	16 Transportation	3813	4363.4	3763.1	3816.6	3979.8	3813.9	3826.7	4477.6	5948.0
(14)	Computed labor requirements	6662.0[2]	6332.7	5620.8	5545.9	5688.3	5598.2	5456.5	6791.0	5883.0
(15)	Labor requirements computed as above but with 1939 labor coefficients throughout	5504.1	5567.7	5501.6	5493.7	5520.7	5505.1	5498.7	5638.1	4979.2[3]
(16)	Excess of computed over actual 1939 labor inputs	+ 1157.9	+ 828.6	+ 116.7	+ 41.8	+ 184.2	+ 94.1	− 47.6	+ 1286.9	+ 378.9
(17)	Excess over actual 1939 labor inputs of requirements computed as above but with 1939 labor coefficients throughout	0	+ 63.6	− 2.5	− 10.4	+ 16.6	+ 1.0	− 5.4	+ 134.0	− 524.9

[1] Outputs are measured in millions of dollars. Labor requirements are measured in millions of man-hours.

[2] Labor requirements with 1939 input structure but 1929 labor coefficients.

[3] Labor required to produce the 1929 bill of goods with 1929 input structure but 1939 labor coefficients.

—with the 1939 bill of final demand but on the basis of the 1929 matrix of technical coefficients.

Column 2 represents the outputs and employment which would be attained if the same 1939 bill of goods were produced, but industry 1 (agriculture) had operated on the basis of its 1929 input ratios while all the other 15 industries were working on the basis of their 1939 structural coefficients. To obtain these figures again a system of 14 equations was used. But the underlying set of technical coefficients was a hybrid one. Its first column, representing the input structure of agriculture, was taken from the 1929 table of technical coefficients (Table 2), while all the other columns, representing the structures of the remaining 12 industries, were taken from the 1939 table (Table 3). Comparing the figures shown in Column 2 with the corresponding entries in Column 1, one can see what differences in outputs and employment (i.e. the total labor requirements) would have resulted in 1939 if agriculture had relapsed to the technical state in which it actually was in 1929. Comparing, on the other hand, the same Column 1 with Column 8, one can measure the 1929-39 technical progress of all the other 16 industries in terms of its effect on outputs and employment. It is important to note that this latter comparison, as the one above, is made on the basis of the 1939 rather than the 1929 composition of final demand.

Columns 3 to 7 are constructed on the same principle as Column 2. Each of them shows what would have happened to total outputs and labor requirements if one particular industry or a group of industries had relapsed to its 1929 techniques while the other sectors of the economy were still operating on the basis of their 1939 input-output ratios.

In interpreting the meaning of these figures it is important to remember that they represent the total outputs and aggregate employment which would be required to produce—on the basis of different technical possibilities—the same fixed, 1939 final bill of goods. Thus an increase in the output of any industry resulting from the change in structural conditions should not be mistakenly interpreted as a larger supply available for 'outside use.' Such an increase means only that in satisfying the same final demand under the new conditions the economic system would have to lean more heavily on that particular industry. This becomes particularly clear when one turns to the examination of the total labor requirements, i.e. employment, corresponding to the various structural states of the economic system. An increase in the total number of man-hours required to produce a given bill of goods signifies a reduction in the productivity of labor, while a fall in labor requirement indicates that man-power has become more productive. Had labor actually been the only limiting factor of production, a 10 per cent reduction in the aggregate number of man-hours required to turn out a given bill of goods would have meant that the

original labor force should be able to produce a 10 per cent larger bill of goods.

It has often been observed that the variations in the amount of labor absorbed by a particular industry per unit of its total output (i.e. the labor-input coefficient of that industry) cannot measure the effects which the other input changes taking place within the same industry have on the productivity of labor 'in the economy as a whole.' To take account of these other changes, it would be necessary to trace their indirect effect on the outputs and labor requirements of all other industries. It is precisely this set of indirect relationships which has been taken into account in the derivation of figures presented in Tables 6 and 7.

In Table 7 the differences between the actual combined 1939 (or, respectively, 1929 in Table 6) employment in the 13 industrial groups under consideration and the figures entered in Row 14 are shown in Row 16. They show clearly the total labor-saving effects resulting from the various alternative and combined (Column 8) structural changes described above. The magnitude of each individual saving depends on the rate of technical progress as it affected the particular sector of the economy, on the one hand, and the size of that sector as compared with the rest of the economic system, on the other. Both in 1919-29 and 1929-39 the sum total of the hypothetical separate labor savings corresponding to the separate technical advances in the individual industries is larger than the actual aggregate effect of the combined change which has actually taken place. All individual changes could, so far as labor productivity is concerned, stand on their own feet with the exception of the single instance of the transportation industry over the 1929-39 period. Its slight retrogression is possibly connected with the shift from rail to automotive transport which in terms of current inputs is certainly more costly. The advantages of faster and otherwise more convenient service might actually be reflected in correspondingly reduced input ratios of the other industries.

In order to obtain a measure of the relative importance of the roundabout, as compared with the direct, effects which the changes in the input structures of various industries have on the over-all productivity of labor, a modified series of employment figures has been computed and is presented in Row 15 of both tables.

These figures are derived in the same way as those entered in Row 14 except that here the 1939 labor input coefficients were used throughout. Thus the corresponding labor savings,[a] entered in Row 17, reflect the cost economy resulting from changed input coefficients of factors other than direct labor. They are naturally smaller than those shown in Row 16 and in some cases even negative. This means that in such instances the reduc-

[a] The difference between the computed figures and actual 1939 and 1929 employment, as given in Column 1, Row 14, in Tables 6 and 7.

tion in direct labor was achieved at the cost of an increased use of other current inputs. Of the total labor saving of 1287 million man-hours achieved between 1929 and 1939, only 134.0 million, i e. approximately 10 per cent, were attained in an indirect way; the corresponding proportion in 1919-29 is nearly 25 per cent.

This does not mean, of course, that a labor saving of 1287 million man-hours per year could actually have been achieved through change of labor coefficients alone without any variation in the other elements of the structural matrix. These other changes, even if not leading to a direct economy themselves, were necessary to permit the achievement of the former.

As has already been mentioned, the total change within an open system can be divided in two parts—one due to its structural variations and the other, within that particular theoretical formulation, necessarily assigned to the change in the bill of goods itself. The 1939 bill of final demand, if produced on the basis of the 1929 techniques, would have required 1287 million man-hours more than it actually absorbed. The same sectors of the American economy actually used, in 1929, 379 million (equals 5883 − 5504) man-hours more than in 1939. Thus, according to these figures, 908 million (equals 1287 − 379) man-hours were employed in 1939 to produce the excess of the 1939 over the 1929 bill of final demand. A similar comparison of 1919 with 1929 indicates that with the 1919 input ratios the production of the 1929 outside demand would require 3321 million additional man-hours. Actually only 92 million more man-hours were used in these industries than in 1929. This implies that 3229 million man-hours would be devoted to produce the excess of 1929 over the 1919 bill of final demand if the two were identical in their composition.

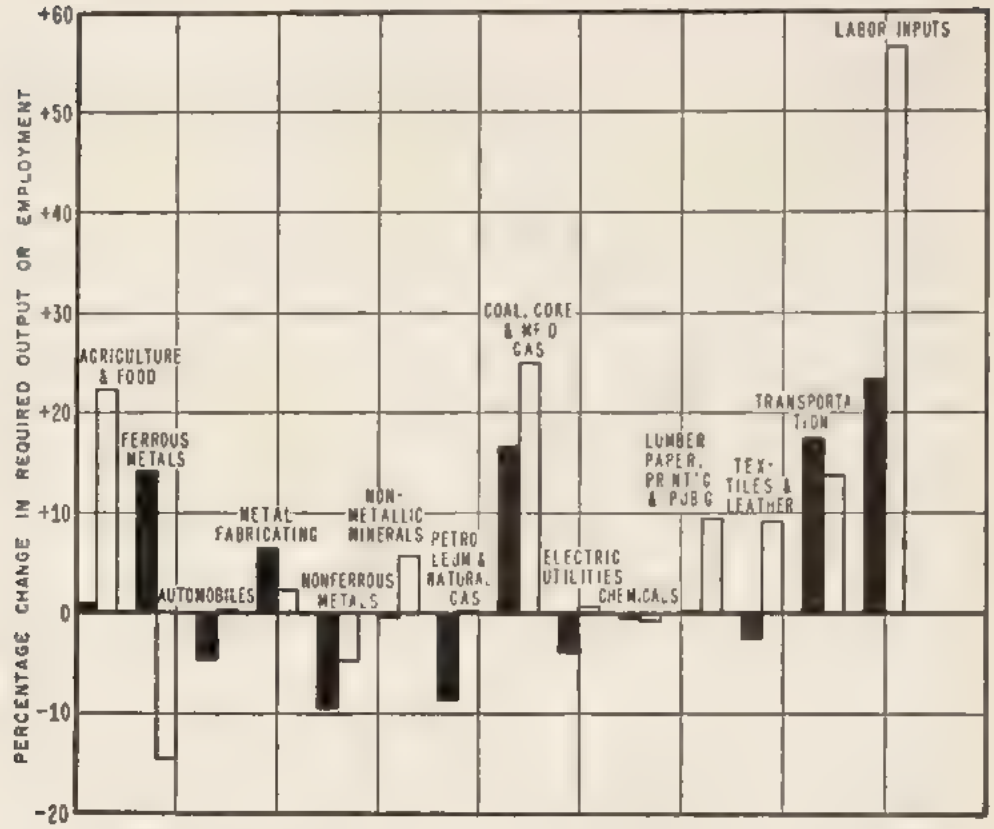
This last qualifying remark is intended to point out that if the 1919 bill of goods—instead of that of the year 1929—had been used as the basis for all the computations the results of which are presented in Table 6, these results would not be quite the same. With a different composition of the final demand the response of the system to any given structural change would also be different. To avoid this problem of arbitrary weights one would have to free the whole empirical analysis from its dependence on any particular bill of goods; in other words, one would have to reformulate the question so as to make the answer independent of any given composition of the outside demand. In principle this could be done by basing the question and the answer on the general formula (2, 4) on page 18, i.e. by computing for each one of the different structural situations a complete set of A's, each showing the dependence of one particular output on one individual component of the final demand. For the limited purpose of the present analysis the computational effort involved in such a generalized approach could hardly be justified.

In Charts 5-11, the separate and combined effects of the structural changes which took place in the American economy during the two interwar decades are depicted in a series of graphs. They are based on figures presented in Tables 8 and 9. Each group of bars shows the repercussion of one particular set of technical changes. Each individual bar measures the increase or the reduction in one particular output. To facilitate comparison all changes are described in percentage terms.

The use of dollar figures for the description of the original input-output flows and of price deflators in their reduction to physical units subjects the results obtained in the course of subsequent numerical analysis to possible distortion caused by utilization of inadequate price data. I refer here not to blurredness inherent in the use of any kind of aggregative data—which is reduced, although certainly not eliminated, in the comparatively differentiated structure of the input-output matrix—but rather to the distortion which would have resulted if the total values (prices times quantities) of a perfectly homogeneous commodity were deflated by the wrong prices. The physical quantity index thus obtained would obviously be also erroneous.

It is inherent in the nature of the theoretical framework of the foregoing analysis of the effects of structural change that the errors resulting from the use of inadequate price figures would distort only those of the numerical findings which are expressed in terms of the particular outputs directly affected by the faulty deflators. If, for example, the 1919-29 price ratio of electric current, which is used to translate the 1919 total output of electric utilities into 1929 dollars is 10 per cent higher than the actual, the corresponding figure entered in Table 1 is 10 per cent too low. This means also that the 1919 input coefficients, describing the use of electricity in all the other industries, are 10 per cent too low, while the input coefficients, showing the use of other goods and services by the electric utilities industry itself, are 10 per cent too high. Carried through all the subsequent computations the consequences of the initial error would show up in Table 6, but only in its fifth row describing the effects of various 1919-29 structural changes on the output of the electric utilities industry. All entries in this row will be 10 per cent larger than they should have been. The output reactions of all other industries cannot be affected by this particular error because in the intermediate computations the 10 per cent increase in one set of the input coefficients would be exactly compensated by a corresponding reduction in the other set. If the use of electric energy in all other industries suddenly became 10 per cent more efficient but the inputs absorbed per unit of its own output rose at the same time and in the same proportion, the total production and consumption of electric

CHART 5
SUBSTITUTION OF INPUT COEFFICIENTS FOR ALL INDUSTRIES



BLACK BARS DENOTE EFFECTS OF SUBSTITUTING 1929 COEFFICIENTS IN THE 1939 MATRIX
WHITE BARS DENOTE EFFECTS OF SUBSTITUTING 1919 COEFFICIENTS IN THE 1929 MATRIX

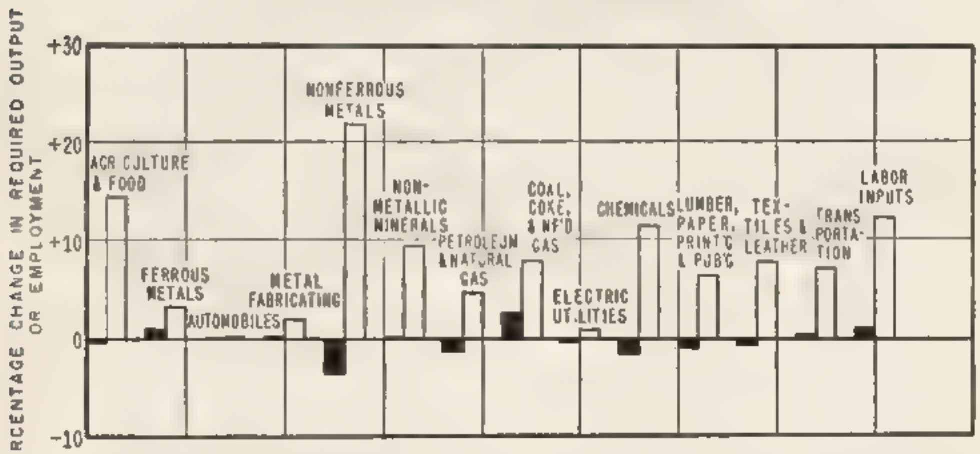
energy would have fallen by exactly 10 per cent, while the outputs of all other branches of the economy would have remained the same.

The same argument can be carried one step further. Instead of 'electric energy' the product under consideration could be 'automobiles.' Instead of a simple error in the price index the original difficulty could have consisted in the introduction of a new model, say an eight cylinder instead of a six. The conservative statistician might base his comparison between the 1919 and 1929 output on the simple assumption that 'a car is a car' while the manufacturers would most likely claim that in the 1929 model they produced, say, the equivalent of $1\frac{1}{2}$ of the 1919 automobiles. The effect of these two different standards of comparison on the final result of our analysis would be quite analogous to those discussed in connection with the ordinary error in price. Whichever of the two alternative units were used to compare the 1919 with the 1929 output in the automobile industry, the computed effects of any structural change upon the production and consumption of all other commodities and services would obviously remain the same.

This conclusion leads us back to the introductory remarks on the nature of structural change and the general methodological problems involved in its possible explanation. As the elements of the economic system change and become finally unidentifiable in historical terms, the scope for the application of analytical and descriptive devices based on simple or even sophisticated numerical comparisons is slowly but inexorably narrowed down. The investigator who refuses to give them up in favor of more general methods of quantitative analysis will find himself finally completely enmeshed in a maze of unintelligible time series and heterogeneous aggregates.

CHART 6

SUBSTITUTION OF INPUT COEFFICIENTS FOR NON-FERROUS METALS AND NON-METALLIC MINERALS

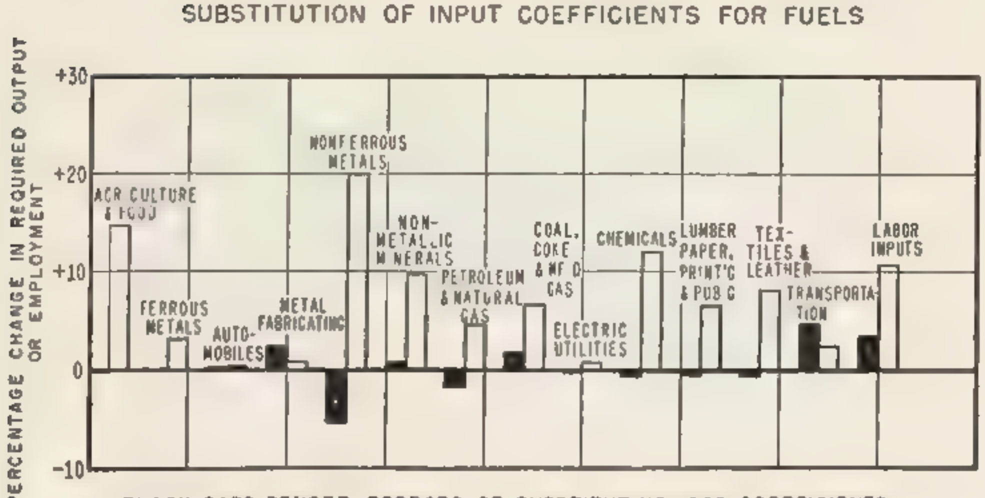


BLACK BARS DENOTE EFFECTS OF SUBSTITUTING 1929 COEFFICIENTS IN THE 1939 MATRIX

WHITE BARS DENOTE EFFECTS OF SUBSTITUTING 1919 COEFFICIENTS IN THE 1929 MATRIX

CHART 7

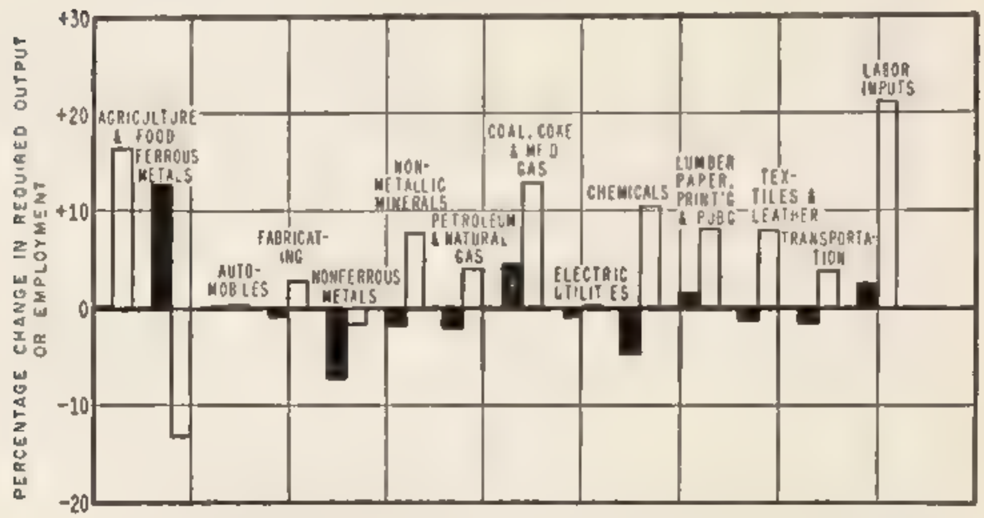
SUBSTITUTION OF INPUT COEFFICIENTS FOR FUELS



BLACK BARS DENOTE EFFECTS OF SUBSTITUTING 1929 COEFFICIENTS IN THE 1939 MATRIX

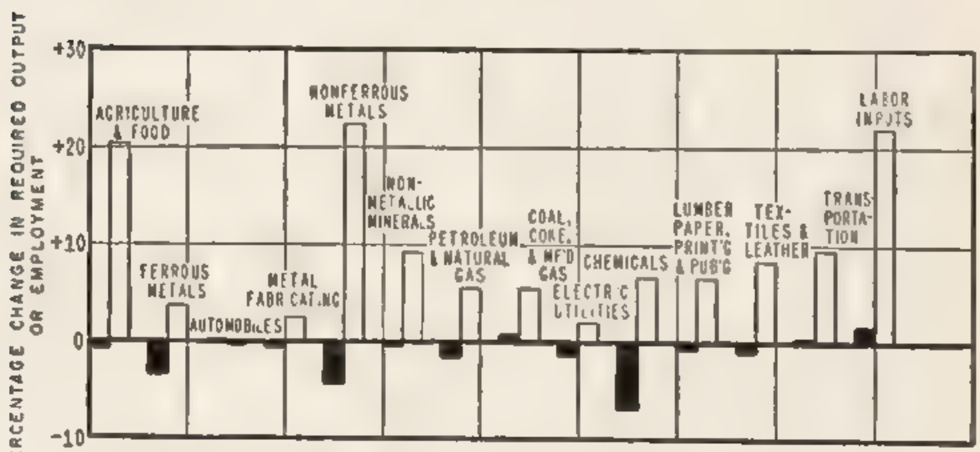
WHITE BARS DENOTE EFFECTS OF SUBSTITUTING 1919 COEFFICIENTS IN THE 1929 MATRIX

CHART 8
SUBSTITUTION OF INPUT COEFFICIENTS FOR HEAVY INDUSTRIES



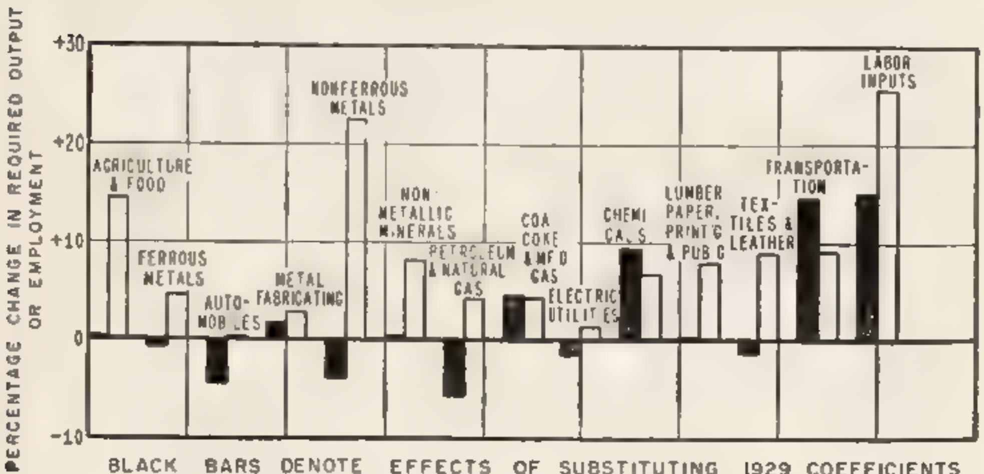
BLACK BARS DENOTE EFFECTS OF SUBSTITUTING 1929 COEFFICIENTS IN THE 1939 MATRIX
WHITE BARS DENOTE EFFECTS OF SUBSTITUTING 1919 COEFFICIENTS IN THE 1929 MATRIX

CHART 9
SUBSTITUTION OF INPUT COEFFICIENTS FOR LIGHT INDUSTRIES



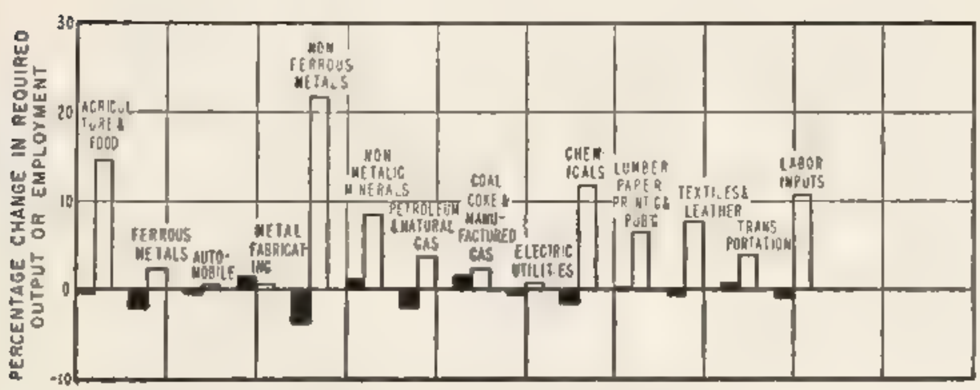
BLACK BARS DENOTE EFFECTS OF SUBSTITUTING 1929 COEFFICIENTS IN THE 1939 MATRIX
WHITE BARS DENOTE EFFECTS OF SUBSTITUTING 1919 COEFFICIENTS IN THE 1929 MATRIX

CHART 10
SUBSTITUTION OF INPUT COEFFICIENTS FOR AGRICULTURE



BLACK BARS DENOTE EFFECTS OF SUBSTITUTING 1929 COEFFICIENTS IN THE 1939 MATRIX
WHITE BARS DENOTE EFFECTS OF SUBSTITUTING 1919 COEFFICIENTS IN THE 1929 MATRIX

CHART 11
SUBSTITUTION OF INPUT COEFFICIENTS FOR TRANSPORTATION



NOTE: BLACK BARS DENOTE EFFECTS OF SUBSTITUTING 1929 COEFFICIENTS IN THE 1939 MATRIX.
WHITE BARS DENOTE EFFECTS OF SUBSTITUTING 1919 COEFFICIENTS IN THE 1929 MATRIX.

TABLE 8

Percentage Changes in Required Outputs of Individual Industries and Labor Inputs with Substitution of 1919 Input Coefficients in the 1929 Matrix, 1929 Bill of Final Demand

	Ind. No.	Industry	Per Cent Excess of Computed over Actual 1929 Output Based on 1919 Coefficients For:						
			Agriculture and Food (1) (1929 coefficients all other)	Heavy Industry (2,3,4,10) (1929 coefficients all other)	Nonferrous Metals and Nonmetallic Minerals (5,6)(1929 coefficients all other)	Fuel and Power (7,8,9) (1929 coefficients all other)	Light Industry (11, 12) (1929 coefficients all other)	Transportation (16) (1929 coefficients all other)	All 1919 coefficients
			(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1)	1	Agriculture and food	+14.6	+16.6	+14.6	+14.6	+20.8	+14.6	+22.1
(2)	2	Ferrous metals	+ 4.3	−13.2	+ 3.1	+ 3.1	+ 3.7	+ 2.1	−14.5
(3)	3	Automobiles	+ 0.2	+ 0.2	+ 0.2	+ 0.2	+ 0.2	+ 0.2	+ 0.2
(4)	4	Metal fabricating	+ 2.6	+ 2.7	+ 1.0	+ 0.7	+ 2.2	+ 0.2	+ 2.3
(5)	5	Nonferrous metals	+22.7	− 1.9	+22.0	+20.0	+22.2	+21.6	− 4.7
(6)	6	Nonmetallic minerals	+ 8.1	+ 7.6	+ 9.6	+ 9.7	+ 9.2	+ 8.5	+ 5.8
(7)	7	Petroleum and natural gas	+ 4.2	+ 4.1	+ 4.9	+ 4.5	+ 5.3	+ 3.7	+ 0.2
(8)	8	Coal, coke, and manufactured gas	+ 4.4	+12.9	+ 8.0	+ 6.7	+ 5.4	+ 2.2	+24.9
(9)	9	Electric utilities	+ 1.3	+ 0.1	+ 0.8	+ 0.4	+ 2.1	+ 0.7	+ 0.8
(10)	10	Chemicals	+ 6.9	+10.3	+11.4	+12.0	+ 6.8	+11.9	− 0.6
(11)	11	Lumber, paper, printing, and publishing	+ 8.0	+ 7.9	+ 6.5	+ 6.5	+ 6.8	+ 6.5	+ 9.5
(12)	12	Textiles and leather	+ 8.8	+ 7.9	+ 7.9	+ 7.9	+ 8.3	+ 7.9	+ 9.1
(13)	16	Transportation	+ 9.1	+ 3.8	+ 7.3	+ 2.4	+ 9.3	+ 3.9	+14.1
(14)		Households (labor inputs)	+25.5	+21.4	+12.6	+10.7	+22.0	+10.7	+56.7

TABLE 9

Percentage Changes in Required Outputs of Individual Industries and Labor Inputs with Substitution of 1929 Input Coefficients in the 1939 Matrix, 1939 Bill of Final Demand

	Ind. No.	Industry	Per Cent Excess of Computed over Actual 1939 Output Based on 1929 Coefficients For:						
			Agriculture and Food(1) (1939 coefficients all other)	Heavy Industry (2,3,4,10) (1939 coefficients all other)	Nonferrous Metals and Nonmetallic Minerals (5,6) (1939 coefficients all other)	Fuel and Power (7, 8, 9) (1939 coefficients all other)	Light Industry (11, 12) (1939 coefficients all other)	Transportation (16) (1939 coefficients all other)	All 1929 Coefficients
			(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1)	1	Agriculture and food	–	+ 0.1	–0.1	–0.1	+0.6	–0.1	+ 1.0
(2)	2	Ferrous metals	– 0.4	+12.7	+0.8	–0.1	–3.1	–2.1	+14.2
(3)	3	Automobiles	– 4.3	–	–	–	–	–0.2	– 4.5
(4)	4	Metal fabricating	+ 1.7	– 0.9	+0.1	+2.2	–0.3	+1.3	+ 6.7
(5)	5	Nonferrous metals	– 3.7	– 7.2	–3.7	–5.5	–4.3	–3.4	– 9.3
(6)	6	Nonmetallic minerals	+ 0.2	– 1.8	–	+0.4	–0.1	+1.0	– 0.2
(7)	7	Petroleum and natural gas	– 5.3	– 2.0	–1.1	–1.9	–1.4	–1.8	– 8.7
(8)	8	Coal, coke, and manufactured gas	+ 4.5	+ 4.1	+2.3	+1.5	+0.6	+1.5	+16.4
(9)	9	Electric utilities	– 1.4	– 0.7	–0.1	–0.2	–1.1	–0.2	– 3.9
(10)	10	Chemicals	+ 9.6	– 4.6	–1.3	–0.6	–6.9	–1.4	– 0.3
(11)	11	Lumber, paper, printing, and publishing	–	+ 1.2	–0.7	–0.3	–0.3	–	+ 0.1
(12)	12	Textiles and leather	– 1.2	– 1.2	–0.6	–0.5	–0.9	–0.5	– 2.2
(13)	16	Transportation	+14.4	– 1.3	+0.1	+4.4	–	+0.4	+17.4
(14)		Households (labor inputs)	+15.0	+ 2.1	+0.8	+3.3	+1.7	–0.9	+23.4

MATHEMATICAL NOTE

The Effect of Changes in the Coefficients of a System of Non-homogeneous Linear Equations upon the Magnitude of Its Roots

The following mathematical consideration of column-by-column changes in the coefficient matrix of a non-homogeneous system of linear equations with m unknowns has direct bearing upon the study of technological change within the framework of the 'input-output' structure of a national economy.

Consider four systems of non-homogeneous linear equations, each containing m equations in m unknowns:

$$\begin{bmatrix} +1 & -a_{12} & \cdots & -a_{1i} & \cdots & -a_{1k} & \cdots & -a_{1m} \\ -a_{21} & +1 & \cdots & -a_{2i} & \cdots & -a_{2k} & \cdots & -a_{2m} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ -a_{i1} & -a_{i2} & \cdots & +1 & \cdots & -a_{ik} & \cdots & -a_{im} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ -a_{k1} & -a_{k2} & \cdots & -a_{ki} & \cdots & +1 & \cdots & -a_{km} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ -a_{m1} & -a_{m2} & \cdots & -a_{mi} & \cdots & -a_{mk} & \cdots & +1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_i \\ \vdots \\ X_k \\ \vdots \\ X_m \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_i \\ \vdots \\ Y_k \\ \vdots \\ Y_m \end{bmatrix} \qquad (2n, 1)$$

$$\begin{bmatrix} +1 & -a_{12} & \cdots & -(a_{1i}+b_1) & \cdots & -a_{1k} & \cdots & -a_{1m} \\ -a_{21} & +1 & \cdots & -(a_{2i}+b_2) & \cdots & -a_{2k} & \cdots & -a_{2m} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ -a_{i1} & -a_{i2} & \cdots & +1 & \cdots & -a_{ik} & \cdots & -a_{im} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ -a_{k1} & -a_{k2} & \cdots & -(a_{ki}+b_k) & \cdots & +1 & \cdots & -a_{km} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ -a_{m1} & -a_{m2} & \cdots & -(a_{mi}+b_m) & \cdots & -a_{mk} & \cdots & +1 \end{bmatrix} \begin{bmatrix} X'_1 \\ X'_2 \\ \vdots \\ X'_i \\ \vdots \\ X'_k \\ \vdots \\ X'_m \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_i \\ \vdots \\ Y_k \\ \vdots \\ Y_m \end{bmatrix}$$

(2n, 2)

$$\begin{bmatrix} +1 & -a_{12} & -\cdots & -a_{1i} & -\cdots & -(a_{1k}+c_1) & -\cdots & -a_{1m} \\ -a_{21} & +1 & -\cdots & -a_{2i} & -\cdots & -(a_{2k}+c_2) & -\cdots & -a_{2m} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ -a_{i1} & -a_{i2} & -\cdots & +1 & -\cdots & -(a_{ik}+c_i) & -\cdots & -a_{im} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ -a_{k1} & -a_{k2} & -\cdots & -a_{ki} & -\cdots & +1 & -\cdots & -a_{km} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ -a_{m1} & -a_{m2} & -\cdots & -a_{mi} & -\cdots & -(a_{mk}+c_m) & -\cdots & +1 \end{bmatrix} \begin{bmatrix} X''_1 \\ X''_2 \\ \vdots \\ X''_i \\ \vdots \\ X''_k \\ \vdots \\ X''_m \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_i \\ \vdots \\ Y_k \\ \vdots \\ Y_m \end{bmatrix} \tag{2n, 3}$$

$$\begin{bmatrix} +1 & -a_{12} & -\cdots & -(a_{1i}+b_1) & -\cdots & -(a_{1k}+c_1) & -\cdots & -a_{1m} \\ -a_{21} & +1 & -\cdots & -(a_{2i}+b_2) & -\cdots & -(a_{2k}+c_2) & -\cdots & -a_{2m} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ -a_{i1} & -a_{i2} & -\cdots & +1 & -\cdots & -(a_{ik}+c_i) & -\cdots & -a_{im} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ -a_{k1} & -a_{k2} & -\cdots & -(a_{ki}+b_k) & -\cdots & +1 & -\cdots & -a_{km} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ -a_{m1} & -a_{m2} & -\cdots & -(a_{mi}+b_m) & -\cdots & -(a_{mk}+c_m) & -\cdots & +1 \end{bmatrix} \begin{bmatrix} X'''_1 \\ X'''_2 \\ \vdots \\ X'''_i \\ \vdots \\ X'''_k \\ \vdots \\ X'''_m \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_i \\ \vdots \\ Y_k \\ \vdots \\ Y_m \end{bmatrix} \tag{2n, 4}$$

The analysis of the interrelation between the magnitudes of the four sets of the dependent variables in the four systems of equations leads to formulation of the following two propositions:

Proposition I

For any set of four corresponding variables, X_1, X'_1, X''_1, and X'''_1,

$$(X_1 - X'''_1) = (X_1 - X'_1)\,\frac{X'''_i}{X'_i} + (X_1 - X''_1)\,\frac{X'''_k}{X''_k}$$

Expressed in more compact notation the four given systems of equations can be written as:

$$[A_1, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]X = Y \qquad (2n, 1a)$$

$$[A_1, A_2, \cdots, (A_i + B), \cdots, A_k, \cdots, A_m]X' = Y \qquad (2n, 2a)$$

$$[A_1, A_2, \cdots, A_i, \cdots, (A_k + C), \cdots, A_m]X'' = Y \qquad (2n, 3a)$$

$$[A_1, A_2, \cdots, (A_i + B), \cdots, (A_k + C), \cdots, A_m]X''' = Y \qquad (2n, 4a)$$

All capital letters represent column matrices; their sequence in each of the above formulae is the same as the sequence of corresponding columns of constants and variables in (2n, 1), (2n, 2), (2n, 3), and (2n, 4). Subtraction of (2n, 2a) from (2n, 1a), (2n, 3a) from (2n, 1a), and (2n, 4a) from (2n, 1a) gives:

$$[A_1, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m][X - X'] = X'_iB \qquad (2n, 5)$$

$$[A_1, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m][X - X''] = X''_kC \qquad (2n, 6)$$

$$[A_1, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m][X - X'''] = X'''_iB + X'''_kC \qquad (2n, 7)$$

Solving the first of these three systems, say, for $(X_1 - X'_1)$, the second for $(X_1 - X''_1)$, and the last for $(X_1 - X'''_1)$ in terms of corresponding determinants we obtain:

$$(X_1 - X'_1) = \frac{\text{Det }[X'_iB, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]}{\text{Det }[A_1, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]} \qquad (2n, 8)$$

$$(X_1 - X''_1) = \frac{\text{Det }[X''_kC, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]}{\text{Det }[A_1, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]} \qquad (2n, 9)$$

$$(X_1 - X'''_1) = \frac{\text{Det }[X'''_iB_i + X'''_kC, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]}{\text{Det }[A_1, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]} \qquad (2n, 10)$$

Multiplying both sides of (2n, 8) by $\dfrac{X'''_i}{X'_i}$ and both sides of (2n, 9) by $\dfrac{X'''_k}{X''_k}$.

$$(X_1 - X'_1)\frac{X'''_i}{X'_i} = \frac{\text{Det }[X'''_iB, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]}{\text{Det }[A_1, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]} \qquad (2n, 11)$$

$$(X_1 - X''_1)\frac{X'''_k}{X''_k} = \frac{\text{Det }[X'''_kC, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]}{\text{Det }[A_1, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]} \qquad (2n, 12)$$

Addition of the left- and the right-hand terms of (2n, 11) and (2n, 12) gives:

$$(X_1 - X'_1)\frac{X'''_i}{X'_i} + (X_1 - X''_1)\frac{X'''_k}{X''_k}$$

$$= \frac{\text{Det }[X'''_iB + X'''_kC, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]}{\text{Det }[A_1, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]} \quad (2n, 13)$$

Since the right-hand term of (2n, 13) is identical with the right-hand term of (2n, 10),

$$(X_1 - X'''_1) = (X_1 - X'_1)\frac{X'''_i}{X'_i} + (X_1 - X''_1)\frac{X'''_k}{X''_k} \quad (2n, 14)$$

The numbering of the variables being obviously arbitrary, X_1 in the equation above can be replaced by any variable X_i.

Proposition II

If (a) the inversions of the m-by-m matrices on the left-hand side of the four systems (2n, 1) (2n, 2) (2n, 3), and (2n, 4) contain only positive elements and (b) the four column matrices X, X', X'', and X''' are also positive, then,

$$[X - X'] + [X - X''] > [X - X'''] \quad (2n, 15)$$

$$[X - X'''] > 0, \quad \text{if} \quad B \geq 0 \quad \text{and} \quad C \geq 0 \quad (2n, 16)$$

$$[X - X'''] < 0, \quad \text{if} \quad B \leq 0 \quad \text{and} \quad C \leq 0 \quad (2n, 17)$$

where $B \geq 0$ and $C \geq 0$ mean that some of the elements in each of the two column matrices are positive and none are negative; while $B \leq 0$ and $C \leq 0$ indicate that some elements are negative and none are positive.

PROOF:

Subtraction of (2n, 4a) from (2n, 2a) and (2n, 4a) from (2n, 3a) gives,

$$[A_1, A_2, \cdots, (A_i + B), \cdots, A_k, \cdots, A_m][X' - X'''] = X'''_kC \quad (2n, 18)$$

$$[A_1, A_2, \cdots, A_i, \cdots, (A_k + C), A_m][X'' - X'''] = X'''_iB \quad (2n, 19)$$

Multiplying both sides of (2n, 18), (2n, 19) and of (2n, 7) by $[A_1, A_2, \cdots, A_m]^{-1}$ we have,

$$[X' - X'''] = [A_1, A_2, (A_i + B), \cdots, A_k, \cdots, A_m]^{-1}X'''_kC \quad (2n, 20)$$

$$[X'' - X'''] = [A_1, A_2, A_i, \cdots, (A_k + C), \cdots, A_m]^{-1}X'''_iB \quad (2n, 21)$$

$$[X - X'''] = [A_1, A_2, \cdots, A_m]^{-1}(X'''_iB + X'''_kC) \quad (2n, 22)$$

By conditions of the proposition to be proved, the matrices $[A_1, A_2, \cdots, A_m]^{-1}$ and the scalars, X'''_i, X'''_k, are all positive; consequently,

$$\left.\begin{array}{l}[X' - X'''] > 0 \\ [X'' - X'''] > 0 \\ [X - X'''] > 0\end{array}\right\} \quad \text{if} \quad B \geq 0 \quad \text{and} \quad C \geq 0 \tag{2n, 23}$$

If $B \leq 0$ and $C \leq 0$, the inequality signs on the left-hand side must be also reversed. Thus (2n, 15) and (2n, 16) are shown to be true.

By adding (2n, 5) and (2n, 6), subtracting from the resulting sum (2n, 7), and multiplying both sides of the equation thus obtained by $[A_1, A_2, A_i, \cdots, A_k, \cdots, A_m]^{-1}$ we arrive at,

$$[X - X'] + [X - X''] - [X - X'''] = [A_1, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]^{-1}$$
$$\{(X'_i - X'''_i)B + (X''_k - X'''_k)C\} \tag{2n, 24}$$

Since $[A_1, A_2, \cdots, A_i, \cdots, A_k, \cdots, A_m]^{-1}$ is positive by assumption while $(X'_i - X'''_i)B$ and $(X''_k - X'''_k)C$ are positive because of (2n, 23), the expression on the left-hand side of the equation above is necessarily positive, which proves that (2n, 15) is correct.

Chapter 3

DYNAMIC ANALYSIS

Wassily Leontief

I. Static and Dynamic Theory

A static theory derives the changes in the variables of a given system from the observed changes in the underlying structural relationships: dynamic theory goes further and shows how certain changes in the variables can be explained on the basis of fixed, i.e. invariant, structural characteristics of the system.

Dynamic theory thus enables us to derive the empirical law of change of a particular economy from information obtained through the observation of its structural characteristics at one single point of time. This possibility, methodologically rather obvious, and practically very important, has unfortunately been obscured by the fact that most of the recent attempts to determine the structural characteristics of actual economic systems have been based on some kind of statistical time-series analysis, thus giving rise to the erroneous impression that the empirical laws of change necessarily must be derived from direct observations of past development.

This would not be true even if one had reason to believe that extended, say forty- or fifty-year, stretches of economic development were explainable in purely dynamic terms, that is, without explicit reference to significant structural change. Our knowledge of Western economic history of the last two hundred years proves such an assumption to be untenable.

The empirical approach designed to derive the operational properties of a national economy through direct and detailed observation of its structural characteristics at one particular point, or at least a relatively short interval, of time seems to commend itself for the purposes of dynamic as well as of static analysis.

Developed with this particular use in mind, the theory described below represents a dynamic extension of the static input-output scene. It is not a general theory if by general one means a formulation which, for the sake of conceptual completeness, incorporates all the hypothetically relevant determinants of the process to be explained, including those which – like the more subtle aspects of entrepreneurial decision-making, for example—will remain beyond the reach of the empirical investigator for a long

time to come. This implies, of course, the assumption that in its specialized form the proposed theory, implemented by accessible factual information, can advance the understanding of the actually observed dynamic processes; it also implies the belief that as additional data become available the theory itself could be refined to take them into account.

Static input-output analysis describes the economic system in terms of mutually interrelated and structurally conditioned, simultaneous flows of commodities and services. The dynamic element—the dependence of the future on the past states of the system—can and usually has been accounted for in the theoretical explanation through the introduction of structural time lags, of structural stock-flow relationships, or of a combination of both.

The methodological emphasis on 'structural' is relevant for the following reason. The accumulation and decumulation of stocks can, in some instances, be shown to result directly from the operation of primary structural lags. The fluctuation in the amount of 'work in progress' (which is a stock of intermediate products) observed in the shipbuilding industry, for example, can be successfully explained on the basis of the structural lag measured by the time which elapses between the laying of the keel of a vessel and its final completion. Conversely an observed lag between the variation in the stream of inputs absorbed by an industry and the corresponding changes in the level of its output can often be traced back to its changing capital requirements based on the technologically determined stock-flow ratio between the amounts, i.e. the stock of equipment, building, and inventory of materials, on the one hand, and the corresponding capacity to turn out the actually observed stream of finished products, on the other.

As is often the case in this type of analysis, the selection of the theoretical scheme depends to a considerable extent on the level of detail at which the empirical inquiry is to be conducted. The stock-flow relationship which, at one such level, would best be considered as a structural datum, might turn out to be reducible on another, deeper level to fixed structural lags and vice versa.

In general stock-flow ratios are more readily observable than lags between flows. It is interesting to note in this connection that the conventional standards of behavior, the rules-of-thumb actually, or at least apparently, adhered to by the practical decision-makers in economic enterprises—be they industrial managers, directors of commercial or financial institutions, or even public budget-making authorities—most often are formulated in terms of some normal period of turnover, desirable inventory ratios and other, similar stock-flow relationships; these conventional rules

hardly ever contain explicit references to desirable or normal time lags.

This is the reason why stock-flow relationships, rather than structural time lags or a combination of both, are relied upon in this initial attempt at a dynamic approach to empirical input-output analysis.[1] A separate note appended to this chapter contains the description of a mathematical procedure which could be used in quantitative analysis of more general dynamic input-output systems involving both structural lags and stock-flow relationships.

II. Statement of Dynamic Theory

The static input-output scheme described in Chapter 2 (p. 18 ff.) explains the mutual interdependence of the distinct sectors of the national economy in terms of a given set of structural coefficients, a_{ik}. Each such coefficient represents the amount of a particular input, i, which is absorbed by industry k per unit of its output. A complete set of such coefficients pertaining to any one particular industry determines the flows of labor, all kinds of materials, fuels, replacement parts, etc., which this industry would have to absorb per unit of time, say per month or per year, in order to be able to produce a given flow of output, i.e. in order to be able to maintain a rate of output defined in terms of so many units of finished product per month or per year.

These input coefficients do not reflect, however, the stock requirements of the economy; they do not and cannot explain the magnitude of those input flows which serve directly to satisfy the capital needs of all its various sectors, either as additions to fixed investment in the form of permanent improvements, building and different kinds of equipment, or as an increase in the necessary inventories of raw material, goods in process, etc. In the open static system, such as is described in the last chapter, these inputs, instead of being assigned to the industries which actually absorb them, are simply considered to be a part of final demand. In equations (2, 1), (2, 2), and (2, 3) all investment demand for the product of any industry i is included in y_i, that means that the effects of investment demand on outputs of all commodities and services are explained, while the observed magnitude of this demand itself, though 'taken in account,' is not explained.

Such an explanation becomes possible as soon as the stock requirements of all the individual sectors of the economy are included in the structural

[1] Other discussions of the same dynamic system based on stock flow relationships will be found in Hawkins, David, 'Some Conditions of Macroeconomic Stability,' *Econometrica*, October 1948, Georgescu Roegen, N., 'Relaxation Phenomena,' in *Activity Analysis of Production and Allocation*, ed. by T. C. Koopmans, New York, 1951; Leontief, W., 'Dynamic Analysis of Economic Equilibrium,' *Proceedings of Second Symposium on Large-Scale Digital Calculating Machinery*, Harvard University Press, 1951, pp. 333-7

map of the system along with its previously described flow requirements.

If $S_{ik}(t)$ represents the stock of a commodity produced by industry i and used by industry k at the time t, the rate of change, i.e. the rate of increase or decrease in that stock at this particular point of time, can be written as $\dfrac{dS_{ik}(t)}{dt}$ or, for short, as $\dot{S}_{ik}$.

The basic balance equation—equation (2, 2) on p. 18—can now be rewritten as follows:

$$X_i - \sum_{k=1}^{m} x_{ik} - \sum_{k=1}^{m} \dot{S}_{ik} = Y_i \qquad \begin{array}{l} i = 1, 2, \cdots, m \\ k = 1, 2, \cdots, m \end{array} \quad (3, 1)$$

The second left-hand term represents here, as before, the sum total of those input flows of commodity i which serve the current production requirements of all the various sectors of the economy, the new, third term describes, on the other hand, the inputs absorbed on 'capital account,' i.e. that part of the total demand for commodity i which is being added to or subtracted (if $\sum_{k=1}^{m} \dot{S}_{ik}$ happens to be negative) from the stocks of that particular good used throughout the economy. All allocations of commodity i to current replacement and maintenance requirements of the capital goods and other stocks are to be thought of as being accounted for by the second term, $\sum_{k=1}^{m} X_{ik}$, unless of course they are included in the final demand, Y_i.

The set of structural equations (2, 2) describing the current input requirements of each sector of the economy on its rate of output, X_k, must now be supplemented by a corresponding set of structural stock-flow relationships:

$$S_{ik} = b_{ik} X_k \qquad \begin{array}{l} i = 1, 2, \cdots, m \\ k = 1, 2, \cdots, m \end{array} \quad (3, 2a)$$

The b_{ik}'s will from now on be referred to as the stock or capital coefficients of the system. If i stands, say, for power tools and k for automobiles, the coefficient, b_{ik}, indicates the amount, i.e. the stock, of power tools used per unit of the annual automobile output. With b_{ik} known, the corresponding equation in (3, 2a) determines the total stock of machine tools which the automobile industry would use if it were to produce at a rate of X_k units of automobiles per year. The over-all balance between the input and the output flows expressed in (3, 1) comprises, however, only additions to and subtractions from stocks rather than the entire stocks themselves. Differentiation of both sides of (3, 2a) in respect to time transforms these structural equations into relations between

changes in specific stocks held by various industries, $\dot{S}_{ik}$, and changes in the rates of output of these industries, $\dot{X}_{ik}$:

$$\dot{S}_{ik} = b_{ik}\dot{X}_k \qquad \begin{array}{l} i = 1, 2, \cdots, m \\ k = 1, 2, \cdots, m \end{array} \qquad (3, 2b)$$

Substitution of the two sets of structural relationships (2, 2) and (3, 2b) in (3, 1) leads to the final system of dynamic input-output equations:

$$X_i - \sum_{k=1}^{m} a_{ik}X_k - \sum_{k=1}^{m} b_{ik}\dot{X}_k = Y_i \qquad \begin{array}{l} i = 1, 2, \cdots, m \\ k = 1, 2, \cdots, m \end{array} \qquad (3, 3)$$

It is a system of m linear differential equations with constant coefficients. Its m 'unknowns' are the same as those of the corresponding static system (2, 3): the output flows of all the various commodities and services, X_1, X_2, $\cdots$, X_k, $\cdots$, X_m. Their mutual interdependence now involves also, however, the rates of change of all these flows, $\dot{X}_1$, $\dot{X}_2$, $\cdots$, $\dot{X}_k$, $\cdots$, $\dot{X}_m$. Because of that the solution of this dynamic system leads to the determination, or perhaps prediction, of the behavior of each one of these variables over time. Once the time shape of a particular output function, $X_k(t)$, has been determined, the corresponding variations in the input flows, $x_{ik}(t)$, absorbed and capital stocks, $S_{ik}(t)$, held in this k^{th} sector of the economy can be found by substituting $X_k(t)$ in the appropriate equations in (2, 2) and (3, 2a).

The complete set of capital coefficients of the m industries described above forms a square matrix,

$$b \equiv \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ b_{m1} & b_{m2} & \cdots & b_{mm} \end{bmatrix} \qquad (3, 4)$$

each column of which lists the stock requirements of one particular sector of the economy. It can be referred to as the capital matrix or the capital structure of the economy. Together with the corresponding previously defined (see p. 19) matrix of the flow coefficients, a, it summarizes the primary structural information required for the derivation of the general dynamic properties of a given economic system.

As explained before, the 'bill of goods' entered on the right-hand side of system (3, 3) is considered to be some known function of time; the history of each individual component, Y_i, of this final demand is assumed to be given, i.e. determined by relationships other than those described in these equations themselves. System (3, 3), in other words, is an open system. To close it one would have to add to it the missing links.

If the Y_i's represented, for example, direct consumers' demand, the missing relations would be those pertaining to the description of the position of the households in their connection with the other parts of the economy. System (3, 3) could then be closed through the introduction of an additional equation (or equations, if households were broken down into several separate categories) with coefficients reflecting the structural properties of this particular sector. The treatment of households as an industry supplying its output, mainly labor services, to other industries and receiving its inputs, consumers' goods, from them has already been considered in connection with static input-output analysis.[2]

The components of final demand, Y_1, Y_2, $\cdots$, Y_m, will appear in the new enlarged system as input flows of the n^{th}, the household, sector. Its structural properties like those of any other industry can, for purposes of dynamic analysis, be described in terms of two sets of parameters—the flow coefficients, a_{1n}, a_{2n}, $\cdots$, and the stock coefficients, b_{1n}, b_{2n}, $\cdots$; the latter determine consumers' investments in housing, automobiles, household appliances, and all other kinds of durables and 'storables.'

Closed through explicit inclusion of this n^{th} sector, the original open system (3, 3) will thus be replaced by the following n $(= m + 1)$ equations:

$$X_i - \sum_{i=1}^{n} a_{ik}X_k - \sum_{i=1}^{n} b_{ik}\dot{X}_k = 0 \qquad \begin{array}{l} i = 1, 2, \cdots, m, n \\ k = 1, 2, \cdots, m, n \end{array} \qquad (3, 5)$$

X_n represents here the output of the households. The final bill of goods, Y_1, Y_2, $\cdots$, of the open system—which one must remember is, for argument's sake, assumed to have comprised household demand only—can now be written as x_{1n}, x_{2n}, $\cdots$ and derived from the following set of relationships:

$$Y_i \equiv x_{in} = a_{in}X_n + b_{in}\dot{X}_n \qquad i = 1, 2, \cdots, n \qquad (3, 6)$$

$a_{in}X_n$ represents the flow of commodities and services absorbed by households on the current consumption accounts while $b_{in}\dot{X}_n$ shows the additions to or—if $\dot{X}_n$ is negative—the subtractions from the consumers' stocks of durables and semi-durables. This investment demand by households obviously must be clearly distinguished from the investment demand of the other sectors of the economy.

III. The Economic Interpretation

A concise technical presentation of the solution of systems of linear differential equations with constant coefficients can be found in any standard

[2] See Leontief, W. W., *The Structure of American Economy*, 1919-1939, New York, 1951, pp. 41, 169-71.

mathematical text on advanced calculus. The following discussion concerns itself primarily with the interpretation of such solutions in specific economic terms.[3]

Closed (the mathematician calls them homogeneous) systems such as (3, 5) are easier to handle than the corresponding open, non-homogeneous systems like (3, 3). Because of that the former will be taken up first. Its solution can be written in the following form:

$$X_i(t) = c_1 k_{i1} e^{\lambda_1 t} + c_2 k_{i2} e^{\lambda_2 t} + \cdots + c_k k_{ik} e^{\lambda_k t} + \cdots + c_n k_{in} e^{\lambda_n t}$$

$$i = 1, 2, \cdots, n \quad (3, 7)$$

Each of these n equations describes the path through time of one of the n different outputs, $X_1(t)$, $X_2(t)$, $\cdots$. The numerical solution of a dynamic input-output system (3, 5) consists in determination of the values of the n 'roots,' $\lambda_1, \lambda_2, \cdots, \lambda_n$, the n^2 coefficients of the type k_{ik} and the n coefficients $c_1, c_2, \cdots, c_n$. Once the numerical values of these $n^2 + n$ coefficients are entered in (3, 7), the level of any output, $X_i(t)$, at any point of time, t_1, can be found by putting $t = t_1$ in the exponentials on the right-hand side of the appropriate equation.

The magnitude of each one of the n roots $\lambda_1, \lambda_2, \cdots$ and of each of the n^2 coefficients, k_{ik}, depends upon the structure of the economy as described by the two sets of the structural coefficients, a and b.[4]

The actual level of all the outputs at any particular point of time, $t = t_1$ (say $t = 1953$), depends, however, not only on these structural properties alone but also on the state of the system, i.e. the level of outputs at some

[3] A simple, step-by-step description and explanation of computations described here in general verbal terms is given in a separate note on p. 76 ff. A reader familiar with the elements of calculus might turn to it for additional clarification.

[4] Given the two square matrices, $\bar{a}$ and b, $\lambda_1, \lambda_2, \cdots, \lambda_n$ are the roots of the 'characteristic equation,' determinant $|\bar{a} - \lambda b| = 0$, where determinant $|\bar{a} - \lambda b|$ is the determinant of the matrix $[\bar{a} - \lambda b]$. In the usual case in which no two of these roots are identical, in the k^{th} column, the coefficients k_{ik}, can be set to be equal (or more generally, proportional) to *any* one column of matrix $[F(\lambda_k)]$, adjoint of matrix $[\bar{a} - \lambda_k b]$, i.e. a matrix in which each element, $F_{ij}(\lambda_k)$, is the co-factor of the element $a_{ji} - \lambda_k b_{ji}$ in $|\bar{a} - \lambda b|$.

The matrix $\bar{a}$ written out in full reads as follows:

$$\begin{bmatrix} 1 - a_{11} & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & 1 - a_{22} & \cdots & -a_{2n} \\ \cdot & & & \\ \cdot & \cdot & & \\ \cdot & & & \\ -a_{n1} & -a_{n2} & \cdots & 1 - a_{nn} \end{bmatrix}$$

and is thus related to the matrix a defined in (2, 5) by the following relationship:

$$\bar{a} = [I - a]$$

where I is a unit matrix. Since the basic set a of all structural flow coefficients enters practically all theoretical equations as a component of $\bar{a}$, the new notation offers the advantage of greater brevity.

original point of time, $t = t_o$ (say $t = 1950$). To predict the development of a dynamic system over a given interval of time, it is not sufficient to know its general law of change; one must have also specific information concerning its position at the start. These, as the mathematician calls them, initial conditions can be described, for example, in terms of the particular levels of output, $X_1(t_o)$, $X_2(t_o)$, $\cdots$, observed at the original point of time, t_o. Which particular point of time is chosen to serve as such a base of reference, in principle at least, does not matter; once some actual state of the system has been observed, the supposedly known dynamic law of change should make it possible to compute its position at any other, either earlier or later, point of time as well. Practically, taking into account the fact that the structural characteristics of an economy and also its law of change does vary with time, so that the actual course of its development is bound to veer away from the theoretically computed course, the initial point of observation should not be too far removed from those for which the indirect prediction is actually being made.

The quantitative information contained in the description of the 'initial conditions' of the given economic system is incorporated in solution (3, 7) through the numerical values of the constants, c_1, c_2, $\cdots$, c_n. Given the levels of all outputs for some initial point of time, t_o, this t_o (say $t = 0$) can be inserted in all the exponents on the right- and the corresponding $X_i(t_o)$'s on the left-hand side of the equations (3, 7). With the roots, λ, and the constants, k, already known, the system is thus transformed into a set of n linear equations which can be easily solved for the n unknown coefficients, c_1, c_2, $\cdots$, c_n.[5] With these inserted in (3, 7) and the time, t, as well as the outputs, $X_i(t)$, considered again as variables, the solution of the closed dynamic system (3, 5) is complete.

[5] There are many other ways to anchor the general structural law of change of a given economic system to the historically emergent circumstances of its existence. Instead of registering the magnitude of all the n variables, i.e. the levels of output of all the separate industries at any single point of time, one could observe the magnitude of one of them, say $X_1(t)$, representing, for example, the output of steel, at n different points of time, t_1, t_2, $\cdots$, t_n. Inserting alternatively the pairs of corresponding values t_1 and $X_1(t_1)$, t_2 and $X_1(t_2)$, and so on up to t_n and $X_1(t_n)$ in the appropriate—in this case it would be the first—equation of system (3, 7), one obtains a system of n linear equations with c_1, c_2, $\cdots$, c_n as the unknowns to be determined by it.

Empirically observed rates of change, $\dot{X}_i(t)$, of outputs even if the $X_i(t)$'s, that is the outputs themselves, are unknown, can also define the initial conditions of the system, sufficient for determination of the constants, c_i. The published Soviet Russian statistics indicate for example the percentage-wise growth of the individual industries in the interval between two years without making any mention of the absolute level of output at either year. Were the basic structural characteristics of the Soviet economy known, given ratios such as $\dfrac{X_1(t_1)}{X_2(t_2)}$, $\dfrac{X_2(t_1)}{X_2(t_2)}$, $\cdots$ combined with information on the actual, absolute level of output, $X_i(t_o)$ reached at any time in any one industry (or on its absolute increment, say $X_i(t_3) - X_i(t_4)$, between two points of time) would suffice for determination of the numerical values of the constants, c_1, c_2, $\cdots$. With these inserted in the dynamic solution (3, 7), the absolute levels of outputs of all industries could be determined for any point of time, t.

The n roots, $\lambda_1, \lambda_2, \cdots, \lambda_n$, depending on the magnitude of the structural coefficients combined in the matrices, $\bar{a}$ and b, will be real or complex. The complex roots occur in conjugate pairs such as $\lambda_k = \alpha_k + i\beta_k$ and $\bar{\lambda}_k = \alpha_k - i\beta_k$, the constants, α and β being structurally determined. The terms on the right-hand side of equations (3, 7) containing such conjugate roots can be combined pairwise and, with imaginary terms cancelled out, they will appear in the following form:

$$c_i k_{ik} e^{\alpha_k t} \cos (\beta_k t + c'_{ik} + k'_i) \qquad (3, 8)$$

As t increases, i.e. as time goes on, $\cos (\beta_k t + c'_{ik} + k'_i)$ will fluctuate between -1 and $+1$.

All exponential terms, $e^{\lambda_k t}$ and $e^{\alpha_k t}$, if λ_k or α_k are positive, will grow in their absolute magnitude as t increases, while the terms with negative λ's and α's will become smaller and smaller tending toward 0. A term with $\lambda = 0$ would, of course, remain constant and a periodic term with $\alpha = 0$ would show a periodic wave-like motion of constant unchanging amplitude.[6]

Considering the solution as a whole, one can see that if all its roots are negative the system, with the passage of time, would shrink to nothing; all exponentials and consequently all the terms on the right-hand side of (3, 7) will tend toward 0. This will not happen if at least one root is positive (or rather, not negative). The largest of the roots, if time went on and all the constants in the system remained unchanged, would finally dominate the direction and the rate of change of all the individual variables: in each one of the equations describing the development of the various parts of the economy, the particular term containing this largest positive root will, sooner or later, begin to grow faster—or fall slower—in its absolute magnitude than all the other terms taken together. If, for example, λ_1 were the largest positive root in (3, 7) in the long run, i.e. after t had become large enough, the outputs of the separate industries could with an ever smaller percentage error be computed from the following truncated set of equations:

$$X_i(t) = c_1 k_{ik} e^{\lambda_1 t} \qquad i = 1, 2, \cdots, n \quad (3, 10)$$

The relative magnitude of the outputs of any two industries would become invariant with time, i.e. constant. From (3, 10), it follows that

$$\frac{X_i(t)}{X_j(t)} = \frac{k_{i1}}{k_{j1}} \qquad (3, 11)$$

[6] In case a root, λ_k, repeats itself s times, the terms containing it will appear on the right-hand side of the i^{th} equation in (3, 7) in the following form:

$$[{}_1c_k\,{}_{11}k_{ik} + {}_2c_k({}_{21}k_{ik} + {}_{22}k_{ik}t) + \cdots + {}_sc_k({}_{s1}k_{ik} + {}_{s2}k_{ik}t + \cdots + {}_{ss}k_{ik}t^s)]e^{\lambda_k t} \qquad (3, 9)$$

The k coefficients are structurally determined while the constants, $c - s$ in number, depend upon the initial conditions.

The relative rate growth of all industries would accordingly become the same and equal to the dominant root, λ_1:

$$\frac{\dot{X}_i(t)}{X_1(t)} = \frac{c_1 k_{ik} \lambda_1 e^{\lambda_1 t}}{c_1 k_{ik} e^{\lambda t}} = \lambda_1 \qquad (3, 12)$$

It is interesting to note that both the long-run rate of growth and the equilibrium proportions between the outputs of the individual industries are dependent upon the structural properties of the economy only; because of that, they can be determined without the knowledge of any initial conditions. To determine the absolute level of outputs even for the state of 'long-run equilibrium' the knowledge of some initial state of the system is, however, indispensable.

Having mentioned these peculiar properties of the theoretical long-run dynamic equilibrium—a concept which plays a considerable role in some of the 'purer' theories of economic growth—one must at once say that. for purposes of empirical analysis of the actual economic development. they most probably will be of very little use.

The more the actual proportion of the outputs of the various industries deviates at any given point of time from the long-run equilibrium ratios corresponding to the existing structural properties of the system, the greater role will the terms in (3, 7) containing the secondary, i.e. non-dominant, roots play in the determination of the actual course of its dynamic development in the immediate future. One can easily visualize situations in which a term containing a relatively small and even the smallest root, λ_k, but carrying the relatively large (positive or negative) coefficient, $c_k k_{ik}$, will—as t increases—contribute more to the change in the output of sector X_i than a term with a large root, λ_j, but a smaller coefficient, $c_j k_{ij}$. In the long run, i.e. with a large enough t, the 'greatest' root would actually come into its own, but by then the empirical significance of the original dynamic equations based on the structural properties of the economy observed at the same time with its initial conditions might long be dead.

In a homogeneous, dynamic system with constant coefficients containing no other than n real roots, and thus showing no periodic fluctuations in its solution, each of the n variables representing the separate outputs could—depending on the initial conditions—reverse the direction of its movement up to $n - 1$ times before settling into a monotonic rise or fall gradually approaching the long-run equilibrium rate of change of the system as a whole.

IV. The Open System

The solution of the original open, i.e. non-homogeneous, system (3, 3) will be of the following general form:

$$\lambda_i(t) = c_1 k_{i1} e^{\lambda_1 t} + c_2 k_{i2} e^{\lambda_2 t} + \cdots + c_k k_{ik} e^{\lambda_k t} + \cdots + c_m k_{im} e^{\lambda_m t} + L_i(t)$$

$$i = 1, 2, \cdots, m \quad (3, 13)$$

All terms, but the last, on the right-hand side are obtained by solving (3, 3) as if it were a closed, homogeneous system, i.e. by assuming that all components of final demand, $Y_1, Y_2, \cdots, Y_m$, are equal to 0. The roots, $\lambda_1, \lambda_2, \cdots, \lambda_m$, and all the k coefficients can thus be determined by the method described above in connection with the solution of system (3, 7).

The functions $L_1(t)$, $L_2(t)$, $\cdots$, $L_m(t)$, on the other hand, represent what is called the particular part of the general solution of the non-homogeneous system (3, 3), the part which incorporates the specific effect of the final demand. The shape of these functions depends not only on the structural coefficients appearing on the left-hand side of (3, 3), but also on the specific shape of the functions, $Y_1(t)$, $Y_2(t)$, $\cdots$, $Y_m(t)$, i.e. the given changes over time of each of the separate components of final demand.

For a comprehensive discussion of the 'particular' solution of non-homogeneous systems of differential equations, the reader must again be referred to standard mathematical texts. The empirically observed or if the analysis is to be used in connection with some kind of policy choices—hypothetically prescribed shapes of the $Y(t)$ functions will often be expressible in some standard functional form. The shapes of the corresponding $L(t)$ functions would then also be of a rather simple form. The numerical computation of the relevant parameters can, in such cases, easily be standardized.

If, for example, the course of final demand for the products of each industry is represented by an exponential polynomial

$$Y_i(t) = g_{i1} e^{\mu_1 t} + g_{i2} e^{\mu_2 t} + \cdots + g_{ik} e^{\mu_k t} + \cdots + g_{iv} e^{\mu_v t}$$

$$i = 1, 2, \cdots, m \quad (3, 14)$$

the particular solution of (3, 3) will be:

$$L_i(t) = w_{i1} e^{\mu_1 t} + w_{i2} e^{\mu_2 t} + \cdots + w_{ik} e^{\mu_k t} + \cdots + w_{iv} e^{\mu_v t}$$

$$i = 1, 2, \cdots, m \quad (3, 15)$$

The numerical values of each of the constants, w_{i1}, w_{i2}, $\cdots$, depend on the magnitude of the coefficients, g_{ik}, occurring in all the given-demand

functions, $L_i(t)$, as well as on the two sets of the structural flow and stock coefficients, a and b.[7]

In case the given bill of goods is described by a set of ordinary polynomials in t:

$$Y_i(t) = g_{i0} + g_{i1}t + g_{i2}t^2 + \cdots + g_{ik}t^k + \cdots + g_{iv}t^v$$

$$i = 1, 2, \cdots, m \quad (3, 17)$$

the corresponding particular solution of (3, 3) will consist also of polynomials of the order v in t:

$$L_i(t) = w_{i0} + w_{i1}t + w_{i2}t^2 + \cdots + w_{ik}t^k + \cdots + w_{iv}t^v$$

$$i = 1, 2, \cdots, m \quad (3, 18)$$

Each of the constants, w_{i0}, w_{i1}, $\cdots$, again depends on the structural matrices $\bar{a}$, b, and the set of constants characterizing the given-demand polynomials (3, 17).[8]

After the particular solution of the open system (3, 3)—that is, the shape of the functions, $L_i(t)$, and the values of all the coefficients occurring in them has been found, the only elements in the solution (3, 13) still to be determined are, c_1, c_2, $\cdots$, c_m. As in the previously discussed solution of a homogeneous system, these constants reflect the specific state which the given system has reached in the course of its development up to some particular point or points of time.

[7] If $\bar{a}$ and b are the previously defined square matrices and g_k the *column* matrix g_{1k}, g_{2k}, $\cdots$, g_{mk} of constants which are associated in each of the equations of (3, 14) with the exponential term, $e^{\mu_k t}$, then the *column*, w_k, of the coefficients, w_{1k}, w_{2k}, $\cdots$, w_{mk}, associated with the same term $e^{\mu_k t}$ in the 'particular' solution (3, 15) can be computed by the formula,

$$w_k = [\bar{a} - \mu_k b]^{-1} g_k \qquad (3, 16)$$

[8] If $\bar{a}$ and b are the previously defined square, structural matrices while g_k represents a *column* matrix of the constants, g_{1k}, g_{2k}, $\cdots$, g_{mk}, which are associated with t^k in each of the equations (3, 17) and if, furthermore, w_k is the *column* matrix of the similarly placed constants, w_{1k}, w_{2k}, $\cdots$, w_{mk}, in solution (3, 18), then all the $(v + 1)$ such columns, w_0, w_1, $\cdots$, w_v, can be computed on the basis of the following recursive formulae:

$$\begin{aligned} w_v &= \bar{a}^{-1} g_v \\ w_{v-1} &= \bar{a}^{-1}(g_{v-1} + vbw_v) \\ &\vdots \\ w_k &= \bar{a}^{-1}(g_k + (k+1)bw_{k+1}) \\ &\vdots \\ w_o &= \bar{a}^{-1}(g_o + bw_1) \end{aligned} \qquad (3, 19)$$

Beginning at the top, one can first compute w_v, insert the magnitude thus found on the right-hand side of the next matrix equation, then compute w_{v-1} and thus work step-by-step down to w_o.

If, for example, at $t = t_o$, $X_1 = X^o{}_1$, $X_2 = X^o{}_2$, $\cdots$, $X_m = X^o{}_m$, these particular values of time and of the output variables can be entered on the right- and left-hand side of the equations (3, 13); the $L_i(t)$ terms on the right having then been written out in their explicit form such as (3, 15) or (3, 18). Thus a system of m linear equations is obtained which can be solved for the m unknown coefficients, c_1, c_2, $\cdots$, c_m.

V. Policy Decisions and the Open System

In dynamic as in the static input-output analysis,[9] consideration of the national economy as an open system offers an analytical tool particularly well suited to the making of appraisals of the material implications of alternative policy decisions.

Questions of policy can have an operational meaning only if one assumes that the structure of certain sectors of the economy can be changed. To examine the possible effects of such changes on the rest of the economy, one can remove the balance equations of the sectors subject to such change from the original, closed set. As soon as the original system is thus opened, the number of equations becomes smaller than the number of the original unknowns. The additional degrees of freedom thus acquired by the explanatory scheme finds its expression in the transfer of certain input items from the left- to the right-hand side of the remaining balance equations. In this position they constitute the given bill of goods of the new open system. If the policies in question were supposed, for example, to modify directly the structure of consumers' demand, the household equations would be those to go from the closed system and the household purchases of the products of all the other sectors of the economy would make up the final bill of goods in the newly constituted open system. If the possible effects of alternative schedules of governmental purchases of military equipment were in question, these schedules would have to appear on the right-hand side of the non-homogeneous, dynamic input-output system (3, 3) to be solved.

In interpreting the operational significance of the final demand in an open system, it is important to emphasize the exclusion of certain sets of structural constants from the left-hand side of the basic dynamic equations rather than the inclusion of some particular kind of demand on their right-hand side. The final bill of goods described by functions Y_1, Y_2, $\cdots$, Y_m in (3, 3) can, strictly speaking, comprise any demand not derivable, i.e. not explainable, on the basis of the structural input-output relationships explicitly accounted for by the flow and stock coefficients appearing on the left-hand side of the particular open system used.

[9] See Leontief, op. cit. pp. 142, 168, 205.

With some basic structural characteristics of the economy considered as given and described by the appropriate sets of the flow and capital coefficients, the problem of possible choice must, first of all, be defined in terms of the particular interval of time within which the direct effects of the policy in question on the course of the dynamic development of the given economy are to be examined. With the beginning, say t_o, and the end—let it be t_g—of this period given, our attention for the purposes of policy decisions can be centered, first of all, on the interdependence between (1) the state of the economy, i.e. the levels of all the outputs, $X^o{}_1$, $X^o{}_2$, $\cdots$, $X^o{}_m$, at the time t_o; (2) the state of the economy, similarly described, at the time t_g, and (3) the development of the final demand in the interval of time between t_o and t_g as depicted by the m functions, $Y_1(t)$, $Y_2(t)$, $\cdots$, $Y_m(t)$.

With the general form of these bill-of-goods functions chosen in advance,[10] their particular time profiles can be described in terms of an appropriate set of constants, such as g and μ in (3, 14) or g in (3, 17).

The desired interrelationship can be obtained from the general solution (3, 13) of the open system (3, 3). Let the final demand, for example, be described in terms of the m exponential polynomials (3, 14). The levels of the m outputs, $X^g{}_1$, $X^g{}_2$, $\cdots$, $X^g{}_m$, characterizing the state of the economy at the final point of time, t_g, are, according to (3, 13), determined by the following set of equations:

$$\begin{aligned} X^g{}_i = {} & c_1(X^o, g, \mu)k_{i1}e^{\lambda_1 t_g} + c_2(X^o, g, \mu)k_{i2}e^{\lambda_2 t_g} + \cdots \\ & + c_m(X^o, g, \mu)k_{im}e^{\lambda_m t_g} + w_{i1}(g, \mu)e^{\mu_1 t_g} + w_{i2}(g, \mu)e^{\mu_2 t_g} + \cdots \\ & + w_{iv}(g, \mu)e^{\mu_v t_g} \quad i = 1, 2, \cdots, m \qquad (3, 20) \end{aligned}$$

The parameters, c, are shown here to be functions of the initial levels of the individual outputs, $X^o{}_1$, $X^o{}_2$, $\cdots$, $X^o{}_m$—represented, in short, by X^o—and of all the bill-of-goods constants symbolized by g and μ; the constants, w, are shown to be dependent on g and μ. Both the c's and w's as well as the roots, λ, depend also, of course, on the magnitude of the basic sets of the structural flow and stock coefficients, a and b. This dependence will necessarily be taken into account when in the course of actual computations the c's and the w's are explicitly replaced on the right-hand side of (3, 20) by numerical functions of the $X^o{}_1$'s, the g's, and the μ's.[11]

With these substitutions completed, system (3, 20) can be considered to represent a set of m equations in $2m + u$ variables: the m initial out-

[10] As already pointed out (p. 63), many alternative types of single-valued functions, $Y_i(t)$, can be used to describe, with any desired degree of approximation, the actually observed or hypothetically assumed time shape of final demand over any given interval of time.

[11] For purposes of actual computation, the right-hand terms of (3, 20) can be simplified by putting $t_g = 0$, i.e. by conventionally putting the origin of the time count at the end of the time interval in question.

puts, $X^o{}_1$, $X^o{}_2$, $\cdots$, $X^o{}_m$; the m final outputs, $X^g{}_1$, $X^g{}_2$, $\cdots$, $X^g{}_m$; and the u constants which determine the particular shape of the m final-demand functions, $Y_1(t)$, $Y_2(t)$, $\cdots$, $Y_m(t)$, between $t = t_o$ and $t = t_g$. Providing only one constant for each such function, the number u cannot be smaller than m. Usually to provide a sufficient range of choice between alternative time paths of final demand, one would use for their description functions in which the total number of constants will be much larger than m. In the particular case discussed above, $u = m \cdot v + v$, where $m \cdot v$ is the total number of the g coefficients and v the number of the μ coefficients in (3, 14).

With $2m + u$ variables, and m relations connecting them, one can generally fix arbitrarily the values of $m + u$ of these variables, and then determine the corresponding values of the remaining m from the m equations.

For purposes of many kinds of policy decisions, the initial state of the system, i.e. its position in the point of time t_o, when the first effects of alternative policies can be expected to set in, must be considered as given; that is, determined by direct observation or possibly by unconditional prediction based on a solution of an appropriate closed dynamic system. With the set X^o of the initial output given, the range of possible alternatives is thus reduced to free determination of the values of any u of the remaining $u + m$ variables, the corresponding magnitudes of the other m being obtained through the solution of system (3, 20).

For certain purposes one would want to prescribe, for example, the final state of the economy, i.e. the levels of all outputs at the time t_g, insert them in (3, 20), and then obtain a quantitative description of all the paths which the final demand could take while leading the economy from its given original to the prescribed final state. This presupposes, of course, that u is larger than m, i.e. that the number of parameters available for the description of the changing levels of final demand between t_o and t_g exceeds the number of equations limiting their admissible courses to those which would actually connect the two fixed positions of the system.

The variety of materially significant alternatives might be very great. A low level of final demand in the first part of the time period under consideration might, for example, be substituted by choice for a high level in its later stretch, or vice versa. Demand for one type of goods can be substituted for that of another kind. Each particular path of final demand would, of course, be associated with a specific solution of the entire dynamic system; the corresponding time paths of all the individual outputs can be found by inserting the alternative bill-of-goods functions in the general solution (3, 13). The associated changes in stocks of all commodities in all the sectors of the economy can finally be computed

by simply multiplying every output function, $X_k(t)$, by all the capital coefficients, b_{1k}, b_{2k}, $\cdots$, b_{mk}.

Conjunctural discussion of special situations which might arise under difficult hypothetical assumptions lies beyond the scope of this chapter and indeed of this entire volume. The empirical description of the capital structure of the American economy in the year 1939, combined with the previously derived matrix of current input-output ratios, provides sufficient basis for the application of the analytical tool described in this chapter to the study of the dynamic properties of an actual economy.

VI. Irreversibility

The straightforward elaboration of the so-called 'acceleration principle' developed above has one particularly serious defect—it neglects the irreversibilities of the accumulation process.

The dynamic balance equations (3, 3), as they have been set up above, provide for a strict and continued maintenance within each and every sector of the economy of the stock-flow ratios determined by the appropriate sets of capital coefficients. In most instances of expanding output under conditions of fully utilized capacity, this seems to be a pretty good description—at least in a first approximation—of the observed reality. Increasing output requires, in such cases, additional 'stocks' of building equipment, of raw material, and of goods in process, all of which constitute input requirements on the investment account which, in the description of the input-output balance of the economy, must be added to the amounts of the same commodities absorbed simultaneously on current production account.

But when the rate of output of an industry declines and stock requirements are reduced accordingly, the structurally determined stock flow ratios can be maintained only through a corresponding reduction in stocks: a reduction which, if it actually is achieved, must be entered in the input-output balance of the economy as part of the positive flow available for coverage of the effective input requirements for the commodity in question. In the dynamic system (3, 3), whenever the rate of output, X_k, of some industry, k, declines, i.e. whenever $\dot{X}_k$ becomes negative, all the terms describing the investment input in that industry, $-b_{1k}\dot{X}_k$, $-b_{2k}\dot{X}_k$, $\cdots$, turn positive; therefore, in striking the input-output balance, they are added to—instead of being charged against—the 'new' production, X_1, X_2, $\cdots$.

At the time of a rapidly declining rate of production, an industry might even discontinue entirely all purchases of raw materials and satisfy its whole current input demand by living off inventories. In other instances, however, the previously accumulated stocks cannot be reduced. So-called permanent improvement to land obviously cannot be used up at all; the same applies to a large extent to buildings. Other kinds of fixed capital, such as machinery and equipment, can be consumed on current account through discontinued or re-

duced maintenance, but even this can happen only to a strictly limited extent, i.e. at a relatively slow rate. The typical irreversibility of the accumulation process finds its expression in idle capacity and surplus inventories, i.e. in unused stocks of fixed and working capital.

Non-transferability of stocks in addition to technical irreducibility constitutes another cause of the irreversibility of the accumulation process. In each equation of the dynamic system (3, 3) the entire output of one particular industry is being balanced against the consumption of its products throughout the entire economy. This implies that the surplus stocks resulting from diminished capacity utilization in any one industry can be transferred to and put to use—either as part of the required stock or as input on current account—in any other expanding, or at least not too rapidly contracting, industry. Sales of surplus machinery, leasing of vacated floor space, and liquidation of surplus inventories of raw and semi-manufactured materials through sale to other more fortunate users are obvious alternatives to the downward adjustment of stockholdings of an industry through under-maintenance and internal consumption on current account. Frequently such transfers from one sector of the economy to another prove, however, to be impracticable. More often than not the balance, or better to say the imbalance, between the available and the required stocks has to be struck for each industry or even each individual establishment separately.[12]

To fix our ideas, let us consider a case in which one and only one kind of stock, say the stock of commodity i used in industry k, i.e. S_{ik}, is irreducible At any time in which $S_{ik} = b_{ik}X_k$ and $\dot{X}_k > 0$, i.e. in which the available stock, S_{ik}, is being fully utilized and the output of X_k increases, or at least remains unchanged, the input-output balance will be adequately described by the k^{th} equation in system (3, 3). In a situation with $\dot{X}_k < 0$, that is, under conditions of falling output, this equation ceases to apply: the reduction of the stock implied by the positive magnitude of the term, $-b_{ik}\dot{X}_k$, cannot actually occur At the time of declining production, this term necessarily must be equal to 0 In other words, $b_{ik} = 0$ whenever $\dot{X}_k < 0$; that means that whenever $\dot{X}_k < 0$ the dynamic input-output balance of the whole economy has to be described not by the system (3, 3) but by another set of equations which can be obtained from the first by simply putting $b_{ik} = 0$.[13] The original solution (3, 13) must, for the time during which $\dot{X}_k < 0$, accordingly be replaced by an analogous but numerically different solution for which all the constants are computed on the assumption that $b_{ik} = 0$, while the rest of the structural b (stock) coefficients and all the a (flow) coefficients remain the same as before.

In some cases the investment process is not entirely irreversible but the previously accumulated stocks can be consumed only at a limited rate.

[12] The unequal degree of transferability of various commodities between different localities constitutes the basis of interregional input-output analysis described in Chapters 4 and 5.

[13] In this particular instance only the i^{th} equation will be affected by this change.

Such a downward limit to negative accumulation could, for example, be set by the rate of normal replacement omitted in order to allow the stocks to run down. If $a_{ik}X_k$ represents such replacement requirements,[14] then, with $\dot{X}_k < 0$, the rate of negative accumulation, $b_{ik}\dot{X}_k$, cannot be smaller in its absolute value than $a_{ik}X_k$. As soon as $-a_{ik}X_k - b_{ik}\dot{X}_k$ becomes positive, i.e. as soon as the absolute magnitude of the second term becomes greater than that of the first, idle capital, that is surplus capacity, is bound to appear. The i^{th} balance equation in system (3, 3) has to be replaced by a new one in which not only the coefficient, b_{ik}, but also a_{ik} is set to equal 0: the rate of stock reduction, $-b_{ik}\dot{X}_k$, will be just large enough to equal the replacement requirements.

In this second phase of dynamic change characterized by the existence of a surplus stock, the capital-output relationship described in (3, 2a) cannot be used any more to determine the magnitude of the total stock, $S_{ik}(t)$. The partly idle stock leads, in these conditions, a quasi-independent existence. In the case of complete irreversibility, it would remain constant through the entire duration of that regime. In the second case it would decline at a rate equal to the current maintenance requirement, $a_{ik}X_k$. The size of total stock at any one point of time, t, can, during that second phase, be computed from the following formula:

$$S_{ik}(t) = S_{ik}(t_1) - a_{ik}\int_{t_1}^{t} \dot{X}_k(t)dt \qquad (3, 21)$$

t_1 marks the point of transition from the first to the second regime, and $S_{ik}(t_1)$ represents the magnitude of stock at the time t_1. The rate of change of the output, $\dot{X}_k(t)$, is determined by the solution (3, 13) of the modified system (3, 3) with $a_{ik} = 0$ and $b_{ik} = 0$.

The difference between the total available and the required stock of commodity i in industry k—let it be called the surplus or idle stock and designated by $\bar{S}_{ik}(t)$—can be computed as follows:

$$\bar{S}_{ik}(t) = S_{ik}(t) - b_{ik}X_{ik}(t) \qquad (3, 22)$$

$\bar{S}_{ik}(t)$ cannot be negative. With $\bar{S}_{ik}(t) = 0$, the economy finds itself in the first phase; with $\bar{S}_{ik}(t) > 0$, in the second.

The transition from the first to the second phase must take place at that point of time in which—if the system had continued to move along the first-phase path—a positive surplus stock, $\bar{S}_{ik}$, would have appeared. In the case of absolute irreversibility discussed above, this would occur whenever the output, $X_k(t)$—with the stock, $S_{ik}(t)$, still fully utilized—turns from expansion to contraction, i.e. at the point of time, t_1, in which the state of industry k is characterized by the following three conditions:

$$\bar{S}_{ik}(t_1) = 0 \quad {}_1\ddot{X}_k(t_1) < 0 \quad {}_1\dot{X}_k(t_1) = 0 \qquad (3, 23)$$

[14] In the case of most fixed capital goods, the entire consumption on current account consists of replacement inputs.

The left-hand subscript '1' indicates that both the first and the second derivatives, ${}_1\dot{X}_k(t)$ and ${}_1\ddot{X}_k(t)$, refer to the 'rate of change' and the 'rate of the rate of change' of output $X_k(k)$ as determined by the dynamic equation governing the behavior of the economy in the first phase, i.e. under conditions of $\bar{S}_1(t) = 0$. And the combined conditions indicate a situation in which the output, X_k, after rising up to the time, t_1, would have begun to fall if allowed to move further along the first-phase path. It is the last equation which in the form ${}_1\dot{X}_k(t) = 0$ must be used to determine the terminal point of time, t_1.

The levels of outputs, $X_1(t_1)$, $X_2(t_1)$, $\cdots$, reached at the end of the just-concluded first phase represent also the initial condition which must be used in the determination of the path to be followed by the economy in the course of the second phase.

That second phase lasts as long as $\bar{S}_{ik}(t) > 0$ and it terminates if and when, at some point of time, t_2,

$$\bar{S}_{ik}(t_2) = 0 \quad {}_2\dot{X}_k(t_2) > 0 \tag{3, 24}$$

Note the left-hand subscript '2' which means that ${}_2\dot{X}_k(t_2)$ is computed from the equations governing the behavior of the economy in the second phase, i.e. under the assumption of $b_{ik} = 0$. These conditions describe a situation in which the output, $X_k(t)$, rises and reaches, at the time t_2, the level at which the entire previously idle stock, $\bar{S}_{ik}$, becomes absorbed into active use.

Since in the course of this second phase, and just up to the time $t_2, \bar{S}_{ik}(t) > 0$, it is the equation $\bar{S}_{ik}(t) = 0$ which can be used to determine the point of time, t_2, marking its end. Here again the outputs, $X_1(t_2)$, $X_2(t_2)$, $\cdots$, supply the initial conditions which must be inserted in the dynamic solution charting the further movement of the economy now again under the original first-phase regime.

To derive the general dynamic law of change for an economy with limited reversibility, one must thus have:

1. A set of general solutions of the alternative dynamic systems appropriate to all the various phases through which it will pass in the course of its development.

2. A set of rules specifying the conditions under which the process is switched from one phase to the next.

With one irreversible stock, the number of possible alternative phases is two; with two irreversible stocks, it is four.[15] In general, with n irreversible stocks, the total number of all possible phases is 2^n.

Actually, starting with a given initial state, an economy will hardly ever pass, in the course of its subsequent change, through all the theoretically possible phases. As an example of a development which would never abandon the original tracks, one can think of a homogeneous system which from the outset finds itself in the position of the 'long-run equilibrium' described on pages 59

[15] (1) Both stocks fully utilized, (2) the first partly, the second fully utilized, (3) both partly utilized, (4) the first fully, the second partly utilized.

to 62. With a positive dominant root, the course of its development will be marked by practically uniform growth of all the outputs. The proportional expansion of fully utilized stocks could, in this case, be interrupted only by fundamental structural variation. Otherwise the original full-utilization phase would continue *ad infinitum*.

VII. Defects of the Multi-phase Theory

The theory of the multi-phase process as presented above promises to become a useful tool in the empirical analysis of economic change. However it has certain defects which, even if they were negligible from the point of view of practical application, deserve special consideration since their examination will throw light on the internal logic of the theory of economic change in general and of the dynamic input-output analysis in particular.

These defects show up in the difficulties which may arise in the application of the rules under which the system is switched from one phase of its development to the next.

Consider an economy with a strictly irreversible stock, S_{ik}, at time t_1, i.e. at the point of transition from the first—the full-utilization—phase to the second phase characterized by the presence of a positive surplus stock, $\bar{S}_{ik}$.

The last two conditions in (3, 23) describe the movement of the output X_k at the time t_1 in terms of the dynamic law governing the first, full-utilization phase which is supposed to end exactly at that point of time, t_1. In its very next step, the economy will already have to follow the alternative, second path derived on the assumption of the presence of a positive surplus stock, $\bar{S}_{ik}$.

If in following this second path the output, X_k, proceeded to fall, idle stock would actually appear and the system would remain under the jurisdiction of the second-phase law up to the time when the idle stocks would again be absorbed in the course of some further developments.

But what would happen if, starting from the initial position of the economy at t_1, the second-phase law were to indicate a rise rather than a fall in the output X_k?

Increasing output at the time of full utilization of stock cannot lead to idle capacity; on the contrary, it implies additional investment. But additional investment can take place only under the rule of the first, not of the second, phase regime. This latter, however, cannot be established since, under conditions existing at time t_1, it would result in a declining output of X_k and a simultaneous appearance of idle stock.

In short, we are facing here a basic contradiction. A rule which—if applied in the existing situation—in effect requires that an industry,

already operating at full capacity, should contract so long as it uses all its capital, but should expand if surplus capacity were to appear, obviously cannot be followed consistently.[16]

[16] Let ${}_1X_k(t_1)$ be the output of commodity k achieved under the law of the first phase at the point of transition, t_1 and let ${}_2X_k(t_1 + \Delta)$ represent the level which that output would reach at a slightly later point of time, $t_1 + \Delta$, if it had followed the second-phase law of change from t_1 onward.

Since the positive increment of time, Δ, is very small, one can use, for the purpose of comparing ${}_1X_k(t_1)$ and ${}_2X_k(t_1 + \Delta)$, the following three-term expansion:

$$ {}_2X_k(t + \Delta) = {}_2X_k(t_1) + {}_2\dot{X}_k(t_1)\Delta + {}_2\ddot{X}_k(t_1)\Delta^2 \cdots \tag{3, 25} $$

${}_2X_k(t_1)$ by definition equals ${}_1X_k(t_1)$. To determine the value of ${}_2\dot{X}_k(t_1)$, let us first solve system (3, 3) for any one $\dot{X}_k$ in terms of the outputs, $X_1, X_2, \cdots, X_m$:

$$ {}_1\dot{X}_k = \frac{\begin{vmatrix} b_{11} & b_{12} & \cdots & f_1(X) & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & f_2(X) & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{i1} & b_{i2} & \cdots & f_i(X) & \cdots & b_{im} \\ \vdots & & & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & f_m(X) & \cdots & b_{mm} \end{vmatrix}}{\begin{vmatrix} b_{11} & b_{12} & \cdots & b_{1k} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2k} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ik} & \cdots & b_{im} \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mk} & \cdots & b_{mm} \end{vmatrix}} \tag{3, 26} $$

The expressions, $f_1(X), f_2(X), \cdots$, which make up the k^{th} column of the determinant on the top are linear functions of $X_1, X_2, \cdots, X_m$:

$$ f_i(X) = Y_i - X_i + \sum_{l=1}^{m} a_{il}X_l \qquad i = 1, 2, \cdots, m \tag{3, 27} $$

The outputs, $X_1, X_2, \cdots, X_m$, refer, of course, to the same point of time, say t_1, as the derivative ${}_1\dot{X}_k$. The left-hand subscript '1' indicates that the relation (3, 26) describes the condition prevailing in the first, full utilization phase. To determine the value which ${}_2\dot{X}_k(t)$ would have had at the same point of time, t_1, under the rules of the second phase, it is only necessary to change the right hand side of (3, 26) to the extent of putting $b_{ik} = 0$ in the k^{th} column of the determinant on the bottom, the top determinant remaining the same as before.

But, according to (3, 23), ${}_1\dot{X}_k(t_1) = 0$; this means that at $t = t_1$, the top determinant in (3, 26) equals 0. Barring the special case in which the bottom determinant vanishes when 0 is substituted for the original value of b_{ik} on the bottom, ${}_2\dot{X}_k(t_1) = 0$ if ${}_1\dot{X}_k(t_1) = 0$.

Returning to (3, 25), we see that with ${}_2X_k(t_1) = {}_1X_k(t_1)$ and ${}_2\dot{X}_k(t_1) = 0$,

$$ {}_2X_k(t_1 + \Delta) < {}_1X_k(t_1) \quad \text{if} \quad {}_2\ddot{X}_k(t_1) < 0 $$

and

$$ {}_2X_k(t_1 + \Delta) > {}_1X_k(t_1) \quad \text{if} \quad {}_2\ddot{X}_k(t_2) > 0 \tag{3, 28} $$

The transition from the first to the second phase can take place without difficulties if ${}_2\ddot{X}_k(t_1) < 0$ but a contradictory situation arises if ${}_2\ddot{X}_k(t_1) > 0$ If ${}_2\ddot{X}_k(t) = 0$ the same inequalities will hold for the third derivative, ${}_2X_k(t_1)$.

A similar contradiction might arise at a point of transition from the second back to the first phase. Having expanded its production up to the point where all the previously unused capacity has been absorbed, an industry conceivably can find itself in a situation in which the new regime, setting in as soon as idle stocks have disappeared, requires not a further increase but rather an immediate reduction of its output.

Before we turn to the consideration of the significance of this theoretical impasse, it is well to observe that in a large system containing many separate sectors the contradictory situations described above are not very likely to occur: the solution of the corresponding large systems of differential equations will depend on the numerical magnitudes of a great many structural coefficients. In the simple example considered above, the difference between the solution of the first- and the second-phase system hinges on the change in the value of one single coefficient, $b_{.k}$, which turns into 0 in the second phase. In other, more complicated cases in which a given industry uses more than one irreversible stock at the same time, two, three, or more coefficients might change their values simultaneously. However, so long as their number remains much smaller than the total number of all the structural constants in the entire system, such change will most likely result only in a relatively small difference between the two solutions corresponding to the two adjoining phases. This means that in the immediate vicinity of the point of transition the movement of the economy in general and of the critical output in particular will experience only a slight change of direction. If, under the first-phase law, the output, X_k, were about to begin to fall immediately after the point of transition, t_1, in a large system it will go down in the initial period of the second-phase rule too. Similar near-continuity can be expected to prevail also at the point of a changeover in the opposite direction.

The methodological significance of the inconsistent switching rules can be better understood if one realizes that the impasse caused by contradiction as described above finds its counterpart in possible indeterminacy resulting from too much choice. Consider, for example, an ecoomic system smoothly moving along the path prescribed by the solution of a set of dynamic balance equations which neither assumes the existence nor provides for a necessary appearance of surplus stocks. So long as all the outputs involving the use of potentially irreducible stocks actually expand, the explanation of the observed dynamic process, given in terms of the corresponding mathematical system of the appropriate differential equations, seems to be entirely satisfactory.

A closer examination of this theoretical system might show, however, that if—at some arbitrarily chosen point of time, t_1—the equations were modified so as to correspond to the assumption that industry k suddenly begins to operate under conditions of surplus capacity, the new path,

followed by the economy from t_1 onward, would actually indicate a reduction in the output of that industry and simultaneous appearance of idle capacity.

Once the existence of this second alternative has been admitted, the original set of equations and its solution can be accepted as an explanation of the actually observed process only in a qualified sense.

One, it could be called the preliminary, justification of such an acceptance might be based on the fact that 'it works,' i.e. on the fact that the observed economic system actually follows the original path without ever deviating into the second. The final vindication of the original theory would have to wait, however, for an explicit statement of the specific reasons why such deviation does not actually happen. This means that it would require the formulation of a more general theory, a theory which would probe into the background of some of the relationships taken for granted by the present theory and explain, that is derive, them in terms of some more fundamental factors.

If, for example, the theory described above referred not to a free or a quasi-free exchange economy but to a centrally guided system, the choice between the two alternative dynamic systems would have to be made on the deeper level of analysis involving consideration of the final objectives of the planning authority and possibly an application of certain maximizing rules. In terms of these principles, the deviation into the second, alternative path would as one actually has reason to believe—be explicitly rejected.

In the case of the apparent contradiction described before, only an appeal to the same, more basic consideration would obviously lead to a correct solution, a solution which incidentally will necessarily involve an actual increase in some stocks which already exceed the capacity requirements computed on the basis of current production rates.

Returning to the unplanned economy that operates in a quasi-automatic manner, the preliminary rejection of the alternative path in the latent indeterminacy case will also have to be ultimately justified in terms of a more general theory, i.e. a theory entering deeper layers of structural relationships. It is very possible that in the explanation of the ordinary course of dynamic change the role of more general analysis could still be limited to that of an arbiter between alternative solutions reached on the lower level of refinement. In certain special circumstances, the coarser approach might fail altogether and the more detailed analysis has to take over entirely. Such is the case of the contradictory switching rule.

A more detailed analysis of the internal operating conditions of the individual sectors of the economy might, for example, show that so long as it is subject to apparently contradictory impulses, the output of the particular industry in question will simply remain constant while the

rest of the system will move on. Such, for example, would be the case if, under conditions of considerable stress, an industry could, for a short time, operate beyond the limits of its normal capacity, i.e. with stocks inferior to those computed on the basis of the standard capital coefficients, but then nearly at once would contract again, over-shoot in the downward direction only to turn back again.

Essentially this means the introduction, in our dynamic system, of a third, intermediary phase, a regime which would last up to the point at which—as a result of changes in the outputs of all the other industries and incidental variations in stocks—the second, the idle capacity, phase could take over without internal contradiction.[17]

Even without entering into a detailed discussion of such a compromise regime, it can be shown that it necessarily will involve accumulation somewhere in the economy of surplus stocks.

The constancy of the critical output, X_k, would have to be described in terms of a separate equation, $X_k = X^1{}_k$, where $X^1{}_k$ represents its level at the beginning of the phase, i.e. at the time t_1. But with the introduction of such an additional equation into the fundamental set (3, 3) of the dynamic balance equations, some other equation would have to be dropped. Otherwise the system as a whole could not be satisfied. Since each equation reflects the balance between the total available outputs and the required inputs of one particular commodity, the omission of any one of them means the admission of the possible and practically the necessary accumulation of a surplus stock of at least one of the commodities in question.

From here on, the theoretical analysis, if pursued any further, is bound to be highly speculative. Only a large amount of additional empirical information—very different in kind from that for the use of which the theory developed in the main part of this chapter has been designed—will be able to give it the necessary direction.

MATHEMATICAL NOTE 1

General and Special Solution of a Simple Dynamic Input-Output System Containing Stocks and Flows but No Lags

This note contains a systematic step-by-step description of the solution of a simple two-part dynamic input-output system. A reader acquainted with elementary algebra and familiar with the rudiments of

[17] The introduction of an additional phase can, under certain assumptions, be described as a recourse to higher-order systems of differential equations. The higher terms of such equations would influence the course of events (i.e. the solution of the system), however, at the critical turning points only.

calculus will find in this example a concrete illustration of the analytical and computational procedures discussed verbally in Chapter 3.

I

To illustrate the analysis of a dynamic system presented in Chapter 3 one can consider the following set of two homogeneous linear differential equations of the first order with constant coefficients representing a dynamic input-output equilibrium of a closed two-sector economy.

$$\begin{aligned} X_1 - a_{11}X_1 - a_{12}X_2 - b_{11}\dot{X}_1 - b_{12}\dot{X}_2 &= 0 \\ -a_{21}X_1 - a_{21}X_2 + X_2 - b_{21}\dot{X}_1 - b_{22}\dot{X}_2 &= 0 \end{aligned} \qquad (3n, 1)$$

To determine the magnitude of the two roots, λ_1 and λ_2, and the various other constants involved in its general solution,

$$\begin{aligned} X_1 &= c_1k_{11}e^{\lambda_1 t} + c_2k_{12}e^{\lambda_2 t} \\ X_2 &= c_1k_{21}e^{\lambda_1 t} + c_2k_{22}e^{\lambda_2 t} \end{aligned} \qquad (3n, 2)$$

let us make an anticipatory assumption that,

$$\begin{aligned} X_1 &= k_1e^{\lambda t} \\ X_2 &= k_2e^{\lambda t} \end{aligned} \qquad (3n, 3a)$$

and consequently that,

$$\begin{aligned} \dot{X}_1 &= \lambda k_1e^{\lambda t} \\ \dot{X}_2 &= \lambda k_2e^{\lambda t} \end{aligned} \qquad (3n, 3b)$$

Substituting from (3n, 3a) and (3n, 3b) in (3n, 1), we have

$$\begin{aligned} e^{\lambda t}\{k_1(1 - a_{11} - b_{11}\lambda) + k_2(-a_{12} - b_{12}\lambda)\} &= 0 \\ e^{\lambda t}\{k_1(-a_{21} - b_{21}\lambda) + k_2(1 - a_{22} - b_{22}\lambda)\} &= 0 \end{aligned} \qquad (3n, 4a)$$

Considered as a system of two homogeneous equations with two unknowns, k_1 and k_2, (3n, 4a) can be rewritten in the following form:

$$\begin{aligned} \frac{k_1}{k_2} &= \frac{a_{12} + b_{12}\lambda}{1 - a_{11} - b_{11}\lambda} \\ \frac{k_1}{k_2} &= \frac{1 - a_{22} - b_{22}\lambda}{a_{21} + b_{21}\lambda} \end{aligned} \qquad (3n, 4b)$$

Comparing these two equations, we see that they can be consistent only if

$$\frac{a_{12} + b_{12}\lambda}{1 - a_{11} - b_{11}\lambda} = \frac{1 - a_{22} - b_{22}\lambda}{a_{21} + b_{21}\lambda} \qquad (3n, 5)$$

Multiplying out and shifting all terms over to the left side, we have

$$[b_{22}b_{11} - b_{12}b_{21}]\lambda^2 - [(1 - a_{11})b_{22} + (1 - a_{22})b_{11} + a_{12}b_{21} + a_{21}b_{12}]\lambda + [(1 - a_{11})(1 - a_{22}) - a_{21}a_{12}] = 0 \quad \text{(3n, 6a)}$$

or, in short,

$$A\lambda^2 - B\lambda + C = 0 \quad \text{(3n, 6b)}$$

where A, B, and C represent the similarly placed expression in structural constants.

According to the well-known formula, this quadratic equation is satisfied by the following two values of λ:

$$\begin{aligned} \lambda_1 &= \frac{1}{2A}\left(B + \sqrt{B^2 - 4AC}\right) \\ \lambda_2 &= \frac{1}{2A}\left(B - \sqrt{B^2 - 4AC}\right) \end{aligned} \quad \text{(3n, 7)}$$

The special case of $\lambda_1 = \lambda_2$ can be disregarded since, as shown below, in a two-part economic system $\sqrt{B^2 - 4AC} > 0$ and thus $\lambda_1 \neq \lambda_2$.

Since both λ_1 and λ_2 will satisfy (3n, 4b), two different ratios of the coefficients, k_1 and k_2, can be computed from that formula. Using a second subscript to distinguish the two pairs and introducing two arbitrary constants, c_1 and c_2, to indicate that a proportional change in both coefficients belonging to the same pair is admissible, since it would not affect their ratios, one can write,

$$\begin{aligned} k_{11} &= c_1(a_{12} + b_{12}\lambda_1) \qquad & k_{12} &= c_2(a_{12} + b_{12}\lambda_2) \\ k_{21} &= c_1(1 - a_{11} - b_{11}\lambda_1) \qquad & k_{22} &= c_2(1 - a_{11} - b_{11}\lambda_2) \end{aligned} \quad \text{(3n, 8)}$$

These are based on the first of the two equations in (3n, 4b); the second, because of (3n, 5), would obviously give the same ratio, $\dfrac{k_1}{k_2}\cdot$

Substitution from (3n, 8) in (3n, 3a) gives two 'constituent' solutions of the original system (3n, 1), one for each of the two roots λ_1 and λ_2.

$$\begin{aligned} X_1 &= c_1k_{11}e^{\lambda_1 t} \qquad & X_1 &= c_2k_{12}e^{\lambda_2 t} \\ &\text{and} \\ X_2 &= c_1k_{21}e^{\lambda_1 t} \qquad & X_2 &= c_2k_{22}e^{\lambda_2 t} \end{aligned} \quad \text{(3n, 9)}$$

Since each pair of these expressions for X_1 and X_2 taken separately satisfies (3n, 1), an additive combination of both will satisfy it too. Thus we arrive at the general solution (3n, 2) with all coefficients—but c_1 and c_2—explicitly described in terms of the two sets of the structural

constants which make up the empirical background of the original dynamic system (3n, 1).

The numerical magnitudes of c_1 and c_2 can be determined only through the introduction of additional empirical data consisting of information on the values of the variables or on the magnitude of their changes at some specific points of time.

For example, let $X_1(t_o)$ and $X_2(t_o)$ represent the values of $X_1(t)$ and $X_2(t)$ observed at the point of time, $t = t_o$. Substituted in (3n, 2) they give,

$$\begin{aligned} X_1(t_o) &= c_1k_{11}e^{\lambda_1 t_o} + c_2k_{12}e^{\lambda_2 t_o} \\ X_2(t_o) &= c_1k_{21}e^{\lambda_1 t_o} + c_2k_{22}e^{\lambda_2 t_o} \end{aligned} \qquad (3n, 10)$$

The equations can be solved for c_1 and c_2:

$$\begin{aligned} c_1 &= \frac{k_{22}X_1(t_o) - k_{12}X_2(t_o)}{(k_{22}k_{11} - k_{12}k_{21})e^{\lambda_1 t_o}} \\ c_2 &= \frac{k_{11}X_2(t_o) - k_{21}X_1(t_o)}{(k_{22}k_{11} - k_{12}k_{21})e^{\lambda_2 t_o}} \end{aligned} \qquad (3n, 11)$$

These formulae can be simplified if one conventionally decides to count the time from the point, t_o, from which the initial state of the system happens to be given: If $t_o = o$, each of the exponential expressions on the right-hand side is reduced to 1.

Substituting for A, B, and C, their explicit definitions in terms of the a's and b's as shown in (3n, 6a), one can verify the following identity:

$$\begin{aligned} B^2 - 4AC = {} & [(1 - a_{11})b_{22} - (1 - a_{22})b_{11} - a_{21}b_{12} + a_{12}b_{21}]^2 \\ & + 4[(1 - a_{11})b_{22}a_{21}b_{12} + (1 - a_{22})b_{11}a_{12}b_{21} + b_{22}b_{11}a_{21}a_{12} \\ & + b_{12}b_{21}(1 - a_{11})(1 - a_{22})] \end{aligned} \qquad (3n, 12)$$

The first term on the right-hand side is positive because it is a square; the second, since $(1 - a_{11})$ and $(1 - a_{22})$, representing the fraction of the outputs of each of the two industries not absorbed on current account, are by themselves also essentially positive.

Systems consisting of three or more separate sectors can, however, have complex roots and thus contain periodic components in their solutions.

II

The general solution of an open, i.e. non-homogeneous, system with 'final demand' functions of an exponential type on the right-hand side:

$$\begin{aligned} X_1 - a_{11}X_1 - a_{12}X_2 - b_{11}\dot{X}_1 - b_{12}\dot{X}_2 - y_1 &\equiv g_{11}e^{\mu_1 t} + g_{12}e^{\mu_2 t} \\ X_2 - a_{21}X_1 - a_{22}X_2 - b_{21}\dot{X}_1 - b_{22}\dot{X}_2 = y_2 &\equiv g_{21}e^{\mu_1 t} + g_{22}e^{\mu_2 t} \end{aligned} \tag{3n, 13}$$

can be written as follows:

$$\begin{aligned} X_1 &= c_1k_{11}e^{\lambda_1 t} + c_2k_{12}e^{\lambda_2 t} + w_{11}e^{\mu_1 t} + w_{12}e^{\mu_2 t} \\ X_2 &= c_1k_{21}e^{\lambda_1 t} + c_2k_{22}e^{\lambda_2 t} + w_{21}e^{\mu_1 t} + w_{22}e^{\mu_2 t} \end{aligned} \tag{3n, 14}$$

The first two right-hand terms in each of these equations are obtained from solution (3n, 2) of the corresponding homogeneous system (3n, 1). the roots λ_1 and λ_2 and the coefficients, k_{11}, k_{12}, k_{21}, and k_{22}, are computed by equations (3n, 7) and (3n, 8).

The last two terms represent the particular solution of (3n, 13). The constants μ_1 and μ_2 are known; to determine w_{11}, w_{12}, w_{21}, and w_{22} assume that,

$$\begin{aligned} X_1 &= w_{11}e^{\mu_1 t} + w_{12}e^{\mu_2 t} \\ X_2 &= w_{21}e^{\mu_1 t} + w_{22}e^{\mu_2 t} \end{aligned} \tag{3n, 15}$$

Substitute these values of X_1 and X_2 and the corresponding values of the derivatives $\dot{X}_1$ and $\dot{X}_2$ in (3n, 13), and group the terms containing $e^{\mu_1 t}$ separately from those containing $e^{\mu_2 t}$. If the two resulting equations,

$$\begin{aligned} &[(1 - a_{11} - b_{11}\mu_1)w_{11} - (a_{12} + b_{12}\mu_1)w_{21} - g_{11}]e^{\mu_1 t} \\ &\qquad + [(1 - a_{11} - b_{11}\mu_2)w_{12} - (a_{12} + b_{12}\mu_2)w_{22} - g_{12}]e^{\mu_2 t} = 0 \\ &[-(a_{21} + b_{21}\mu_1)w_{11} + (1 - a_{22} - b_{22}\mu_1)w_{21} - g_{21}]e^{\mu_1 t} \\ &\qquad + [-(a_{21} + b_{21}\mu_2)w_{12} + (1 - a_{22} - b_{22}\mu_2)w_{22} - g_{22}]e^{\mu_2 t} = 0 \end{aligned} \tag{3n, 16}$$

are to hold for all possible values of t, each one of the four expressions enclosed in square brackets taken separately must equal 0. Thus we have four equations of which the first and the third can be solved for w_{11} and w_{21} while the second and the fourth can be solved for w_{12} and w_{22}:

$$\begin{aligned} w_{11} &= \frac{g_{11}(1 - a_{22} - b_{22}\mu_1) + g_{21}(a_{12} + b_{12}\mu_1)}{(1 - a_{11} - b_{11}\mu_1)(1 - a_{22} - b_{22}\mu_1) - (a_{12} + b_{12}\mu_1)(a_{21} + b_{21}\mu_1)} \\ w_{21} &= \frac{g_{21}(1 - a_{11} - b_{11}\mu_1) + g_{11}(a_{21} + b_{21}\mu_1)}{(1 - a_{11} - b_{11}\mu_1)(1 - a_{22} - b_{22}\mu_1) - (a_{12} + b_{12}\mu_1)(a_{21} + b_{21}\mu_1)} \\ w_{12} &= \frac{g_{12}(1 - a_{22} - b_{22}\mu_2) + g_{22}(a_{12} + b_{12}\mu_2)}{(1 - a_{11} - b_{11}\mu_2)(1 - a_{22} - b_{22}\mu_2) - (a_{12} + b_{12}\mu_2)(a_{21} + b_{21}\mu_2)} \\ w_{22} &= \frac{g_{22}(1 - a_{11} - b_{11}\mu_2) + g_{12}(a_{21} + b_{21}\mu_2)}{(1 - a_{11} - b_{11}\mu_2)(1 - a_{22} - b_{22}\mu_2) - (a_{12} + b_{12}\mu_2)(a_{21} + b_{21}\mu_2)} \end{aligned} \tag{3n, 17}$$

The constants c_1 and c_2 are finally determined on the basis of the given initial conditions. If, for example, at $t = 0$, $X_1 = X^o{}_1$, and $X_2 = X^o{}_2$, the insertion of these particular values of the three variables in (3n, 14) gives:

$$
\begin{aligned}
X^o{}_1 &= c_1 k_{11} e^{\lambda_1 t_0} + c_2 k_{12} e^{\lambda_2 t_0} + w_{11} e^{\mu_1 t_0} + w_{12} e^{\mu_2 t_0} \\
X^o{}_2 &= c_1 k_{21} e^{\lambda_1 t_0} + c_2 k_{22} e^{\lambda_2 t_0} + w_{21} e^{\mu_1 t_0} + w_{22} e^{\mu_2 t_0}
\end{aligned} \qquad (3n, 18)
$$

Solved for c_1 and c_2 these two equations yield,

$$
\begin{aligned}
c_1 &= \frac{(X^o{}_1 - w_{11} e^{\mu_1 t_0} - w_{12} e^{\mu_2 t_0})k_{22} - (X^o{}_2 - w_{21} e^{\mu_1 t_0} - w_{22} e^{\mu_2 t_0})k_{12}}{(k_{11}k_{22} - k_{12}k_{21}) e^{\lambda_1 t_0}}, \\
c_2 &= \frac{(X^o{}_2 - w_{21} e^{\mu_1 t_0} - w_{22} e^{\mu_2 t_0})k_{11} - (X^o{}_1 - w_{11} e^{\mu_1 t_0} - w_{12} e^{\mu_2 t_0})k_{21}}{(k_{11}k_{22} - k_{12}k_{21}) e^{\lambda_2 t_0}}
\end{aligned} \qquad (3n, 19)
$$

III

An analogous procedure can be used in solving a non-homogeneous system in which the bill of goods is represented by ordinary polynomials. The solution of

$$
\begin{aligned}
X_1 - a_{11}X_1 - a_{12}X_2 - b_{11}\dot{X}_1 - b_{12}\dot{X}_2 &= y_1 \equiv g_{10} + g_{11}t + g_{12}t^2 \\
X_2 - a_{21}X_1 - a_{22}X_2 - b_{21}\dot{X}_1 - b_{22}\dot{X}_2 &= y_2 \equiv g_{20} + g_{21}t + g_{22}t^2
\end{aligned} \qquad (3n, 20)
$$

is of the following form:

$$
\begin{aligned}
X_1 &= c_1 k_{11} e^{\lambda_1 t} + c_2 k_{12} e^{\lambda_2 t} + w_{10} + w_{11}t + w_{12}t^2 \\
X_2 &= c_1 k_{21} e^{\lambda_1 t} + c_2 k_{22} e^{\lambda_2 t} + w_{20} + w_{21}t + w_{22}t^2
\end{aligned} \qquad (3n, 21)
$$

Again λ_1, λ_2, k_{11}, k_{12}, k_{21}, and k_{22} are defined by (3n, 7) and (3n, 8).

To determine the constants occurring in the last three terms of the two equations, let

$$
\begin{aligned}
X_1 &= w_{10} + w_{11}t + w_{12}t^2 \\
X_2 &= w_{20} + w_{21}t + w_{22}t^2
\end{aligned} \qquad (3n, 22)
$$

and substitute those values of X_1, X_2 and of the corresponding derivatives in (3n, 20). In each of the two resulting equations (which are not being written out here because of their great length), the terms associated with t^2, t, and those not containing the variable, t, can be segregated in separate brackets. Each one of these bracketed expressions (three in each equation) must equal 0, if the two equations are to hold for all values of t. Thus a system of six equations is obtained, which can be solved for w_{10}, w_{11}, w_{12}, w_{20}, w_{21}, and w_{22}.

To determine the magnitudes of the two 'constants of integration,' c_1 and c_2, corresponding to the initial condition of the system described,

for example, in terms of the particular levels of output $X^o{}_1$ and $X^o{}_2$ reached by the two industries at the time $t = t_o$ it is only necessary to insert $t = t_o$ on the right- and $X_1 = X^o{}_1$ and $X_2 = X^o{}_2$ on the left-hand side of the equation (3n, 21).

MATHEMATICAL NOTE 2

Formulation and Solution of Dynamic Input-Output Systems Containing Flows, Stocks, and Structural Lags

I

The similarity which exists between some of the more obvious properties of dynamic systems described in terms of difference—and those defined with the help of differential—equations has led to a nearly interchangeable use of these two types of mathematical formulation in modern business-cycle theory and the recently much debated theory of long-run economic growth. So far as conventional model building is concerned, that is, so long as the theoretical argument is aimed primarily at deriving general implications of certain simple theoretical assumptions, one of these two alternative methods of introducing dynamics into the system can serve as well as the other. Both can lead to periodic solutions, i.e. generate cyclical fluctuations or—with a properly chosen structural assumption—produce progressively expanding or evenly contracting long-run 'trends.'

Whenever the theoretical framework is designed with the specific purpose of incorporating in its mathematical formulation directly observed empirical parameters, the distinction between the difference and the differential equations acquires crucial importance.

In the particular instance of dynamic input-output analysis, the empirically observed structural stock-flow ratios, which are so fundamental to the explanation of the investment process, lead directly to relationships between the *levels* of output of the various industries and the *rates of change* of these outputs, i.e. to differential equations. On the other hand, other observations establish the existence throughout the system of structural lags. Specifically in each particular production process the determination of the levels of required inputs by the level of finished output involves, more often than not, structurally necessary time gaps between the two—a relationship to which one can give direct analytical expression only by setting up corresponding sets of difference equations.

To preclude a possibly over-simplified interpretation of these remarks, it is necessary to admit that the notion of immediate observation like that of direct measurement is conditioned by the general level of concreteness on which the particular analysis is being carried out. A rela-

tionship looked upon as a basic structural characteristic of the system in the light of one given kind of empirical information will, in the context of a closer, more differentiated description, be treated as a derived property, explainable in terms of more detailed primary data. Some of the structural lags used in current descriptions of empirical input-output relationships imply, for example, the existence of causation operating over a gap of time—a somewhat mysterious relationship which in terms of a more detailed fine-grained observation will prove to be reducible to intuitively more satisfying and mathematically more manageable differential formulation.

Whatever the future may bring, in the present state of factual information, a simple method of solving simultaneously mixed difference-differential equations can most decidedly increase the empirical validity of dynamic input-output theory.

II

In order to present the proposed solution of this specific mathematical problem without unnecessary complication, it is best to consider a very simple example of a dynamic input-output system incorporating both lags and stock-flow relationships.

Let it be an open economy consisting of only two industries. The output of each one of them is used first to provide for the investment requirements of the other, and second to satisfy the final, i.e. outside, demand. In accordance with the well-known implication of the acceleration principle, the investment requirements of each industry are determined by the rate of change of its output. Structural lags enter the system through fixed 'lead times,' which—for technological and organizational reasons—have to elapse between the delivery of the capital goods to the investing industry and the increase in its output ensuing from actual utilization of the newly created productive facilities.

The dynamic input-output balance of this simple system is described by the following two mixed difference and differential equations:

$$X_1(t) - b_{12}X'_2(t + \tau_{12}) = Y_1(t) \qquad (3n, 23)$$

$$X_2(t) - b_{21}X'_1(t + \tau_{21}) = Y_2(t) \qquad (3n, 24)$$

$X_1(t)$, $X_2(t)$ represent the production rates of the two industries; while $Y_1(t)$ and $Y_2(t)$ describe the development of the final, i.e. independently determined, demand for their respective outputs; b_{12} is the technical capital coefficient of the second industry; it shows the stock of the product of the second industry used by the first industry per unit of its output, $X_1(t)$. Similarly, b_{21} is the capital coefficient of the first industry; τ_{12} and τ_{21} are the two investment lead periods as defined above.

In order to eliminate one of the two variables—say, $X_2(t)$—differentiate (3n, 24), substitute in the resulting equation, $t + \tau_{12}$ for t and insert the corresponding expression for $X'_2(t + \tau_{12})$ in (3n, 23):

$$X_1(t) - b_{12}b_{21}X''_1(t + \tau_{12} + \tau_{21}) = Y_1(t) + b_{12}Y'_2(t + \tau_{12}) \quad (3n, 25)$$

This equation can be finally rewritten as:

$$X''(t) - bX(t - \tau) = Y(t) \quad (3n, 26)$$

where the time count is shifted by the substitution of $t - \tau$ for t and

$$b \equiv \frac{1}{b_{12}b_{21}},$$

$$Y(t) \equiv \frac{Y'_2}{b_{12}}(t - \tau_{21}) + Y_1(t - \tau)\frac{1}{b_{12}b_{21}}, \text{ and} \quad (3n, 27)$$

$$\tau \equiv \tau_{12} + \tau_{21}.$$

All subscripts can obviously be conveniently omitted.

III

The method of solving this kind of mixed difference-differential equation shown below is based on application of the Laplace transform.[18] It consists of three consecutive steps. First the original variable $X(t)$ is replaced in the given equation by a new variable $x(s)$. This transformation is based on the definitional relationship,

$$x(s) \equiv L[X(t)] = \int_0^\infty e^{-st}X(t)dt \quad (3n, 28)$$

In the second step the transformed—and, incidentally, greatly simplified—equation is solved for $x(s)$ and finally this solution is changed back into the terms of the original variable $X(t)$ with the help of the inverse transformation symbolically described by

$$X(t) \equiv L^{-1}[x(s)] \quad (3n, 29)$$

IV

The solution of a dynamic system must generally provide for the introduction of independent, i.e. exogenously determined, 'initial conditions.' In our particular case these would comprise, for example,

[18] A systematic exposition of the application of the Laplace transform to solution of difference and differential equations can be found in such standard texts as Churchill, Ruel V., *Modern Operational Mathematics in Engineering*, McGraw-Hill, New York, 1944; or Van der Pol, B., and Bremmer, H., *Operational Calculus*, Cambridge University Press, Cambridge, England, 1950.

a. the magnitude $X(0)$ of the level of output $X(t)$ at some initial point of time, $t = 0$,
b. the rate of change $X'(0)$ of that output at $t = 0$,
c. the shape of the output function $X(t)$ over the period of time extending from $t = -\tau$ up to $t = 0$.

The introduction of this third condition is made necessary by the presence of the structural lag, τ: The level of output, $X(t)$, from $t = 0$ and up to $t = \tau$ would, for example, be undetermined so long as the 'past history' of the system from $t = -\tau$ up to $t = 0$ remained unknown.[19]

Applying to the mixed difference-differential equation (3n, 26) the Laplace transform as defined in (3n, 28) we have:

$$\int_0^\infty e^{-st}X''(t)dt - b\int_0^\infty e^{-st}X(t-\lambda)dt = \int_0^\infty e^{-st}Y(t)dt \qquad (3n, 30)$$

In the middle term the first or lower part of the integral—which depends on the given 'initial history' of $X(t)$, i.e. on the interval of time stretching from $t = -\lambda$ to $t = 0$ can be conveniently separated from its upper part extending over the later period of time, $0 \leq t \leq \infty$:

$$\int_0^\infty e^{-st}X(t-\lambda)dt = \int_{-\lambda}^\infty e^{-s(\tau+\lambda)}X(\tau)d\tau =$$

$$e^{-\lambda s}\left\{\int_{-\lambda}^0 e^{-s\tau}X(\tau)d\tau + \int_0^\infty e^{-s\tau}X(\tau)d\tau\right\} \qquad (3n, 31)$$

The new time variable τ is defined by $\tau = t - \lambda$; in the last two integrals it obviously can be simply replaced by t.

It follows that (3n, 30) can be written as,

$$L[X''(t)] - be^{-\lambda s}L[X(t)] = L[Y(t)] + be^{-\lambda s}\int_{-\lambda}^0 e^{-st}X(t)dt \qquad (3n, 32)$$

Applying to the first term the standard transformation formula,[20]

$$L[X^{(n)}(t)] = s^nL[X(t)] - s^{n-1}X(0) - s^{n-2}X^{(1)}(0) - \cdots - X^{(n-1)}(0) \qquad (3n, 33)$$

and using the simplified notation introduced in (3n, 28), the equation above can be rewritten as,

[19] The removal of structural lags from the original system, i.e. the reduction of the mixed difference-differential equation to a simple differential equation, would lead to elimination of the last but not the first two of these three initial conditions as formulated above. For this reason the otherwise possible combination of a and b with c is inadvisable.

[20] See Churchill, op. cit. p. 8.

$$s^2x(s) - sX(0) - X'(0) - be^{-\lambda}x(s) = y(s) + be^{-\lambda s}\int_{-\lambda}^{0} e^{-st}X(t)dt$$

or, if solved for $x(s)$, as,

$$x(s) = \{y(s) + sX(0) + X'(0) + be^{-\lambda s}\int_{-\lambda}^{0} e^{-st}X(t)dt\}\,\frac{1}{s^2 - be^{-\lambda s}} \quad (3n, 34)$$

The final step consists in the application of the inverse transform, L^{-1}, to both sides of the last equation.

The actual operation of inversion is facilitated if the fraction, $\frac{1}{s^2 - be^{-\lambda s}}$, is put in the form of the convergent infinite series,

$$\frac{1}{s^2}\left(1 + \frac{be^{-\lambda s}}{s^2} + \frac{b^{-2\lambda s}}{s^4} + \cdots\right),$$

of which it represents the sum,[21] i.e. if (3n, 34) is rewritten as,

$$x(s) = \{y(s) + sX(0) + X'(0) + be^{-\lambda s}\int_{-\lambda}^{0} e^{-st}X(t)dt\}\,\frac{1}{s^2}\sum_{n=0}^{\infty}\frac{b^n e^{-n\lambda s}}{s^{2n}} \quad (3n, 35)$$

V

Now this method of solving a mixed difference and differential equation will be applied to the particular case in which the final demand functions $Y_1(t)$ and $Y_2(t)$ occurring in the original system (3n, 23) and (3n, 24) are represented by two constants, C_1 and C_2, and the initial history of the economy, as reflected in the output level of the first industry between $t = -\lambda$ and $t = 0$, is described by a straight line.

Equation (3n, 26) must be solved for $X(t)$ under the conditions that

$$Y(t) = C \quad (3n, 36)$$

$$X(t) = A + Bt \quad \text{for} \quad -\lambda \leq t \leq 0 \quad (3n, 37)$$

First the integral occurring in (3n, 35) can be explicitly computed on the basis of (3n, 37).

Integrating by parts and omitting the constant of integration, we have,

$$\int e^{-st}X(t)dt = \int e^{-st}(A + Bt)dt = -A\frac{e^{-st}}{s} - B\frac{e^{-st}}{s^2}(st + 1)$$

Consequently,

$$\int_{-\lambda}^{0} e^{-st}X(1)dt = \frac{1}{s}\{A(e^{st} - 1) - B\lambda e^{s\lambda}\} + \frac{1}{s^2}B(e^{s\lambda} - 1) \quad (3n, 38)$$

[21] The convergence of the series is secured since the arbitrary parameter, s, can always be assumed to be large enough to make of $\frac{be^{-\lambda s}}{s^2}$ a true positive fraction.

The L-transformation of the constant, C, is obtained on the basis of the following general formula, which incidentally will also be resorted to at the last stage of the argument,

$$L\left[C\frac{(t-k)^{\mu-1}}{(\mu-1)!}U(t-k)\right] = C\frac{e^{-ks}}{s^{\mu}} \tag{3n, 39}$$

where

C and K are arbitrary constants,

μ is a positive integer, and

$U(t-k)$ is the so-called unit function, defined by

$$U(t-k)\begin{cases} = 1 & \text{when } t > k \\ = 0 & \text{when } t \leq k \end{cases} \tag{3n, 40}$$

Putting $k = 0$ and $\mu = 2$ in (3n, 39) gives,

$$y(s) \equiv L[CU(t)] = \frac{C}{s} \tag{3n, 41}$$

It must be noted that this transformation is only valid for $t > 0$. Since the history of the system between $t = -\lambda$ and $t = 0$ is described in the form of the given initial conditions, (3n, 39), and because we are interested only in the determination of its subsequent path, for $t > 0$, this constitutes no objectionable limitation on the final solution.

Substitution from (3n, 38) and (3n, 41) in (3n, 35) gives,

$$\begin{aligned} x(s) = &\left\{\frac{1}{s}C + sX(0) + X'(0) + \frac{1}{s}b\{A(e^{s\lambda}-1) - Bb\lambda e^{s\lambda}\} + \frac{1}{s^2}bB(e^{\lambda s}-1)\right\} \\ &\frac{1}{s^2}\sum_{n=0}^{\infty}\frac{b^n e^{-n\lambda s}}{s^{2n}} \\ = &X(0)\sum_{n=0}^{\infty}\frac{b^n e^{-n\lambda s}}{s^{2n+1}} + X'(0)\sum_{n=0}^{\infty}\frac{b^n e^{-n\lambda s}}{s^{2n+2}} + (C - Ab)\sum_{n=0}^{\infty}\frac{b^n e^{-n\lambda s}}{s^{2n+3}} \\ &- Bb\sum_{n=0}^{\infty}\frac{b^n e^{-n\lambda s}}{s^{2n+4}} + (A - \lambda B)\sum_{n=0}^{\infty}\frac{b^{n+1}e^{-\lambda s(n+1)}}{s^{2n+3}} \\ &+ B\sum_{n=0}^{\infty}\frac{b^{n+1}e^{-\lambda s(n+1)}}{s^{2n+4}} \end{aligned} \tag{3n, 42}$$

Now it only remains to perform the inverse transformation, L^{-1}, i.e. to determine the equation in t of which equation (3n, 42) in s represents a Laplace transform. The previously used special transformation formula, (3n, 39), applied to every term under each summation sign leads thus to

the following solutions of the original mixed difference-differential equation (3n, 26): [22]

$$L^{-1}[x(s)] = X(t) = X(0) \sum_{n=0}^{\infty} \frac{b^n(t-n\lambda)^{2n}}{(2n)!} U(t-n\lambda)$$

$$+ X'(0) \sum_{n=0}^{\infty} \frac{b^n(t-n\lambda)^{2n+1}}{(2n+1)!} U(t-n\lambda)$$

$$+ (C - bA) \sum_{n=0}^{\infty} \frac{b^n(t-n\lambda)^{2n+2}}{(2n+2)!} U(t-n\lambda) - bB \sum_{n=0}^{\infty} \frac{b^n(t-n\lambda)^{2n+3}}{(2n+3)!} U(t-n\lambda)$$

$$+ (A - \lambda B) \sum_{n=0}^{\infty} \frac{b^{n+1}(t-n\lambda-\lambda)^{2n+2}}{(2n+2)!} U(t-n\lambda-\lambda)$$

$$+ B \sum_{n=0}^{\infty} \frac{b^{n+1}(t-n\lambda-\lambda)^{2n+3}}{(2n+3)!} U(t-n\lambda-\lambda) \quad (3n, 43)$$

For any given positive finite t and λ the number of terms under each summation sign will necessarily be also finite.[23] The cut-off point is determined in each instance by the argument of the corresponding unit function, $U(\quad)$. Under the first four summation signs, for example, for any given t, the largest n, let it be called m, is determined by the combination of the following two inequalities:

$$t - m\lambda > 0 \quad \text{and} \quad t - (m+1)\lambda < 0$$

which can be rewritten as

$$\frac{1}{\lambda} - 1 < m < \frac{t}{\lambda} \quad (3n, 44)$$

For the last three terms the corresponding formula is

$$\frac{t}{\lambda} < m < \frac{t}{\lambda} + 1 \quad (3n, 45)$$

[22] Let it be noted that for $n = 0$, $(2n)! = 1$.

[23] A positive λ is, because of obvious formal reasons, essential for the following argument. Should λ as it appears in the original dynamic equation (3n, 26) be negative, i.e. represent a 'lead' rather than a 'lag,' the foregoing formal requirement will still be satisfied if the same relationship is rewritten as:

$$X''(t-\alpha) - bX(t) = Y(t-\alpha)$$

where α stands for $-\lambda$ and thus is positive The solution of this equation can be obtained by a procedure quite analogous to that described above and based on the same special transformation formulae (3n, 33), (3n, 39), and (3n, 40).

VI

Substitutions of $X(t)$, as explicitly described on the right-hand side of (3n, 31), in (3n, 26) show that this indeed is the solution of the original mixed difference-differential equation.

The function $X(t)$ as defined by (3n, 43) represents the right-hand portion of the $X(t)$ curve shown in the schematic Figure 1. Its left-hand part covering the stretch from P_1 to P_2 depicts the past history of $X(t)$. The jump from P_2 to P_3 reflects the fact that the 'initial conditions' of our problem define the shape of $X(t)$ as described by (3n, 37) between

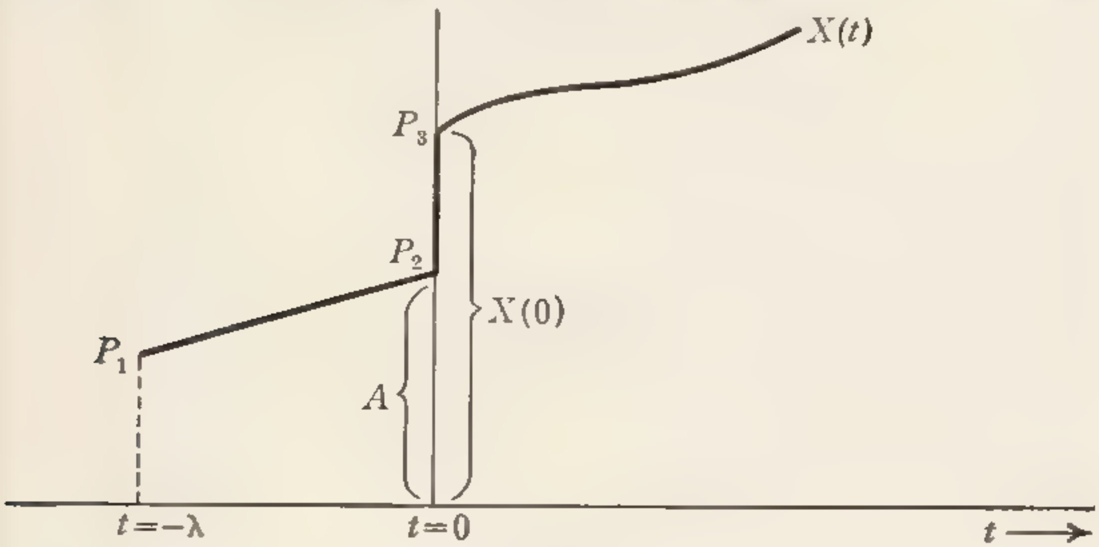


FIG. 1

$t = -\lambda$ and $t = 0$— including its level and its 'left-hand slope,' at P_2—separately from the values of $X(0)$ and $X'(0)$ as they appear in (3n, 43) describing the level and the 'right-hand slope' of the curve in point P_3, i.e. its slope at the origin of its right-hand part.

These additional degrees of freedom of the initial conditions will prove to be useful in special applications, such, for example, as comparison of various policy alternatives: Past history is unalterable and must be considered as given; action as a change in initial conditions can be taken in the future which begins with the present, that is at $t = 0$. It is at that point that an alteration and the consequent discontinuity in the initial condition are likely to occur.

The discontinuity in the initial condition at $t = 0$ will, of course, be eliminated if the constants $X(0)$ and $X'(0)$, as they appear in (3n, 43), are identified with the level and, respectively, the rate of change reached by the output, $X(t)$ at the end of its initial history at P_2. In the special case discussed above in which the history of the system is described by

(3n, 37), the jump between P_2 and P_3 as shown in Figure 1 would vanish if the magnitude of $X(0)$ as entered in (3n, 43) be defined by

$$X(0) = A \tag{3n, 46}$$

The difference between the right-hand and the left-hand slope of the curve at $t = 0$ would disappear if its derivative as entered in (3n, 43) were determined by

$$X'(0) = B \tag{3n, 47}$$

The use of the Laplace transform also allows introduction of any number of discontinuities in the final bill of goods. The last term in (3n, 26) can, for example, be of the following form:

$$Y(t) = Y_1(t)U(t) + Y_2(t)U(t - k) \tag{3n, 48}$$

where

$Y_2(t)$, according to (3n, 40), enters or rather jumps into the picture only at the time, $t = k$,

and where

k represents an empirically given shift constant, i.e. an observed or prescribed lag.

PART II

THE EXTENSION OF INPUT-OUTPUT TECHNIQUES TO INTERREGIONAL ANALYSIS

Chapter 4

INTERREGIONAL THEORY[1]

Wassily Leontief

I. Intra-national Relationships

In this chapter and the next the input-output approach is applied to the empirical study of interregional, or better to say intra-national, relationships within the United States; the first part is concerned with the development of the conceptual framework; the second, with its application in factual analysis.

The movements of commodities and services between separate geographic areas and the internal economic structure of such regions are mutually interdependent. A truly comprehensive theory would consider the spatial distribution of economic activities to be an integral aspect of a single system of economic relationships. Such a theory has not yet been developed, and, if it were, the lack of adequate factual information would make its empirical implementation impossible for a long time to come.

The analytical scheme developed for the purposes of the present study is a partial equilibrium theory. In this theory, relationships which, within the framework of a more general analysis, would be derived, for example, from interregional cost comparisons, from detailed studies of transportation expenses, etc., are treated as primary data. Specifically it combines the conceptual framework of simple straight input-output analysis with the obvious but important observation that some commodities are produced not far from where they are consumed, while the others can and do travel long distances between the place of their origin and that of their actual utilization. In the first instance, the physical production-consumption balance must be struck on a local, regional basis; in the second, it is achieved only on a national, or even international, level.

The simple distinction between only two classes of commodities—the 'regional' and the 'national' is intended to be used as a first approximation to a more graduated scale of differences between commodities with respect to distance shipped. The theoretical treatment of the latter will

[1] The author gratefully acknowledges the valuable assistance which he has received from Mrs Judith Balderston in going over all formulae and particularly in standardizing the mathematical notation used in this chapter and the appended note, as well as in Chapter 3.

emerge at the end of the following argument as an immediate generalization of the elementary dichotomous scheme.

The finer the regional breakdown of the over-all area, the fewer goods will be recognized as being purely regional. The problem of a proper definition of regions, in terms of which all subsequent analysis is to be conducted, is quite analogous to the question of a proper distinction between separate industries, i.e. the familiar problem of industrial classification within the framework of ordinary input-output analysis.

The locational production pattern of all commodities belonging to the 'national,' i.e. easily transportable, category is assumed to be known and to be constant. That means that the percentage-wise split of the total output of each such commodity—motor vehicles, for example—by regions in which it actually is being produced, is considered as given; furthermore, it is also assumed that these percentages remain the same even when the level of the total country-wide output goes up or down. That implies proportional expansion and contraction in all regions.

It hardly needs to be stated that this latter assumption is incorrect. There is good reason to believe that the comparative levels of regional outputs of such a commodity as aluminum or cotton fabric does, to a certain extent, depend upon the level of its total national output. In so far as such dependence can be approximated by linear relationships, it could even be easily incorporated in the subsequent numerical computations. Since, however, no reliable empirical measures of a sufficient number of such relationships are yet available, no useful purpose can be served by incorporating them in the present theoretical formulation. Instead, we include in it a set of locational constants showing the percentage-wise participation of each individual region in the total output of every 'national' commodity.

Once the geographic distribution of the production of all the nationally traded goods is fixed, the demand for the products of the regionally balanced types of commodities and services becomes determined and from it the corresponding regional distribution of their total inputs can also be derived.

With the regional distribution of all outputs, i.e. national as well as regionally balanced, known, the corresponding consumption figures can be computed on the basis of the given input-output structures of the individual industries.

To determine the import-export structure of each region it will finally be only necessary to compare the previously derived output with the corresponding consumption figures. In the case of regionally balanced categories the two will necessarily coincide; for other goods, active and passive regional balances with the rest of the country will be shown Balances 'with the rest of the country' rather than balances 'with other

regions' are referred to in this connection since the latter have no real significance within the framework of the present analytical scheme: The simple but artificial distinction between the regionally balanced and the nationally balanced commodities does not recognize the preferential paths which they might actually follow in their movements from one area to another. They are thrown by the producers into a common national pool to be taken out by the various consumers without reference to specific regional origin. In the final interpretation of the numerical results obtained, one might make the reasonable assumption that the imports of a given commodity into some particular region will most likely come from the nearest surplus area. This is why the present theoretical scheme, strictly speaking, should be referred to as a theory of intranational but not of interregional relationships.[2]

Upon closer theoretical examination this and similar rules would prove, however, to be much more complicated than they appear to be on first sight. Even a purely theoretical analysis of optimum transportation patterns presents a serious mathematical problem. The idiosyncrasies of regional rate structures, combined with the more subtle aspects of quality differentiation, make the full incorporation of such rules into the framework of empirical general equilibrium analysis conditional on large amounts of additional factual and theoretical research.[3]

II. The Analytical Scheme

The theoretical approach described above will be applied to the determination of the differential regional impact of a change in the final, i.e. outside, demand for the products of a given national economy. The exact derivation of the computational formulae is relegated to the mathematical appendix; the following discussion is limited to a general description of the underlying quantitative argument in its successive steps.

The open system of static input-output equations with a given bill of goods as described in set (2, 1) of Chapter 2 and its explicit solution represented by set (2, 2) can be adapted to treatment of regional problems through the introduction of the following more differentiated notation:

The total number of commodities and services is m,

$$i = 1, 2, \cdots, h, h + 1, h + 2, \cdots, m$$

Of these the first h are regional, i.e. they are balanced regionally; while

[2] Theoretical refinement which would transform this analysis into a truly interregional theory has already been proposed by Dr. Walter Isard (see 'Interregional and Regional Input-Output Analysis: A Model of a Space-Economy,' in the November 1951 *Review of Economic Statistics*).

[3] See pp. 176-81, ch. 5.

the last m-h are national, i.e. their production and consumption are balanced only on the national level. Accordingly, let

$$l = 1, 2, \cdots, h$$

$$g = h + 1, h + 2, \cdots, m$$

The whole economy contains n distinctly numbered regions,

$$j = 1, 2, \cdots, n$$

X_i, X_l and X_g represent the total national outputs of various commodities and services.

${}_jX_i$, ${}_jX_l$ and ${}_jX_g$ refer to the corresponding regional outputs.

Y_i, Y_l and Y_g represent the total final, i.e. given, demand for various commodities and services.

${}_jY_l$ refers to the final, i.e. given, demand for the regional goods and services in region j.

The structure of the system is determined by two sets of constants: a_{ik} is the technical input coefficient, i.e. the number of units of commodity i used per unit of output of commodity k,

$$a_{ik} = \frac{x_{ik}}{X_k}$$

and ${}_jr_g$ represents the proportion of the total national output of the 'national' commodity g produced in region j, i.e.

$${}_jr_g = \frac{{}_jX_g}{X_g} \qquad \begin{matrix}(g = h + 1, h + 2, \cdots, m) \\ (j = 1, 2, \cdots, n)\end{matrix}$$

The over-all system of input-output equations for the economy as a whole,

$$X_i - \sum_{k=1}^{m} a_{ik}X_k = Y_i \quad (i = 1, 2, \cdots, h, h + 1, \cdots, m) \quad (4, 1)$$

can be solved for the total national outputs X_i, of all commodities.

$$X_i = \sum_{k=1}^{m} A_{ik}Y_k \quad (i = 1, 2, \cdots, h, h + 1, \cdots, m) \quad (4, 2)$$

where

A_{ik} = elements of the inverse of the matrix of input coefficients (see footnote 1, Chapter 2).

The regional outputs ${}_jX_g$ of any *national* commodity can be determined by multiplying the previously derived national output by the appropriate regional percentages,

$$ {}_jX_g = {}_jr_g\, X_g \quad (g = h+1, h+2, \cdots, m) \quad (4, 3)$$
$$ (j = 1, 2, \cdots, n)$$

To derive the regional outputs of regional commodities, one has to separate from system (4, 1) the first h equations describing the over-all production-consumption balance of this particular category of goods and then split each one of them into n regional balance equations. Thus one obtains for every region, i.e. for each given j, a separate set of input-output equations for all the regionally balanced goods and services:

$$ {}_jX_l - \sum_{i=1}^{m} a_{li}\, {}_jX_i = {}_jY_l \quad (l = 1, 2, \cdots, h) \quad (4, 4)$$
$$ (j = 1, 2, \cdots, n)$$

Each such system contains h equations—one for every regionally balanced output—and $m+1$ variables, since the total regional consumption of any regional good or service depends—in addition to the direct final demand—upon the input requirements of all the industries (regional as well as national) operating in that region. But the final demand located in the particular region, ${}_jY_l$, belongs to the data of our problem while the $m-h$ outputs of the national commodities produced in the region have already been determined through the sets (4, 2) and (4, 3) above. Solving system (4, 4) for the remaining h outputs of regional goods we have,

$$ {}_jX_l = \sum_{g=h+1}^{m} B_{lg}\, {}_jX_g + \sum_{k=1}^{h} C_{lk}\, {}_jY_k \quad (l = 1, 2, \cdots, h) \quad (4, 5)$$
$$ (j = 1, 2, \cdots, n)$$

The two sets of constants B_{lg} and C_{lk} are computed from the basic input coefficients, a_{ik}. The ${}_jX_g$'s in the first right-hand term can be expressed as linear functions of the final demand $Y_1, Y_2, \cdots, Y_m$ through substitutions from (4, 3) and (4, 2). Thus every regional output of each regional commodity can finally be derived from a given bill of goods. That bill must be described in terms of the total outside demand for each nationally balanced group of commodities and separate regional demand for all regional goods and services. In the special case in which the bill of goods comprises only commodities of the first kind, the computation is somewhat simplified since the last right-hand term in equation (4, 5) vanishes.

The actual procedure, which is described in full detail in the Appendix, involves essentially a rather intricate combination of the results of two separate operations. One of these consists in the computation of the effects of a given bill of final demand on all the industries; and the other,

the determination of the corresponding effects on the regional industries taken separately.

With the outputs of both the regional and the national commodities thus determined region by region, the total regional inputs of all individual commodities can also be derived. Such regional consumption can be visualized as consisting of two parts. The first is accounted for by the input requirement of all the industries located in the particular region. By multiplying the regional output of each industry by each one of its various input coefficients one determines all their separate input requirements first. A summation of the input requirements for each particular commodity gives total 'derived' consumption within the given region. To this, however, must be added the consumption directly accounted for by that part of the final demand, both national and regional, which happens to be located in the particular region under consideration.

If 'net investment' is considered to be a part of the final bill of goods, the total regional demand (equals regional output, since it is a typically regional industry) for the product of the construction industry will comprise, first, the replacement requirements on construction account of all the industries—both regional and national—producing in that region and, second, the construction component of the regional 'new investment.' An explicitly dynamic approach to the analysis of regional investment is described in section v below.

The information on the locational distribution of the final demand for *regionally* balanced outputs has already been used for determination of the locational output pattern. The locational distribution of the final demand for *national* commodities has, however, not been required for that purpose. Since they are defined as being movable between regions, their regional outputs were derived by application of the independently given constants, $_jr_g$, to total final demand for nationally balanced products. Thus the knowledge of the locational pattern of this part of the final demand represents additional information which is required for determination of total regional consumption of national commodities.

To determine all regional balances of trade, i.e. the export or import surpluses, one has only to compute the difference between the previously derived production and consumption of each kind of goods in each region. The internal logic of the theoretical relationships on which this entire analysis is based guarantees, of course, the equality of combined regional export and import surpluses of each kind of nationally balanced commodities.[4]

[4] This presupposes that either foreign countries are treated as a separate industry or that all exports to these foreign countries are included in the final bill of goods.

III. Successive Regional Breakdowns

As intimated in the introductory discussions of the distinction between the nationally and regionally balanced commodities and services, the theoretical framework of regional input-output analysis as described above can be expanded from a two- to a many-storied structure. The regions can, for example, be divided into sub-regions and the latter into even smaller local areas. To simplify the terminology one might refer to the total national area and then to regions of the first, second, third, and any further order. Accordingly, a distinction can be made between commodities balanced on the national level, those balanced within the regions of the first, second and third order, the reference always being made to the lowest order of regional subdivision within which the balance between total output and the aggregate inputs of a particular commodity or service is being achieved. The theoretical scheme developed above in the analysis of the relation between the national and regional (first order) commodities can be applied directly and without any essential modification to the description of the corresponding interrelation between the input-output structures of a region of the first order and the second-order regions comprised within it. Starting with the total outputs of all industries located in the particular first-order region—determined in the previous computation as functions of the given final demand the second round of computation involves first of all the introduction of a distinction between goods whose input-output balance is established within the confines of each second-order region and all the other commodities which cross such borders. This latter category includes both goods balanced only on the national level as well as those whose balance is achieved on the level of the first but not that of the smaller second-order regions.

The locational distribution of the outputs of all commodities belonging to these national and first-order groups is then considered as open. It is described in terms of fixed percentage figures, showing the participation of each second-order region in the total output of the particular commodity originating within the entire first-order region to which it belongs. The local outputs of all commodities balanced on the lower level of the separate second-order regions are finally described on the basis of the same formulae which were used before to compute the local outputs of the 'regional' commodities from a given bill of final demand and a known locational pattern of nationally balanced goods

This procedure can be repeated again and again in passing from any previously reached level of the locational pattern to the next layer with its more finely differentiated pattern. The ordering of commodities and the corresponding ordering of economic regions represent, as has been stated before, two different aspects of the same analytical operation.

The formal consistency of the system does not actually require that in breaking down, say, two different economic regions into their respective sub-regions one would draw the distinction between the regionally and sub-regionally balanced commodities necessarily along the same line. Since the input-output relation within each of the two sets of sub-regions are treated separately—their mutual relations within the system as a whole being established in terms of the two larger areas before they have been subdivided—one of these might not, for example, be broken down at all while the other could be reduced to smaller and smaller area units through repeated subdivisions. Neither is it necessary to assume as it has been done above that the input structure of an industry does not vary from one region or sub-region to another. Computational formulae permitting the use of different sorts of input coefficients for each industry in every territorial unit can be easily derived.

In actual empirical analysis, a uniformly symmetrical treatment of the individual branches of successive regional breakdowns does save a considerable amount of computational work. Because of that, in the following study of the regional structure of the American economy, one single classification of commodities belonging to successively more and more localized orders is used throughout. The possibility of using regionally diversified cost structures is for the present time excluded by the lack of systematic and sufficiently detailed information on the variation of the input coefficients of the same industry from one region to another.

IV. Households in the Matrix

As in any other analysis involving the study of an open system, a decision has to be made, also, in the present case as to how open the system is actually to be, i.e. what sections of the economy are to be described through the given set of structural constants and which are to be left on the outside and represented through the prescribed bill of final demand. In many input-output studies only the productive parts of the economy have been included in the structural matrix with households left on the outside to be represented through the bill of final consumers' demand. But already in the analysis of the impact of foreign trade on a national economy, it has been found advisable, instead of keeping the households on the outside, to include them in the structural matrix of the domestic system and thus to trace through the secondary, the 'multiplier' effects of increased employment via the additional demand for consumers' goods.[5]

Similar considerations recommend the inclusion of the relation between employment and household demand for consumers' goods in the

[5] See *The Structure of American Economy*, 1919-1929, 2nd ed., Part IV, B.

present study of the regional repercussions of changes in final demand. With household purchases treated as any other industrial inputs and related to the 'output' of labor services through a set of consumption coefficients, the definition of outside demand must correspondingly be narrowed down. It includes now only investment demand, the purchases of governmental agencies and exports. Household purchases which exceed the normal input-output ratios, described by consumption coefficients and thus incorporated in the structural matrix of the economy, would also fall into this category of final demand determined from the outside.

Since households constitute a very large sector of the whole system, a relatively small error in the magnitude of the consumption coefficient used in subsequent input-output computations is bound to have a marked effect on their numerical results. This is as a matter of fact the principal reason why, pending a further and more detailed analysis of consumers' behavior,[6] household demand has, in most applications of the input-output technique, been treated as an outside datum. Applied to the study of the present investigation such an approach would miss, however, one of the most important aspects of the regional reaction to variations in outside demand—the change in the output of local service industries resulting from increases and decreases in the level of local employment.

Even if the estimate of the absolute magnitude of such reactions will in the following analysis be affected by possible errors in the consumption coefficients used, the relative magnitude of such responses in regions with different industrial structures will be disturbed to a much lesser extent; and it is these differential reactions of structurally dissimilar regions which constitute the principal object of the following empirical study.

V. Dynamic Regional Theory

The simple schematic theory developed above and actually applied in the next chapter to the empirical study of the regional relationships within the American economy is entirely static. Looking a few steps ahead one can easily perceive the outlines of a similar approach to the study of the regional aspects of economic change. The distinction between structural variation and dynamic change in the narrow sense, as developed in the previous chapters (see p. 17 and p. 53), can also be applied here.

Changes in the structural characteristics of the economy obviously will have regional repercussions. In addition to the variation in the structures of the individual industries, the locational pattern of the economy as presented in the regional input-output formulae will be also affected by the shifts in the coefficients in the regional distribution of the national industries, the sub-regional distribution of the regional industries, and so

[6] See, for example, Chapter 12.

on down the line. In so far as these coefficients simply represent a crude reflection of the basic factors determining the advantages and disadvantages of various industrial locations, any meaningful inquiry aimed in this direction will have to concern itself with the question of the difference in the regional input coefficients. In complete analogy with the analytical inquiry into the general problem of technological change (see Chapter 3) the explanation of regional shifts will have to be explicitly stated as a problem of choice between alternative sets of input coefficients. I say explicitly since implicitly the problem of comparative costs already finds its partial solution in the basic distinctions between national, regional, and sub-regional commodities, a classification which obviously also will be affected by introduction of new goods and services and, which as we have previously seen (p. 20) is essentially the same change in the production and consumption methods of the old.

The static scheme of regional relationships as developed above can be directly hitched on to the solution of the dynamic input-output system described in Chapter 3. The latter enables us to derive the time paths of the total outputs of all individual industries. Inserting these total outputs—now described as known functions of time—into the three static regional equations described above, one arrives at a set of corresponding regional and sub-regional production and consumption figures, each one expressed as a function of time.

In addition and above this simple one-sided dependence on the overall dynamic solution, the regional structure of the economy can in its turn contribute to the understanding of its basic dynamic characteristics. The course of the dynamic development of an economic system has been shown (see Chapter 3, p. 68) to depend on the irreversibilities of certain kinds of investment. Some of the practically most significant irreversibilities are actually determined by the locational pattern of the regionally or sub-regionally balanced industries.

A shift in the demand for electric power from, say, the North Atlantic to the Pacific region would require additional investment in electric stations even if the combined national demand went down and fell below the combined utilities capacity for the country as a whole. The surplus capacity available in the one region cannot be used to satisfy the increased demand in the other region. In terms of the dynamic input-output scheme, this means that the stocks of various capital goods used in one region cannot be reduced as their utilization rate declines and allocated as investment inputs in another region. This particular irreversibility, if taken into account in the solution of the fundamental dynamic system, will lead to output paths automatically providing for an appropriate

amount of additional investment in the public utilities industry as a whole. It hardly needs to be explained that the over-all dynamic system must be translated—with the help of the relationships mentioned above—into regional terms. Only in this new form will its solution be able to incorporate the regional irreversibility, or rather non-transferability, conditions.[7]

VI. Empirical Application

One of the three different kinds of basic empirical ingredients used in the static analytical scheme of regional input-output relations is the complete set of the input coefficients of all the separate industries. The structural matrix of the entire system has already been discussed and requires no further comment. The sets of location coefficients, the r's of our formulae, describing the percentage-wise distribution of the total outputs of various industries according to their regional origin can be derived directly from the Census and other official statistics through the output of all commodities and services by individual states.

The distinction between the national, regional, and sub-regional sets of commodities and services of necessity involves us in a rather elaborate subsidiary piece of empirical inquiry. This inquiry consists of two parts. The first represents a cross-cut study of the balance of production, consumption, exports, and imports by all the 39 separate groups of commodities and services for each of the 48 states of the Union. The second part—based on the findings of the first—is the actual assignment of each commodity group to one of the various regional categories in terms of which the regional input-output structure of the American economy will finally be analyzed. Since this subsidiary inquiry might prove to be of interest to students of location of economic activities quite independently of the particular purpose for which they are here derived, its method and principal findings are described with some detail in the next chapter. The present discussion is only intended to explain the general method of attack.

The statistical information on the levels of outputs of each of the 96 industries of our basic industrial classification—subsequently consolidated into 39 groups—in each of the 48 states for the year 1939, constitutes the principal set of new data now added to the basic information contained in the 1939 input-output table.

No similar direct figures are obtainable on state-by-state consumption of the same commodities and services. These had to be computed indirectly from the output figures on the basis of the previously derived technical coefficients of all industrial inputs and consumption coefficients of all commodities absorbed directly by the households.

[7] For concise formulae, see the mathematical note at the end of this chapter.

The total output of each industry in each state was multiplied successively by all the structural input coefficients of that industry. The resulting columns of figures show the amounts of all the various commodities and services absorbed by the corresponding industries state by state. If added together, the inputs of any one kind consumed by all the industries and households in one particular state plus the state's contribution to the corresponding bill-of-goods item show the total consumption of that commodity or service within that state. The difference between these combined input and bill-of-goods figures and the corresponding output figures show the net imports or the net exports of each commodity for each state. In other words, this computation yields a balance of trade for each of the 48 states in terms of each of the 39 consolidated commodities and service groups. Let it be noted, however, that the resulting figures do not show the specific origin of the imports received by any one state or the particular destination of its exports.

In so far as the input coefficients used to derive the consumption requirements of the various industries refer only to inputs on 'current account' and do not include additional investment requirements, the computed input-output balances reflect the 'external trade' of the particular areas only to the extent to which they are determined by the current input-output relation. Household consumption, as has been pointed out before, is included among these but not new investment, government purchases, or for that matter any other items included in the bill of final demand.

In these instances in which the state by state distribution of the final bill of goods is actually known, it must be added to the input-output figures on current account in order to obtain the complete input-output balances of the individual areas. In the cases of such strictly regionally balanced goods as the product of the construction industry, the directly computed regional discrepancy between regional outputs and the corresponding inputs on current account can be used to determine the amount of regional new investment.

It hardly needs to be added that, as in all other instances of indirect computations, any errors in basic figures, particularly those in the set of the technical and consumption coefficients on the basis of which all the input requirements are derived, will naturally affect the final numerical results. As will be shown in the subsequent discussion of the actual empirical findings, some of these errors can be detected indirectly and partially corrected with the help of additional outside information. A fundamental improvement in this respect can, however, be brought about only through systematic development of detailed studies of the input structure of individual industries on a regional basis.

VII. Regional Division

The quantitative description of the input-output structure of the elementary territorial units—in the present instance these are the separate states—constitutes the principal basis for the simultaneous division of economic areas and the corresponding ranking of commodities and services. For the purposes of the following study, the United States was divided into two large regions and each of these partitioned into a number of sub-regions. For argument's sake, let a territorial classification of this kind be considered as given; the assignment of each individual industry with its respective products to the national, regional, and sub-regional group represents, then, the problem at hand.

The basic state figures can be easily consolidated into sub-regional input-output tables and these latter combined further into regional. In particular production, consumption, export, and import figures can now be examined for each commodity in each of the previously defined territorial units.

In the improbable case in which a perfect input-output balance of some commodities and services proves to be established within each of the sub-regional units, while among those which do move across the sub-regional borders some would observe a similarly perfect balance on a regional basis leaving the third remaining group to be included in a nationally (or even internationally) balanced category; the ranking problem would have been already solved. Actually such perfect separation can hardly be expected,[8] which means that the actual division between the three types of goods will have to be based on the distinction in degree of their relative sub-regional, regional, or national balance. A simple index suggestive of such balance for any one commodity on a given level of territorial division is represented by a ratio of its combined exports and imports—from and into all the areas of the particular rank—and its aggregate consumption.

Arranged from the highest to the lowest a series of such ratios—one for each commodity or service—form a descending scale along which breaks may occur which can be helpful in separating the more from the less localized commodities.

A comparison of two or more of such localization spectrums all comprising the same commodity groups and referring to the same over-all economic area, but based on necessarily more refined territorial subdivisions, can be used as a means to test the internal consistency—or better the appropriateness—of the underlying geographic split-ups.

[8] The nearest approximation to such clear cut delineation of distinct economic regions can be found in separation along the national borders. Tariffs and other political impediments to free flow of trade combined with geographical obstacles often result in this case in sharply drawn and more easily discernible distinctions between 'domestic' and 'international' commodities.

As one passes from larger to smaller areas the volume of 'external trade' in any commodity should increase in its relation to the magnitude of its local consumption (or output). The relative position of the different commodities within the localization might, however, change. The shift in the position of individual commodities along the scale accompanying the passage from one territorial split-up to another can be used as a useful criterion in definitive analytically appropriate territorial subdivisions of the constituent, geographic areas on either one or both levels of territorial division.[9]

The theoretical formulae developed above make it possible to compute for any given bill of final demand the territorial distribution of the outputs of all commodities which would be obtained if all the simplified assumptions built into this elementary input-output scheme were actually 100 per cent correct. A comparison of the actual outputs of various commodities and services with the distribution indirectly computed from the same empirically observed final bill of goods will necessarily reveal a discrepancy—a discrepancy which can be used as a valuable guide for revision and improvement of the original assumption.

In appraising the significance of such practically unavoidable differences between the real and the stylized picture of a given economy it is important to realize that in determining the effects of an assumed or observed *change* in the final demand, the theoretical computations are likely to give a more accurate result, if the thus computed increases and decreases are described in percentage terms. The over- or under-estimates of the absolute levels of regional or sub-regional outputs will very probably be proportionally about the same in the situation (as it existed) before and after the assumed change. Thus the discrepancy will tend to cancel out when the computed change is described in relative, i.e. percentage, terms.

MATHEMATICAL NOTE

A general discussion of all definitions and assumptions underlying the mathematical argument presented in this note, as well as the economic interpretation of the final computational formulae derived in it, can be found in Chapter 4 (and to a certain extent, in Chapters 1 and 3).

Since the systems with which we are dealing involve large sets of linear interrelationships, matrix notation is used throughout.

[9] Incongruity means in this connection only the lack of correspondence with the requirements of the formal scheme of interregional input-output relations used in subsequent analysis.

I. Formulation and Solution of an Open Static System of Interregional Input-Output Equations

A. DETERMINATION OF REGIONAL OUTPUTS

1. *Notation*

There are m commodities:

$$\left.\begin{array}{l} \left.\begin{array}{l} 1 \\ 2 \\ \cdot \\ \cdot \\ \cdot \\ h \end{array}\right\}\text{balanced in region} \\ h+1 \\ h+2 \\ \cdot \\ \cdot \\ \cdot \\ m \end{array}\right\}\text{balanced in nation}$$

for which the outputs and final demands are defined as follows:

$$X_L = \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_h \end{bmatrix} \quad X_N = \begin{bmatrix} X_{h+1} \\ X_{h+2} \\ \cdot \\ \cdot \\ \cdot \\ X_m \end{bmatrix} \quad X = \begin{bmatrix} X_L \\ X_N \end{bmatrix}$$

$$Y_L = \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_h \end{bmatrix} \quad Y_N = \begin{bmatrix} Y_{h+1} \\ Y_{h+2} \\ \cdot \\ \cdot \\ \cdot \\ Y_m \end{bmatrix} \quad Y = \begin{bmatrix} Y_L \\ Y_N \end{bmatrix}$$

$${}_jX_L = \begin{bmatrix} {}_jX_1 \\ {}_jX_2 \\ \cdot \\ \cdot \\ \cdot \\ {}_jX_h \end{bmatrix} \quad {}_jX_N = \begin{bmatrix} {}_jX_{h+1} \\ {}_jX_{h+2} \\ \cdot \\ \cdot \\ \cdot \\ {}_jX_m \end{bmatrix} \quad {}_jX = \begin{bmatrix} {}_jX_L \\ {}_jX_N \end{bmatrix} \quad \text{for all } j$$

$$
{}_jY_L = \begin{bmatrix} {}_jY_1 \\ {}_jY_2 \\ \cdot \\ \cdot \\ \cdot \\ {}_jY_h \end{bmatrix} \quad
{}_jY_N = \begin{bmatrix} {}_jY_{h+1} \\ {}_jY_{h+2} \\ \cdot \\ \cdot \\ \cdot \\ {}_jY_m \end{bmatrix} \quad
{}_jY = \begin{bmatrix} {}_jY_L \\ {}_jY_N \end{bmatrix} \quad \text{for all } j
$$

$$
(j = 1, 2, \cdots, n)
$$

The coefficients of production are:

$$
a_{LL} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1h} \\ a_{21} & a_{22} & \cdots & a_{2h} \\ \cdot & \cdot & & \cdot \\ \cdot & & & \\ \cdot & \cdot & & \cdot \\ a_{h1} & a_{h2} & \cdots & a_{hh} \end{bmatrix} \qquad
a_{LN} = \begin{bmatrix} a_{1,h+1} & a_{1,h+2} & \cdots & a_{1m} \\ a_{2,h+2} & a_{2,h+2} & \cdots & a_{2m} \\ \cdot & & & \cdot \\ \cdot & \cdot & & \\ \cdot & \cdot & & \cdot \\ a_{h,h+1} & a_{h,h+2} & \cdots & a_{hm} \end{bmatrix}
$$

$$
a_{NL} = \begin{bmatrix} a_{h+1,1} & a_{h+1,2} & \cdots & a_{h+1,h} \\ a_{h+2,1} & a_{h+2,2} & \cdots & a_{h+2,h} \\ \cdot & \cdot & & \cdot \\ \cdot & & & \cdot \\ \cdot & \cdot & & \cdot \\ a_{m1} & a_{m2} & \cdots & a_{mh} \end{bmatrix} \qquad
a_{NN} = \begin{bmatrix} a_{h+1,h+1} & a_{h+1,h+2} & \cdots & a_{h+1,m} \\ a_{h+2,h+1} & a_{h+2,h+2} & \cdots & a_{h+2,m} \\ & & & \cdot \\ \cdot & & & \cdot \\ a_{m,h+1} & a_{m,h+2} & \cdots & a_{mm} \end{bmatrix}
$$

$$
a = \begin{bmatrix} a_{LL} & a_{LN} \\ a_{NL} & a_{NN} \end{bmatrix}
$$

and the regional coefficients:

$$
{}_jR = \begin{bmatrix} {}_jr_{h+1} & 0 & \cdots & 0 \\ 0 & {}_jr_{h+2} & \cdots & 0 \\ \cdot & \cdot & & \cdot \\ & & & \cdot \\ \cdot & \cdot & & \cdot \\ 0 & 0 & \cdots & {}_jr_m \end{bmatrix} \quad \text{so that, } {}_jRX_N = {}_jX_N.
$$

Furthermore, the inverse of $[I - a]$ can be written as

$$[I - a]^{-1} = A \equiv \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1h} & A_{1,h+1} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2h} & A_{2,h+1} & \cdots & A_{2m} \\ \cdot & & & \cdot & & & \\ & & & \cdot & & & \\ & & & \cdot & \cdot & & \\ A_{h1} & A_{h2} & \cdots & A_{hh} & A_{h,h+1} & \cdots & A_{hm} \\ A_{h+1,1} & A_{h+1,2} & \cdots & A_{h+1,h} & A_{h+1,h+1} & \cdots & A_{h+1,m} \\ \cdot & & & \cdot & \cdot & & \\ \cdot & & & & & & \\ \cdot & \cdot & & & \cdot & & \\ A_{m1} & A_{m2} & \cdots & A_{mh} & A_{m,h+1} & \cdots & A_{mm} \end{bmatrix}$$

$$= \begin{bmatrix} {}_LA \\ {}_NA \end{bmatrix}$$

where any element

$$A_{k_1k_2} = \frac{\text{co-factor of the element containing } a_{k_2k_1} \text{ in the determinant of } [I - a]}{\text{determinant of } [I - a]}$$

and the inversion of A_{LL}

$$A_{LL}{}^{-1} = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1h} \\ w_{21} & w_{22} & \cdots & w_{2h} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \\ \cdot & & & \\ w_{h1} & w_{h2} & \cdots & w_{hh} \end{bmatrix}$$

where any element

$$w_{k_1k_2} = \frac{\text{co-factor of the element containing } a_{k_2k_1} \text{ in the determinant of } [I - a_{LL}]}{\text{determinant of } [I - a_{LL}]}$$

2. *Sets of equations*

The balance equation for the system as a whole is:

$$[I - a]X = Y \tag{4n, 1}$$

and its solution for all total outputs

$$X = A\,Y \tag{4n, 2}$$

The balance equation for the regional outputs in any given region j is

$$[I - a_{LL},\ -a_{LN}] \begin{bmatrix} {}_jX_L \\ {}_jX_N \end{bmatrix} = {}_jY_L \tag{4n, 3}$$

Multiplied out on the left side it gives

$$[I - a_{LL}]_j X_L - a_{LN\ j}X_N = {}_jY_L \tag{4n, 3a}$$

$$[I - a_{LL}]_j X_L = {}_jY_L + a_{LN\ j}X_N \tag{4n, 4}$$

and solving for ${}_jX_L$

$${}_jX_L = A_{LL\ j}Y_L + A_{LL}\, a_{LN\ j}X_N \tag{4n, 5}$$

But we assume that the proportions of regional outputs of national goods are given, i.e.

$${}_jX_N = {}_jRX_N \tag{4n, 6}$$

Furthermore the $m - h$ equations in (4n, 2) determine the output of national goods,

$$X_N = {}_NA\ Y \tag{4n, 7}$$

Substitution from (4n, 6) and (4n, 7) in (4n, 5) leads to the final computation formula,

$${}_jX_L = A_{LL\ j}Y_L + A_{LL}\, a_{LN\ j}R\ {}_NA\ Y \tag{4n, 8}$$

B. DETERMINATION OF SUB-REGIONAL OUTPUTS

We now extend the theoretical scheme of the previous section to include production within sub-regions of sub-regionally, regionally, and nationally balanced commodities.

1. *Notation*

There are m commodities:

$$\left.\begin{array}{l}\left.\begin{array}{l}\left.\begin{array}{l}1\\ 2\\ \cdot\\ \cdot\\ \cdot\\ f\end{array}\right\}\text{balanced in each sub-region}\\ f+1\\ f+2\\ \cdot\\ \cdot\\ \cdot\\ h\end{array}\right\}\text{balanced in each region}\\ h+1\\ h+2\\ \cdot\\ \cdot\\ \cdot\\ m\end{array}\right\}\text{balanced in nation}$$

We define the following variables:

OUTPUT AND FINAL DEMAND	COMMODITIES		
	Sub-regionally Balanced	Regionally but Not Sub-regionally Balanced	Nationally Balanced
In sub-region i	${}_iX_S$, ${}_iY_S$	${}_iX_T$, ${}_iY_T$	${}_iX_N$, ${}_iY_N$
In region j	${}_jX_S$, ${}_jY_S$	${}_jX_T$, ${}_jY_T$	${}_jX_N$, ${}_jY_N$
In nation	X_S, Y_S	X_T, Y_T	X_N, Y_N

We also have:

$${}_iX_N = {}_iR_N\, X_N$$

where

$${}_iR_N = \begin{bmatrix} {}_ir_{h+1} & 0 & \cdots & 0 \\ 0 & {}_ir_{h+2} & \cdots & 0 \\ \cdot & & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 0 & 0 & \cdots & {}_ir_m \end{bmatrix}$$

and

$${}_iX_T = {}_iR_T\, {}_jX_T$$

where

$${}_iR_T = \begin{bmatrix} {}_ir_{f+1} & 0 & \cdots & 0 \\ 0 & {}_ir_{f+2} & \cdots & 0 \\ & & & \cdot \\ \cdot & & & \\ \cdot & & & \cdot \\ 0 & 0 & \cdots & {}_ir_h \end{bmatrix}$$

We therefore have the following matrices:

$${}_iX_S = \begin{bmatrix} {}_iX_1 \\ {}_iX_2 \\ \cdot \\ \cdot \\ \cdot \\ {}_iX_f \end{bmatrix} \quad {}_iX_T = \begin{bmatrix} {}_iX_{f+1} \\ {}_iX_{f+2} \\ \cdot \\ \cdot \\ \cdot \\ {}_iX_h \end{bmatrix} \quad {}_iX_N = \begin{bmatrix} {}_iX_{h+1} \\ {}_iX_{h+2} \\ \cdot \\ \cdot \\ \cdot \\ {}_iX_m \end{bmatrix} \quad \text{for all } i$$

$$
{}_jX_S = \begin{bmatrix} {}_jX_1 \\ {}_jX_2 \\ \cdot \\ \cdot \\ \cdot \\ {}_jX_f \end{bmatrix} \quad
{}_jX_T = \begin{bmatrix} {}_jX_{f+1} \\ {}_jX_{f+2} \\ \cdot \\ \cdot \\ \cdot \\ {}_jX_h \end{bmatrix} \quad
{}_jX_N = \begin{bmatrix} {}_jX_{h+1} \\ {}_jX_{h+2} \\ \cdot \\ \cdot \\ \cdot \\ {}_jX_m \end{bmatrix} \quad \text{for all } j
$$

$$
X_S = \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ X_f \end{bmatrix} \quad
X_T = \begin{bmatrix} X_{f+1} \\ X_{f+2} \\ \cdot \\ \cdot \\ X_h \end{bmatrix} \quad
X_N = \begin{bmatrix} X_{h+1} \\ X_{h+2} \\ \cdot \\ \cdot \\ X_m \end{bmatrix}
$$

(and similarly for the Y's).

$$
a_{SS} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1f} \\ a_{21} & a_{22} & \cdot & a_{2f} \\ \cdot & \cdot & & \cdot \\ & \cdot & & \cdot \\ a_{f1} & a_{f2} & \cdots & a_{ff} \end{bmatrix} \qquad
a_{ST} = \begin{bmatrix} a_{1,f+1} & a_{1,f+2} & & a_{1h} \\ a_{2,f+1} & a_{2,f+2} & & a_{2h} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \cdot \\ a_{f,f+1} & a_{f,f+2} & & a_{fh} \end{bmatrix}
$$

$$
a_{SN} = \begin{bmatrix} a_{1,h+1} & a_{1,h+2} & \cdot & a_{1m} \\ a_{2,h+1} & a_{2,h+2} & \cdots & a_{2m} \\ \cdot & \cdot & & \cdot \\ \cdot & & & \\ & \cdot & & \\ a_{f,h+1} & a_{f,h+2} & \cdots & a_{fm} \end{bmatrix}
$$

$$
A_{SS} = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1f} \\ v_{21} & v_{22} & \cdot & v_{2f} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \cdot \\ v_{f1} & v_{f2} & \cdot & v_{ff} \end{bmatrix}
$$

where

$$
v_{k_1k_2} = \frac{\text{co-factor of the element containing } a_{k_2k_1} \text{ in the determinant of } [I - a_{SS}]}{\text{determinant of } [I - a_{SS}]}
$$

Finally A^{-1} can now be written as:

$$
A = \begin{bmatrix}
A_{11} & A_{12} & \cdot & A_{1f} & A_{1,f+1} & & A_{1h} & \cdot & A_{1m} \\
A_{21} & A_{22} & & A_{2f} & A_{2,f+1} & \cdot & A_{2h} & \cdots & A_{2m} \\
\vdots & & & & & & & & \cdot \\
A_{f1} & A_{f2} & \cdot & A_{ff} & A_{f,f+1} & & A_{fh} & \cdots & A_{fm} \\
\vdots & & & \cdot & & & \vdots & & \cdot \\
A_{h1} & A_{h2} & \cdots & A_{hf} & A_{h,f+1} & \cdots & A_{hh} & \cdot & A_{hm} \\
\vdots & & & & & & \cdot & & \vdots \\
A_{m1} & A_{m2} & \cdot & A_{mf} & A_{m,f+1} & \cdots & A_{mh} & \cdots & A_{mm}
\end{bmatrix}
$$

$$
= \begin{bmatrix} {}_SA \\ {}_TA \\ {}_NA \end{bmatrix} = \begin{bmatrix} {}_LA \\ {}_NA \end{bmatrix}
$$

2. *Sets of equations*

By definition,

$$ {}_iX_N = {}_iR_N \; X_N \tag{4n, 9} $$

As before,

$$ X_N = {}_NA \; Y \tag{4n, 10} $$

Therefore,

$$ {}_iX_N = {}_iR_N \; {}_NA \; Y \tag{4n, 11} $$

gives the output of nationally balanced goods in sub-region i.

We also defined

$$ {}_iX_T = {}_iR_T \; {}_jX_T \tag{4n, 12} $$

and from (4n, 8) we can write

$$ {}_jX_T = {}_TA_{LL} \; {}_jY_L + {}_TA_{LL} \; a_{LN} \; {}_jR \; {}_NA \; Y \tag{4n, 13} $$

Substituting (4n, 13) in (4n, 12), we get the output of regionally balanced goods in sub-region i:

$$ {}_iX_T = {}_iR_T({}_TA_{LL} \; {}_jY_L + {}_TA_{LL} \; a_{LN} \; {}_jR \; {}_NA \; Y) \tag{4n, 14} $$

where the expression $a_{LN} \; {}_jR \; {}_NA \; Y$ has already been computed for (4n, 8).

The balance equation for the sub-regional outputs in any sub-region i is

$$[I - a_{SS}, \ -a_{ST}, \ -a_{SN}]\begin{bmatrix} {}_iX_S \\ {}_iX_T \\ {}_iX_N \end{bmatrix} = {}_iY_S \qquad (4n, 15)$$

Multiplied out on the left side it gives

$$[I - a_{SS}]\, {}_iX_S - a_{ST}\, {}_iX_T - a_{SN}\, {}_iX_N = {}_iY_S \qquad (4n, 15a)$$

Transposing and multiplying by A_{SS} we get

$${}_iX_S = A_{SS}\, {}_iY_S + A_{SS}\, a_{ST}\, {}_iX_T + A_{SS}\, a_{SN}\, {}_iX_N \qquad (4n, 16)$$

$${}_iX_S = A_{SS}\, {}_iY_S + A_{SS}\, (a_{ST}\, {}_iX_T + a_{SN}\, {}_iX_N) \qquad (4n, 16a)$$

Substituting from (4n, 14) and (4n, 11) for ${}_iX_T$ and ${}_iX_N$ we get the final computation formula for the outputs of sub-regionally balanced goods in the sub-region i:

$$\begin{aligned} {}_iX_S = A_{SS}\, {}_iY_S + A_{SS}[a_{ST}\, {}_iR_T({}_TA_{LL}\, {}_jY_L \\ + {}_TA_{LL}\, a_{LN}\, {}_jR\, {}_NA\ Y) + a_{SN}\, {}_iR_N\, {}_NA\ Y] \end{aligned} \qquad (4n, 17)$$

which can also be written

$$\begin{aligned} {}_iX_S = A_{SS}\, {}_iY_S + A_{SS}\, a_{ST}\, {}_iR_T\, {}_TA_{LL}\, {}_jY_L \\ + A_{SS}(a_{ST}\, {}_iR_T\, {}_TA_{LL}\, a_{LN}\, {}_jR_N + a_{SN}\, {}_iR_N)\, {}_NA\ Y \end{aligned} \qquad (4n, 18)$$

II. Open Dynamic System of Interregional Input-Output Relations

This system represents a combination of the dynamic scheme developed in Chapter 3 with sets of interregional relationships described in Part I of this note. The analysis is conducted in terms of two groups of commodities, the nationally and the regionally balanced.

A. NOTATION

In addition to the definition given in the first part of this note, the following structural constants will be used.

b_{ik} = stock coefficient of commodity i in industry k

$$b_{LL} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1h} \\ b_{21} & b_{22} & \cdots & b_{2h} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ b_{h1} & b_{h2} & \cdots & b_{hh} \end{bmatrix} \qquad b_{LN} = \begin{bmatrix} b_{1,h+1} & b_{1,h+2} & \cdots & b_{1m} \\ b_{2,h+1} & b_{2,h+2} & \cdots & b_{2m} \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ b_{h,h+1} & b_{h,h+2} & \cdots & b_{hm} \end{bmatrix}$$

$$b_{NL} = \begin{bmatrix} b_{h+1,1} & b_{h+1,2} & \cdots & b_{h+1,h} \\ b_{h+2,1} & b_{h+2,2} & \cdots & b_{h+2,h} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ b_{m1} & b_{m2} & \cdots & b_{mh} \end{bmatrix} \qquad b_{NN} = \begin{bmatrix} b_{h+1,h+1} & b_{h+1,h+2} & \cdots & b_{h+1,m} \\ b_{h+2,h+1} & b_{h+2,h+2} & \cdots & b_{h+2,m} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ b_{m,h+1} & b_{m,h+2} & \cdots & b_{mm} \end{bmatrix}$$

$$b = \begin{bmatrix} b_{LL} & b_{LN} \\ b_{NL} & b_{NN} \end{bmatrix}$$

B. SETS OF EQUATIONS

$$X_L = \sum_{j=1}^{h} {}_jX_L \qquad (4n, 19)$$

These are definitional equations which state that the aggregate national output of each regional commodity is the sum of its regional outputs. There are h equations in this set.

$${}_jX_N = {}_jR\, X_N \qquad (j = 1, 2, \cdots, n) \quad (4n, 20)$$

These $(m - h)n$ equations describe the fixed regional distribution of outputs of all the nationally balanced commodities.

$$[I - a_{NN}]X_N - a_{NL}\, X_L - \sum_{j=1}^{n} {}_jb_{NN}\, \dot{X}_N - \sum_{j=1}^{n} b_{NL}\, {}_j\dot{X}_L = +{}_jY_L \quad (4n, 21)$$

This set comprises the $m - h$ dynamic balance equations for all the national commodities. The subscripts j do not modify the values of the matrices (i.e. ${}_jb_{NN} \equiv b_{NN}$ for all j). They are used only to identify the same capital coefficient as used to determine the capital requirements of the same industry in different regions.

$$[I - a_{LL}]_jX_L - a_{LN}\, {}_jX_N - {}_jb_{LL}\, {}_j\dot{X}_L - {}_jb_{LN}\, {}_j\dot{X}_N = +{}_jY_L$$
$$(j = 1, 2, \cdots, n) \quad (4n, 22)$$

This set comprises the $h \cdot n$ dynamic balance equations for all the regional commodities.

The four sets (4n, 19-22) comprise $m(n + 1)$ first-order linear differential equations with constant coefficients. They can be solved for the set of $m(n + 1)$ unknowns comprising the m total national and $m \cdot n$ regional outputs of all the m industries included in the system.

The irreversibility of regionally invested stocks must be taken into account through suppression of the stock coefficients, b_{ik}, in those elements $b_{ik}\, {}_j\dot{X}_k$ (in the third and fourth terms in sets (4n, 21) and (4n, 22)) in which either ${}_j\dot{X}_k < 0$ or $S_{ik} - b_{ik}\, {}_jX_k > 0$, i.e. in which the output goes down or the available stock, ${}_jS_{ik}$, exceeds the current regional capital requirements, $b_{ik}\, {}_jX_k$, of the corresponding industry.

Chapter 5

SOME EMPIRICAL RESULTS AND PROBLEMS OF REGIONAL INPUT-OUTPUT ANALYSIS [1]

Walter Isard

THE PREVIOUS chapter has developed an input-output model for regional and intra-national analysis. In the first section of this chapter we shall examine the empirical implications of certain hypothetical operations with the model. In succeeding sections we shall discuss how the existing data have been classified and processed into the appropriate forms for use in the model. Though, in fact, the classification and processing of the data preceded investigation into empirical implications, it seems more advisable to present the empirical implications first in order that the reader may have clearly in mind some of the purposes to which the model may be put.

I. Empirical Implications

One of the basic assumptions of this scheme is that the percentage-wise distribution of the total output of each national commodity by regions in which it is produced remains the same whether output rises or falls. Regions expand and contract their output of any national commodity proportionately. Clearly, then, the model has most usefulness when actual conditions are such that this assumption approximates reality, *ceteris paribus;* or more important, when the conditions of any problem are such that the most realistic assumption one can make is to postulate that the percentage-wise geographic distribution of the output of any national commodity remains unchanged. These latter conditions obtain particularly when the problem is to examine the regional implications of national projections of various national industries, when, as is frequently the case, there is no advance knowledge of how any new projected capacity will be distributed geographically and when locational and regional comparative cost studies are lacking. We may ask, for example, what are the

[1] The author is indebted to Leon Moses and Robert A Kavesh for their invaluable services in carrying through the statistical operations. Others who at one time or another have rendered valuable assistance are Robert L Allen, Paul McGouldrick, Ruth Metzger, William Mennick.

regional implications of a 10 per cent increase in the national output of the chemicals industry, or of textiles, or of tank production, or of the aircraft industry. Without knowing beforehand where new production will be located, the model as it is now designed is particularly valuable in answering such questions.[2]

On the other hand, this scheme is not as useful when the basic question is turned around to read: What are the national implications of regional projections? For here, consideration of any change in the regional output of a national commodity without a corresponding percentage change in its output in other regions tends to be inconsistent with the above assumption. This inconsistency can be largely removed by separating out first-round effects.[3] However, to answer this type of question without such separation, models of somewhat different design are required.[4] They will engage us in the future.

Accordingly, the empirical material presented is designed to answer the first general type of question: What are the regional implications of given national projections? Since any projection of a national industry can be translated into a set of projections of input requirements, and thus as a set of increases in the corresponding final demands, Table 1 has been constructed to reveal regional implications of changes in the final demand for any separate bill-of goods item, given the structural relations of the American economy in 1939. More specifically, we have asked, for example, what a 10 per cent change in the final demand for agriculture would have implied for the output of each of the four regional industries and households in both Major Regions Id and IId, and for the five sub-regional industries in Sub-Region A.[5]

[2] Or we may wish to determine beforehand what the impact of a 10 per cent increase in national production of chemicals might be upon power requirements in the several regions, assuming that the percentage-wise distribution of chemical output among regions does not change. Given this knowledge, we are in a position to judge at least to some extent where resulting power shortages might occur and to plan a regional distribution of new chemical output so as to minimize future power shortages.

[3] If a limited number of regional projections are made, these regional projections can be translated into regional input requirements and thus the required changes in the regional bills of goods. Once the direct, first round effects are accounted for and put into the regional bills of goods, the model can be applied once again.

[4] For elaboration of this point, see Isard, W., and Freutel, G., 'Regional and National Product Projections and their Interrelations,' forthcoming in *Studies in Income and Wealth* (Conference on Research in Income and Wealth, 1951); and Isard, W., 'Interregional and Regional Input-Output Analysis: A Model of a Space-Economy,' *Review of Economics and Statistics,* November 1951.

[5] Major Region Id comprises the New England, Middle Atlantic, South Atlantic, and East South Central census regions and the state of Ohio; Major Region IId comprises the remainder of the United States; and Sub-Region A embraces part of Major Region IId, namely, Washington, Oregon, California, Nevada, and Arizona.

TABLE 1

Effect of a 10 Per Cent Increase in the Final Demand for the Output of Each National Industry Upon the Output of Households and Each Regional Industry of Major Regions Id and IId and Upon the Output of each Sub-Regional Industry of Sub-Region A (in per cent figures)

	1 Agriculture and Fishing	2 Food Processing	3 Ferrous Metals	4 Iron and Steel Foundry Products	5 Shipbuilding	6 Agricultural Machinery
	(1)	(2)	(3)	(4)	(5)	(6)
Major Region Id						
(1) 35 Trade	0.191	0.064	0.046	0.002	0.107	0.072
(2) 21 Communications	0.202	0.069	0.052	0.003	0.108	0.082
(3) 37 Eating and drinking places	0.200	0.064	0.049	0.003	0.120	0.075
(4) 38 Unallocated	0.206	0.077	0.062	0.003	0.096	0.097
(5) 39 Households	0.200	0.064	0.049	0.003	0.120	0.075
Major Region IId						
(6) 35 Trade	0.277	0.082	0.035	0.002	0.066	0.128
(7) 21 Communications	0.284	0.088	0.040	0.002	0.071	0.150
(8) 37 Eating and drinking places	0.292	0.082	0.037	0.002	0.070	0.129
(9) 38 Unallocated	0.273	0.094	0.046	0.003	0.075	0.196
(10) 39 Households	0.292	0.082	0.037	0.002	0.070	0.129
Sub-Region A						
(11) 36 Business and personal services	0.269	0.083	0.036	[1]	0.083	0.103
(12) 32 Construction	0.000	0.000	0.000	[1]	0.000	0.000
(13) 34 Steam railroad transportation	0.295	0.093	0.043	[1]	0.076	0.108
(14) 20 Manufactured gas and electric power	0.244	0.081	0.043	[1]	0.083	0.121
(15) 39 Households	0.248	0.074	0.033	[1]	0.083	0.095

[1] No figure available since the effects upon Sub-Region A were derived under conditions where Sub-Region A was part of Major Region II for which iron and steel foundry products was designated non-national. *See below* for further explanation.

TABLE 1 (Continued)

7 Engines and Turbines	8 Motor Vehicles	9 Aircraft	10 Transportation Equipment, n.e.c.	11 Industrial and Heating Equipment, n.e.c.	12 Machine Tools	13 Merchandising and Service Machines	14 Electrical Machinery, n.e.c.
(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
0.015	0.221	0.074	0.034	0.280	0.080	0.050	0.172
0.018	0.219	0.088	0.036	0.340	0.102	0.060	0.206
0.016	0.232	0.079	0.036	0.297	0.083	0.054	0.181
0.023	0.263	0.110	0.038	0.426	0.143	0.074	0.261
0.016	0.232	0.079	0.036	0.297	0.083	0.054	0.181
0.010	0.300	0.072	0.033	0.276	0.066	0.050	0.153
0.012	0.321	0.085	0.034	0.330	0.079	0.059	0.179
0.010	0.312	0.076	0.035	0.290	0.067	0.054	0.159
0.014	0.271	0.105	0.036	0.412	0.105	0.073	0.221
0.010	0.312	0.076	0.035	0.290	0.067	0.054	0.159
0.012	0.236	0.117	0.032	0.280	0.067	0.054	0.148
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.012	0.262	0.114	0.031	0.269	0.067	0.046	0.154
0.014	0.252	0.107	0.034	0.331	0.082	0.055	0.172
0.011	0.220	0.123	0.031	0.264	0.061	0.044	0.136

TABLE 1 (Continued)

Effect of a 10 Per Cent Increase in the Final Demand for the Output of Each National Industry Upon the Output of Households and Each Regional Industry of Major Regions Id and IId and Upon the Output of each Sub-Regional Industry of Sub-Region A (in per cent figures)

15 Iron and Steel n.e.c.	16 Nonferrous Metals and Their Products	17 Nonmetallic Minerals and Their Products	18 Petroleum Products and Refining	19 Coal Mining and Manufactured Solid Fuel	22 Chemicals	23 Lumber and Timber Products	24 Furniture
(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
0.068	0.082	0.026	0.069	0.022	0.062	0.011	0.071
0.081	0.095	0.031	0.080	0.024	0.070	0.013	0.078
0.072	0.088	0.028	0.073	0.025	0.060	0.012	0.072
0.099	0.113	0.037	0.093	0.022	0.084	0.015	0.087
0.072	0.088	0.028	0.073	0.025	0.060	0.012	0.072
0.062	0.091	0.025	0.114	0.015	0.053	0.016	0.071
0.072	0.106	0.029	0.145	0.016	0.059	0.018	0.078
0.065	0.098	0.026	0.119	0.016	0.053	0.017	0.072
0.088	0.126	0.034	0.203	0.017	0.070	0.021	0.086
0.065	0.098	0.026	0.119	0.016	0.053	0.017	0.072
0.064	0.100	0.027	0.117	0.015	0.057	0.023	0.078
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.073	0.122	0.050	0.208	0.019	0.075	0.080	0.116
0.074	0.155	0.035	0.129	0.016	0.065	0.023	0.081
0.060	0.099	0.026	0.109	0.014	0.050	0.025	0.076

Column 1 of Table 1 provides the answer; namely, that:

(1) In Major Region Id the output of trade would have increased by 0.19 per cent, that of communications by 0.20 per cent, that of eating and drinking places by 0.20 per cent, and that of households (i.e. wages and salaries and income of unincorporated businesses) by 0.20 per cent;

(2) In Major Region IId, the output of trade would have increased by 0.28 per cent, that of communications by 0.28 per cent, that of eating and drinking places by 0.29 per cent, and that of households by 0.29 per cent;

(3) In Sub-Region A, the output of business and personal services would have increased by 0.27 per cent, that of construction by 0.00 per cent,[6] that of steam railroad transportation by 0.29 per cent; that of manufactured gas and electric power by 0.24 per cent and that of households by 0.25 per cent.

[6] That construction activity remains unaffected by any change in the final demand for agriculture and for any other bill-of-goods item (except construction itself) merely

TABLE 1 (Continued)

25 Wood Pulp and Paper	26 Printing and Publishing	27 Textile Mill Products	28 Apparel and other Finished Textile Products	29 Leather and Leather Products	30 Rubber	31 All Other Manufacturing	33 Miscellaneous Transportation
(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
0.018	0.034	0.053	0.024	0.026	0.023	0.057	0.015
0.019	0.042	0.060	0.025	0.032	0.026	0.077	0.016
0.017	0.036	0.055	0.024	0.026	0.024	0.068	0.017
0.022	0.041	0.070	0.026	0.044	0.031	0.092	0.014
0.018	0.036	0.055	0.024	0.026	0.024	0.068	0.017
0.016	0.032	0.032	0.014	0.020	0.017	0.056	0.017
0.017	0.038	0.035	0.016	0.024	0.019	0.064	0.018
0.016	0.034	0.033	0.015	0.020	0.018	0.057	0.020
0.020	0.037	0.038	0.016	0.030	0.022	0.074	0.017
0.016	0.034	0.033	0.015	0.020	0.018	0.057	0.020
0.016	0.033	0.037	0.015	0.020	0.022	0.066	0.020
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.024	0.030	0.039	0.016	0.020	0.020	0.054	0.016
0.019	0.035	0.040	0.016	0.022	0.024	0.080	0.019
0.015	0.032	0.034	0.014	0.018	0.019	0.066	0.019

In a similar manner, Column 2 traces out the regional implications of a 10 per cent increase in the final demand for food processing alone, Column 3 of a 10 per cent increase in the final demand for ferrous metals alone, and so forth. Actually, a projection will generally involve several concomitant and unequal percentage increases in the final demands for the outputs of several industries, and thus will in turn involve the summation of the implications of these increases.

The reader will note that the data have been put into percentage form. This tends to minimize the distortions that may arise out of the imperfections of the model which will be discussed in subsequent sections. It is reasonable to presume that the distortions will be present approximately to the same relative extent when modest changes in final demands

reflects the fact that the entire demand for construction activity has been viewed as final demand and as part of the bill of goods. Thus any meaningful projection of a bill-of-goods item other than construction must be accompanied by a consistent projection of the construction bill-of-goods item. This one would normally do to some extent when an industry projection is translated into a set of input requirements and hence into a set of projections of bill-of-goods items.

are hypothesized. As a consequence, changes expressed percentage-wise tend to reflect distortions less than when put in absolute dollar terms.[7]

As will be apparent later, the empirical information on sub-regional implications is on a firmer basis than that on major regional implications, since the distortions from imperfections of the model are smaller for the sub-region than for the major region. Nevertheless the reader is cautioned against the use of this empirical information as it now stands. Improvement in the design of the model and in sources of data are required before the computed results or implications can be recommended for consideration for policy purposes.

At the moment, however, we can use the empirical information in Table 1 to focus attention upon contrasts in the reactions of various regions to any given change in final demand. These contrasts will have considerable significance when more reliable empirical information is forthcoming from improved models. They can be illustrated by Charts 1a and 1b. Chart 1a shows within the framework of our scheme the differential effect of a 10 per cent increase in the final demand for machine tools upon households and each of the four regional industries of both major regions. The impact upon Major Region Id is the greater in the case of each of these five industries. The reverse situation is revealed in Chart 1b which depicts the differential impact of a 10 per cent increase in the final demand for petroleum products and refining upon each of these five in-

[7] It may be of interest, however, to have in mind the absolute dollar increase in output which corresponds to the 10 per cent increase in each of the final demands. They are presented in the following table.

Absolute Dollar Increases of Output Corresponding to 10 Per Cent Increases in Final Demand of Each National Industry (in millions of dollars)

	Industry	
1	Agriculture and fishing	$ 73.6
2	Food processing	28.6
3	Ferrous metals	17.9
4	Iron and steel foundry	0.9
5	Shipbuilding	37.1
6	Agricultural machinery	37.2
7	Engines and turbines	4.8
8	Motor vehicles	100.1
9	Aircraft	25.9
10	Transportation equipment, n.e.c.	17.5
11	Industrial and heating equipment, n.e.c.	105.2
12	Machine tools	36.1
13	Merchandising and service machines	19.5
14	Electric equipment, n.e.c.	58.5
15	Iron and steel, n.e.c.	25.5
16	Nonferrous metals and their products	49.3
17	Nonmetallic minerals and their products	$ 10.4
18	Petroleum products and refining	50.2
19	Coal mining and manufactured solid fuel	7.5
22	Chemicals	24.3
23	Lumber and timber products	5.0
24	Furniture and other manufactured wood	22.5
25	Wood pulp and paper	6.8
26	Printing and publishing	12.2
27	Textile mill products	14.9
28	Apparel and other finished textile products	6.6
29	Leather and leather products	5.8
30	Rubber	9.9
31	All other manufactures	23.1
33	Misc. & transoceanic trans.	10.1

dustries of the two major regions.[8] Also, in Chart 2 we illustrate the hypothetical effect of a 10 per cent increase in the final demand for the output of each of the national industries upon employment in the five western states contained in Sub-Region A.

II. Classification of Industries

With this preliminary statement of empirical implications as a background, we can now discuss the more salient problems, namely, how the existing data were classified and processed for the operation of the model, and the derivation of these implications.

The first major step was to obtain for every state data on value of production for each of the component parts of the 96 industries used in the 1939 input-output table. Though it was clear at the start that such a fine classification would not be used in its full detail for the purposes of subsequent computation, this classification was adopted for two reasons. The extent to which the number of industries would ultimately have to be reduced was not apparent at the outset. More important, it was recognized that the standard conversions previously employed by Professor Leontief and the Bureau of Labor Statistics—e.g. the consolidations of the 96 industries into 43 and 22 respectively for the 1939 data[9]—might not be the most desirable for regional input-output analysis.

Regional analysis introduces another vital consideration. Industries in any consolidated category should be homogeneous with respect to market areas. To include in the food-processing category (1) the slaughtering and meat-packing industry whose market area tends to be national or regional of a low order[10] and (2) the bread and bakery products industry whose market area is much more limited in scope, blurs results. The classification of the food-processing category as a regional commodity of a certain order becomes not only more difficult, but also less significant. Development of a more appropriate consolidation, and one which will incidentally throw much more light than is currently shed on the channels through which interregional impulses are transmitted, is then an area for future research. Our initial data can serve as basic raw material.

The chief sources of output data by states were the *Census of Agriculture*, *Census of Manufactures*, *Census of Mineral Industries*, *Census of Business*, and various *I.C.C. Reports*. In several cases state data had to be

[8] These contrasts, both by region and by industry, can in turn be broken down into contrasts in terms of direct effects alone, and those in terms of indirect effects alone. The particular set of contrasts which is of most value in any given study depends obviously upon the purpose and scope of such a study.

[9] See Chapter 9, Appendix 2, Table 1, for these consolidations.

[10] Our general classification of commodities runs from national, regional of the first order, regional of the second order $\cdots$, regional of the $(n-1)$ order, and finally local (or regional of the n^{th} order).

CHART IA

EFFECTS OF A 10% INCREASE IN THE FINAL DEMAND FOR MACHINE TOOLS UPON HOUSEHOLDS AND FOUR REGIONAL INDUSTRIES OF MAJOR REGIONS I D AND II D

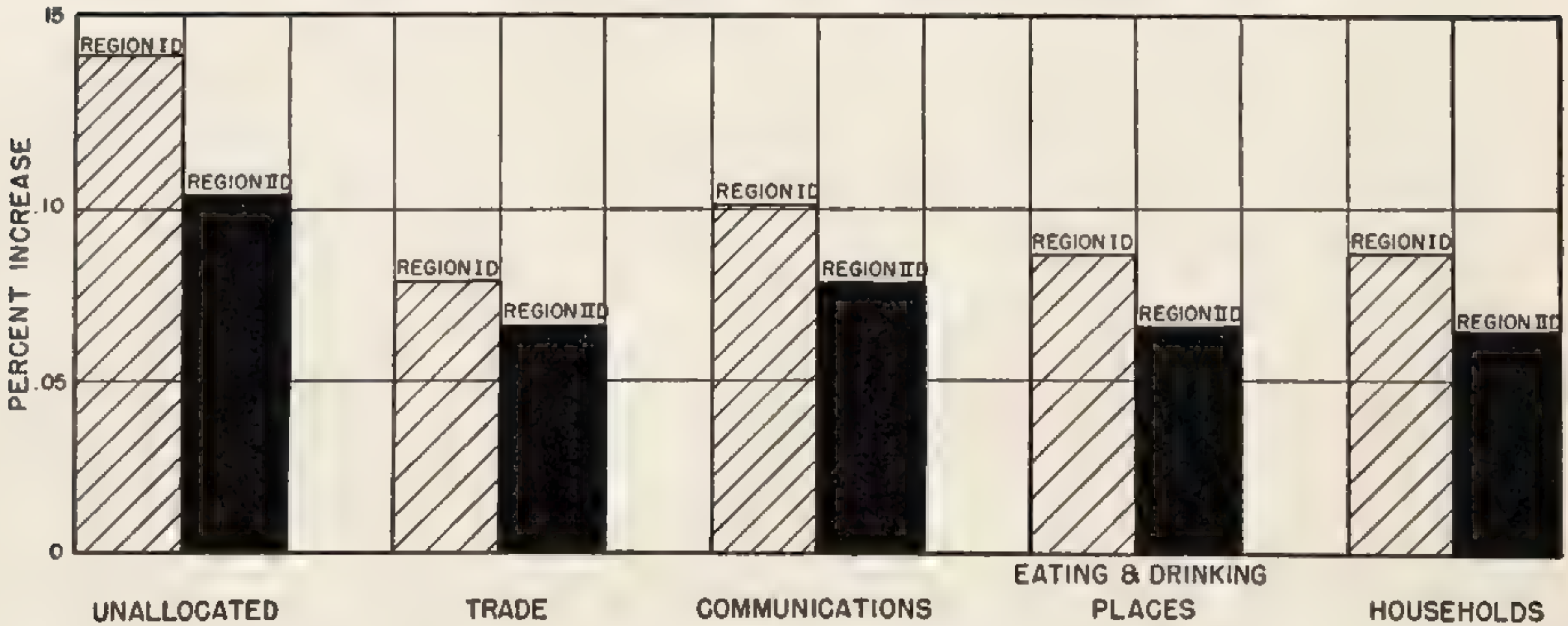


CHART I B

EFFECTS OF A 10% INCREASE IN THE FINAL DEMAND FOR PETROLEUM PRODUCTS AND REFINING UPON HOUSEHOLDS AND FOUR REGIONAL INDUSTRIES OF MAJOR REGIONS I D AND II D

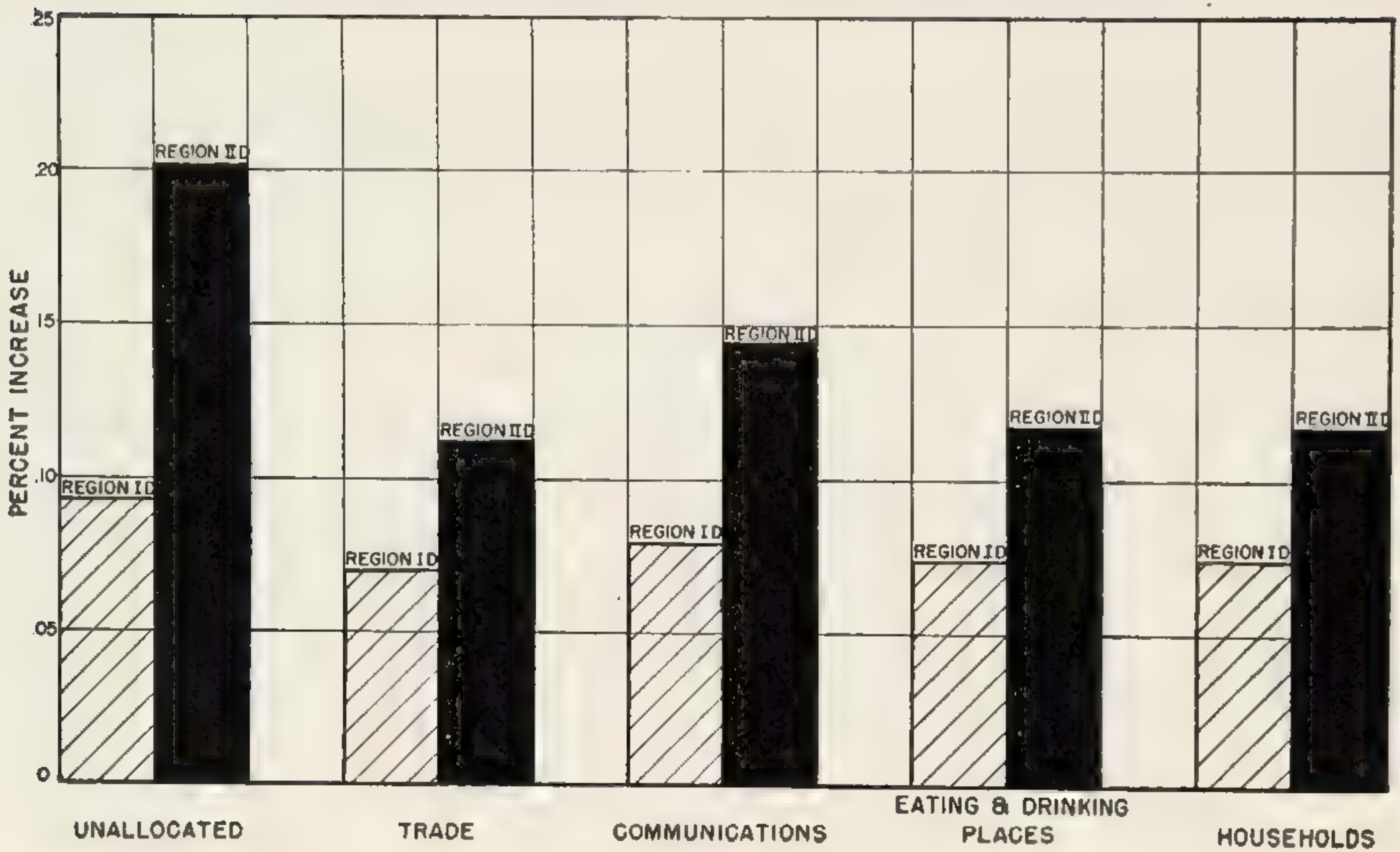
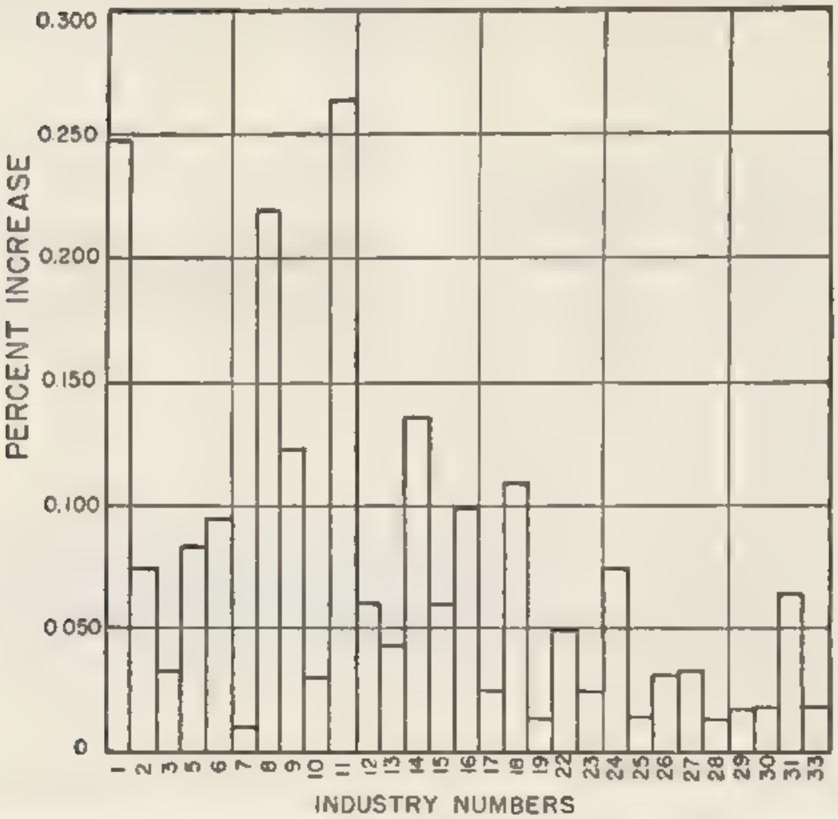


CHART 2

EFFECTS OF A 10% INCREASE IN THE FINAL DEMAND FOR EACH NATIONAL INDUSTRY UPON HOUSEHOLDS (EMPLOYMENT) IN SUB-REGION A



estimated from information on employment, income, costs, standard margins, and so forth.[11]

A major difficulty stemmed from the failure of the censuses of business and manufactures to report value of production by states when this would have disclosed production data relating to individual companies. Rather than set up a fictitious state, 'others,'[12] we chose to distribute among the

[11] For example, we used operating expenses by states as given in the *Census of Business*, 1939, *Wholesale Trade*, p. 48, as indicative of the value of wholesale trade services by states.

[12] We actually carried through an extensive experiment in setting up such a fictitious state analogous to the 'undistributed' industry in the ordinary input-output table. We found that in certain industries a large fraction of their output would be assigned to 'others' (for example, approximately 51 per cent in the case of shipbuilding, and 75 per cent in the case of aluminum products). In other industries, only an insignificant fraction of their output would be theoretically produced in the state, 'others' (for example, approximately 0.04 per cent in petroleum and natural gas, and 0.17 per cent in communications). Obviously we would then have a state with *nonsensical* structural relations. This in turn would detract from the meaning of structural relations in actual states and regions formed from these states. In a preliminary operation of the model with the fictitious state 'others' as a region, the results obtained when we compared computed outputs with actual outputs (to be described below) were poor and did not seem to warrant any further experimentation along this line.

individual states for which data were unreported the total assigned by the Census to any group of those states. Sources of data other than Census were used to provide guidance.[13] Where appropriate information from additional sources was not available, the following procedures were used. First, the distribution among states was made on the basis of employment in a given industry. Since the *Census of Manufactures* reports total employment for each state in each industry in which there are three or more establishments, the allocation of total values for groups of unreported states was made mostly on the basis of employment in an unreported state expressed as a percentage of total employment in the relevant group of unreported states. Second, where unreported states possessed fewer than three establishments, allocation was made on the basis of number of establishments in an unreported state expressed as a percentage of total number of establishments in the relevant group of unreported states. Finally, as with certain *Census of Business* items, when even data on number of establishments were lacking, 'others' was divided equally among all the unreported states.[14]

Limited research resources early compelled us to fall back upon the use of the 38×38 inverted matrix formerly derived for the United States economy, 1939.[15] Hence, a consolidation into 39-industry groups was performed. Households was included as the 39th industry for reasons enumerated in the previous chapter.[16]

[13] For example, in distributing sugar refining among states included in 'others' we used physical product data from 'Report of the United States Beet Sugar Association,' and 'Sugar Cane and Cane Sugar Production,' *Facts About Sugar*, April 1940, p. 49, and May 1940, p. 38, respectively.

[14] Value for any industry or service reported for Washington, D. C., was arbitrarily distributed equally between Virginia and Maryland. It should be noted that Census data may not reveal figures on individual enterprises when only the total for a consolidated category is reported for a given state. This type of data which might thus be publicly available is not used, however. To do so would interfere with the analysis of the market areas of the component industries which is crucial in the early stage of regional input-output model development.

[15] A comparison of the 96-industry and the 38-consolidated industry classifications is given in the first two columns of Table 1, Appendix 2, Chapter 9. The classification used in this chapter is the same as the 38-industry classification noted above, except that (1) households is considered as industry 39 (rather than as industry 41); (2) foreign trade is put into the bill of goods along with government, stocks, and capital goods; and (3) no bill-of-goods industry is given a number.

[16] See pp. 100-1. The technique for entering households into the structural system by bordering the 38×38 inverted matrix and thus converting it into a 39×39 inverted matrix is described in any standard treatise on matrix algebra.

The elements for the households (39th) column of the coefficient matrix were derived from corresponding data in Column 41 in the table facing page A-14, Appendix A, *Full Employment Patterns*, 1950 (U. S. Department of Labor, Bureau of Labor Statistics, May 1946). The elements for the households (39th) row of the coefficient matrix were derived from estimates of wages and salaries and income of unincorporated businesses generated by each of the 38 other industries. These estimates were based upon data from *Survey of Current Business, National Income Supplement*, July 1947; *Statistical Abstract* 1941; and censuses of mineral industries, manufactures, and construction.

At this stage it was necessary to adjust our state data on value of output so that for any industrial group the total value for all states would be consistent with the data recorded on a national basis alone. These latter data are used by the Bureau of Labor Statistics (B.L.S.) and are basic to the computation of the elements of the above inverted matrix.

Aside from inaccuracies in computations and differences in sources, these two sets of data diverge on two counts. First, the B.L.S. data are in purchasers' value, including the value of transportation services and trade margins on the product of the industry. Ours are in manufacturers' value. Thus, an upward adjustment of our data is required. Second, the B.L.S. data are *net* in the sense that the figure on any industry's value of national production does not include the value of its own product which it consumes. Ours are *gross* in that intra-industry transfers are not eliminated from the state data. Here, a downward adjustment is required. To make these adjustments properly would have necessitated detailed and time-consuming studies which we were not in a position to pursue. Lacking any more reasonable short-cut procedure we adopted the expedient of adjusting the data for each state to the same relative extent. For any given industry, each state's value of output was multiplied by the same constant factor The adjusted data on value of output for the state of Ohio, for example, are recorded for the 39 consolidated industries in Column 2 of Table 2.[17]

After we had adjusted the data on value of output for each consolidated industry for each state, the next step was to designate commodities as national, regional of various orders, and local (regional of the n^{th} order). The ideal procedure would have been to construct maps on the spatial flows of each commodity and service within the United States This task, which logically is one for the economic geographer, was beyond our means; and for 1939 the full array of required information was not available.[18] Hence, we adopted an indirect procedure involving an approximation of these flows. This, for reasons to be detailed below, required first the derivation of estimates of the consumption of each commodity and service by states.

Consumption estimates were derived in two steps. First, the requirements of various inputs by the industries of any given state were calcu-

[17] For each state, the amount of production in Undistributed was obtained by multiplying the total amount of undistributed for the nation by the ratio of the value of output for all industries (excluding households) recorded for each state to the value of production of these industries for the nation as a whole.

[18] More and better data are now being published: for example, the I.C.C. quarterly reports on state to state shipments of individual commodities by Class I railroads based on a 1 per cent sample. In addition, geographers have now intensively undertaken the mapping of flow phenomena. Another area for future research, therefore, lies in the development of techniques and methods to utilize the expanding range of data on flows in models for current and future years.

lated, since obviously all these input requirements represent consumption by that state. For any Industry A, the value of output required of it by all other industries in a given state was calculated by: (1) multiplying the value of the output of each of these industries by the corresponding technical production coefficients (a_{iA}) which represent the value of the product of Industry A absorbed by each of these industries per dollar value of output;[19] and (2) summing these products.

Second, the direct allocation of various inputs to the bill-of-goods sector for which any given state is responsible were approximated. In our model the national bill of goods for 1939 consists of the input requirements of four activities: production of goods for foreign trade, government, production of capital goods, and production of goods for inventories (stocks). The per cent of each of these activities performed in each state was estimated: production for foreign trade on the basis of data on exports (including re-exports) by customs districts;[20] production of

[19] Since the technical production coefficients are based upon national indices (averages), this operation assumes that for each industry production practices among regions are alike. For many industries this postulate approximates reality when one considers industrial inputs at the site of production. An area for future research, however, is the isolation of industries for which this assumption is not warranted (e.g. the power industry) and the determination for these industries of differences in production practices among regions.

[20] As obtained from the *Statistical Abstract*, 1942, p. 568. A number of adjustments had to be made. Where a customs district includes two adjacent states, exports for the customs district were divided equally between the two states. Where a customs district is listed by port of origin, the data for such a district were assigned to the state in which the port is located. Excluded from the totals were parcel post and exports from Puerto Rico, Virgin Islands, Alaska, and Hawaii.

The per cent allocation of production for foreign trade by states is crude on two scores. First, exports which are assigned to a state, because a customs district is designated by the state's name or by the name of a port which lies within the state, may actually be shipped from a port outside that state. For example, part of New Jersey falls within the New York customs district. By our procedure any exports from northeastern New Jersey ports are incorrectly assigned to the state of New York.

Second, aside from the previous point, the state in which a port of origin is located or after which a customs district is named is not necessarily the state of origin of any particular export. Pittsburgh steel shipped out of New York is, by our procedure, erroneously credited to New York as the state of origin. In general, the production for foreign trade of interior states is understated; that of coastal states, overstated. However, where states are combined into meaningful regions which tend to coincide with the hinterlands served by chief ports, as frequently they are combined, then the errors of under- and over-statement for states of a region balance out for regional analysis.

Since foreign trade is a relatively minor sector of the bill of goods, and since a considerable amount of export sales data has been collected in the 1947 *Census of Manufactures and Census of Business* which should permit direct assignment among states of production of goods for foreign trade for future models based on 1947 data, it was deemed inexpedient to spend the considerable resources which would be required for developing a finer, indirect technique of assigning 1939 production for foreign trade among states.

TABLE 2

Production, Consumption, Absolute, and Percentage Surpluses and Deficits, by Industries In Ohio, 1939

Industry	Recorded Production ($000) (1)	Adjusted Production ($000) (2)
1 Agriculture and Fishing	420168	420168
2 Food Processing	573944	599943
3 Ferrous Metals	757503	568809
4 Iron and Steel Foundry Products	76482	81262
5 Shipbuilding	2878	3730
6 Agricultural Machinery	9591	10093
7 Engines and Turbines	19855	19708
8 Motor Vehicles	285719	182174
9 Aircraft	5258	5070
10 Transportation Equipment, n.e.c.	33012	33137
11 Industrial and Heating Equipment, n.e.c.	320705	317562
12 Machine Tools	118936	118400
13 Merchandising and Service Machines	58072	58780
14 Electrical Machinery, n.e.c.	307271	267172
15 Iron and Steel, n.e.c.	281175	286321
16 Nonferrous metals and Their Products	126148	81996
17 Nonmetallic Minerals and Their Products	207481	228561
18 Petroleum Products and Refining	94563	111338
19 Coal Mining and Manufactured Solid Fuel	77342	105185
20 Manufactured Gas and Electric Power	142465	153677
21 Communications	80201	79784
22 Chemicals	268415	241600
23 Lumber and Timber Products	19341	19294
24 Furniture	87289	88406
25 Wood Pulp and Paper	145904	123654
26 Printing and Publishing	158545	146940
27 Textile Mill Products	56819	51898
28 Apparel and Other Textile Products	110581	100120
29 Leather and Leather Products	68007	48435
30 Rubber	304825	303118
31 All Other Manufacturing	68654	65207
32 Construction	571066	581916
33 Miscellaneous Transportation	136999	170345
34 Steam Railroad Transportation	328573	290261
35 Trade	947003	907608
36 Business and Personal Services	1277391	1019230
37 Eating and Drinking Places	247376	250171
38 Unallocated	1032688	1387210
39 Households	4154000	4008610

TABLE 2 (Continued)

Consumption excluding Bill of Goods ($000)	Bill of Goods Requirement ($000)	Total Consumption ($000)	Surplus (+) or Deficit (-) Col. 2 – Col. 5	Surplus or Deficit as a Per Cent of Consumption
(3)	(4)	(5)	(6)	(7)
463883	22693	486576	- 66408	- 13.65
757227	4516	761743	-161800	- 21.24
208292	2887	211179	+357630	+169.35
40375	182	40557	+ 40705	+100.36
3735	21432	25167	- 21437	- 85.18
4394	18373	22767	- 12674	- 55.67
3581	2250	5831	+ 13877	+237.99
94216	47033	141249	+ 40925	+ 28.97
626	10879	11505	- 6435	- 55.93
5822	9700	15522	+ 17615	+113.48
71321	53532	124853	+192709	+154.35
6356	15503	21859	+ 96541	+441.65
8167	9586	17753	+ 41027	+231.10
76162	28743	104905	+162267	+154.68
127848	11575	139423	+146898	+105.36
98468	21249	119717	– 37721	– 31.51
124882	3108	127990	+100571	+ 78.58
268748	8785	277533	–166195	– 59.88
137397	1038	138435	– 33250	– 24.02
178317	6821	185138	– 31461	– 16.99
89025	353	89378	– 9594	– 10.73
190044	4360	194404	+ 47196	+ 24.28
72027	755	72782	– 53488	– 73.49
58969	12248	71217	+ 17189	+ 24.14
100886	2114	103000	+ 20654	+ 20.05
122356	5934	128289	+ 18650	+ 14.54
154479	3925	158404	–106506	– 67.24
199378	2492	201870	–101750	– 50.40
54546	2196	56742	– 8307	– 14.64
49209	4089	53298	+249820	+468.72
82683	10659	93342	– 28135	– 30.14
000000	588153	588153	– 6237	– 1.06
143796	929	144725	+ 25620	+ 17.70
286085	5880	291965	– 1704	– .58
912397	52941	965338	+ 57730	– 5.98
1098796	1235	1100031	– 80801	– 7.35
244297	----	244297	+ 5874	+ 2.40
1437694	58800	1496494	–109284	– 7.30
3433024	699132	4132156	–123546	– 2.99

capital goods on the basis of state data on construction activity;[21] government on the basis of state income data;[22] and production for inventories on the basis of state income data.[23] For a given state, the value of the output of any Industry *A* absorbed by the state's bill-of-goods

[21] Construction activity dominates the input requirements of the capital-goods sector of the bill of goods At the same time there is no indication of the extent to which any given state furnished these several input requirements, aside from construction. Hence we were compelled to use the admittedly crude procedure of estimating the requirement from any given state of each of the items absorbed by the capital goods sector by multiplying the total national requirement by the per cent of national construction activity in the given state.

[22] An alternative method of allocating government activities among states was also seriously considered This method involves use of employment as well as income data, and a different procedure for distributing among states government requirements of households than for distributing government requirements of non household industries.

Total government requirements of households ($11,890 million for the nation in 1939) can be broken down into requirements of services of public-emergency employees, of services of regular government employees, and of services corresponding to transfer and interest payments. Government requirements from any given state of services of public-emergency employees can be crudely estimated by multiplying total earnings of persons employed under the federal work programs (*Statistical Abstract of the United States*, 1940, p. 377) by the per cent of national public-emergency employees in that state (1940 *Census of Population*, Vol. II). Government requirements from any given state of services of regular public employees can be determined: (1) by subtracting total earnings of persons employed under the federal work programs from total national wages and salaries plus supplements to wages and salaries paid by government (*Survey of Current Business*, July 1947, 'National Income Supplement,' pp. 27, 28); and (2) by multiplying the remainder by the per cent of the national regular public employees in that state. Government requirements from any given state of services corresponding to transfer and interest payments can be roughly estimated by multiplying total government transfer and net interest payments (ibid. pp. 23, 46) by the per cent of national income originating in that state (*Survey of Current Business*, August 1949, p. 14). For any given state the three types of government requirements of services of households can be aggregated and adjusted by a constant factor (in order that total government requirements of households for the nation derived in this manner be consistent with our figure of $11,890 million) to yield government requirements from this state of households.

Government requirements from any given state of the output of industries other than households, a considerably less important item in the bill of goods, may be crudely estimated by multiplying government input requirements of these industries by the per cent of national regular and emergency public employees in that state.

Since wages per government employee have varied considerably from state to state, e.g. in 1942 average monthly salary per state and local employee for New York was $168 while that for South Dakota was $61 (U S Bureau of the Census, *Governmental Finances in the United States*, 1942, p. 116), it is clear that the use of the technique above involves a fair amount of error. For this reason it did not seem preferable to use this elaborate technique instead of the simple procedure which was actually used, namely, allocation among states of all government requirements of industry on the basis of the percentage distribution of national income among states.

[23] Input requirements from any given state for the production of goods for inventories were roughly estimated by multiplying national requirements by the per cent of national income generated in that state. An alternative procedure would be to multiply national requirements by the per cent of total (national) value of output for all industries recorded for each state. Since in actual practice, both procedures yield approximately the same results, the former, simpler one was employed.

sector was then determined: (1) by multiplying the value of the output of Industry *A* absorbed nationally by each of the four bill-of-goods activities by the respective per cents of these activities in the given state;[24] and (2) by summing the four products.

Adding the requirements of Industry *A* by the 39 industries in any given state (the first step) to the requirements of Industry *A* by the four bill-of-goods activities in the same state (the second step) yields total requirements or consumption in terms of value of output of Industry *A* by the given state. This was done for all industries and states. Columns 3, 4, and 5 of Table 2 show, for example, estimated consumption excluding bill-of-goods requirements, estimated bill-of-goods requirements, and estimated total consumption, respectively, in terms of value of output of each of the 39 industries for the state of Ohio.

The next steps were to determine the surplus (production less consumption) or deficit corresponding to each industry for each state; and finally to express the surplus or deficit as a per cent of the total consumption of the corresponding industry of the corresponding state. Columns 6 and 7 give the resulting data for the state of Ohio.

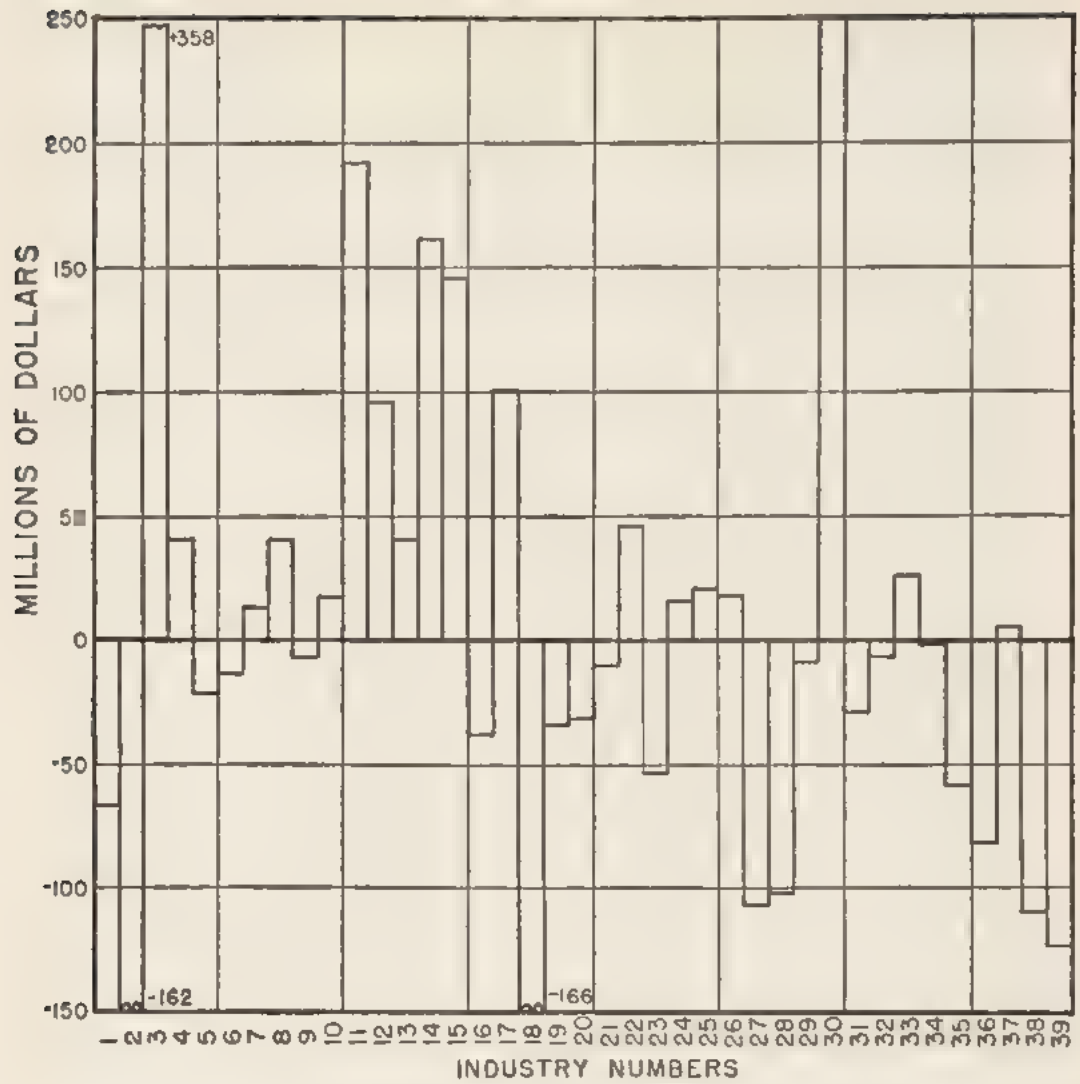
A preliminary classification of some industries as national, regional of various orders, and local was now possible. At this point, however, it should be observed that the figures thus obtained might be of considerable interest independent of the particular use to which they have been put in this study. With a knowledge of production and consumption of the output of each industry by states, it is possible to set up for each state, and for each region formed from states, a commodity balance of trade. Table 2, for example, presents the estimated commodity balance of trade for Ohio for 1939. Chart 3 shows the contribution, positive or negative, of each of the 39 industries to the commodity imbalance of Ohio's trade. Chart 4 expresses each industry's contribution as a per cent of total value of consumption of its product by Ohio. Examination of Table 2 and these charts and similar tables and charts for each state and group of states opens up interesting areas for investigation which we cannot pursue here.

For each industry the state data on production, consumption, surplus and deficit, and surplus and deficit as a per cent of consumption were combined in one table. The state percentages representing relative sur-

[24] This operation involves the implicit assumption of regionally alike production practices in providing the output of each of the bill-of-goods sectors. In many instances, this may be quite unrealistic. For example, more than 10 per cent of ferrous metals exports is assigned to the Pacific Coast when actually the Pacific Coast exported in 1939 little, if any, ferrous metal products; in contrast, Maryland is assigned less than 3 per cent of ferrous metals export when actually in 1939 Maryland exported considerably more.

These unrealities embodied in the model point up the need for more and better regional data, which, as already indicated, will be forthcoming, to some extent at least, in current Census releases for years 1947 and 1948 and in other publications.

CHART 3



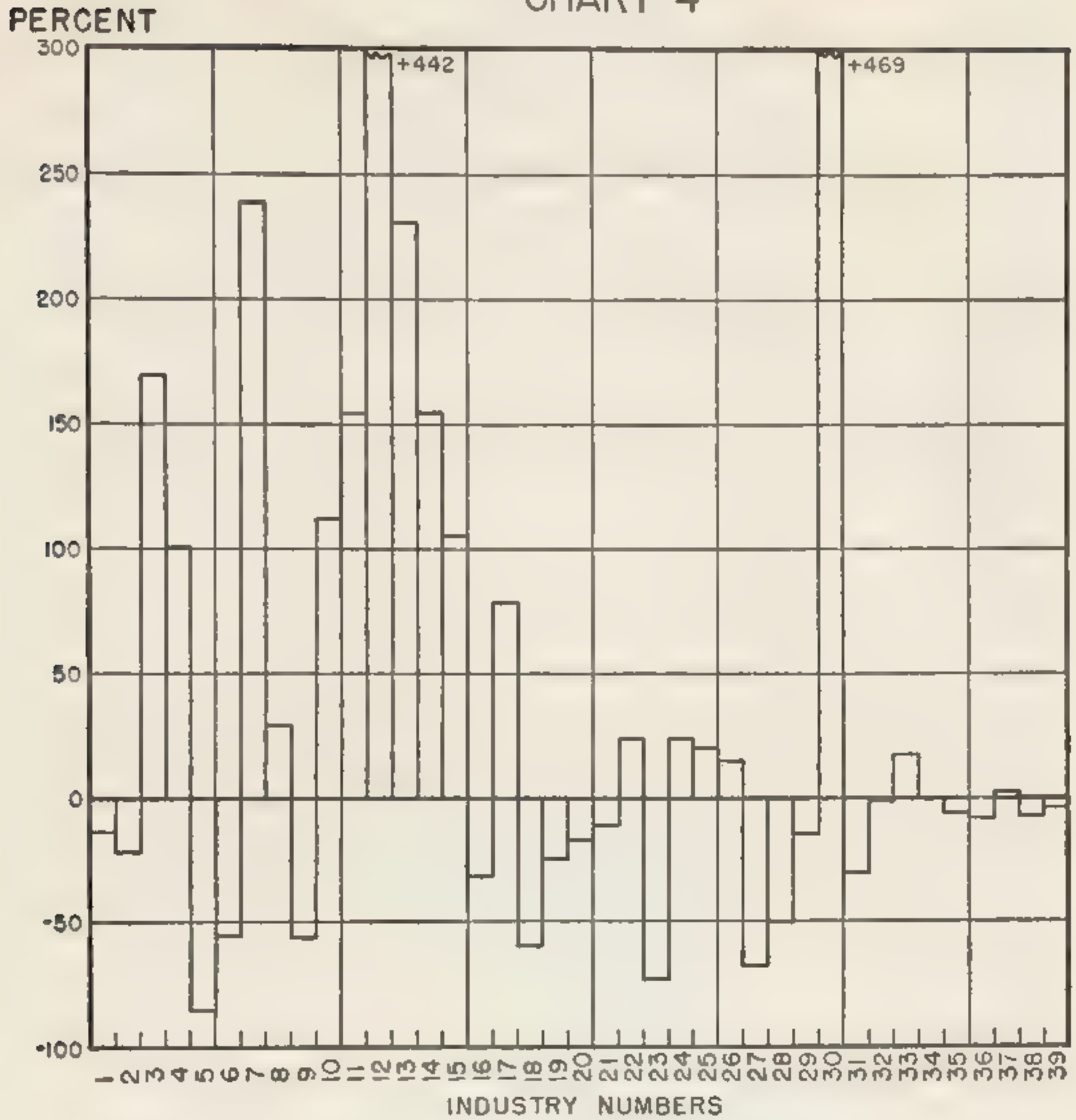
OHIO: ABSOLUTE SURPLUSES AND DEFICITS BY INDUSTRIES

pluses and deficits for each industry were also plotted in a separate graph. See, for example, Chart 5 relating to motor vehicles (Industry 8).

It was fully recognized that these data and the accompanying charts were of themselves insufficient to determine the classification of commodities as regional of various orders. They could only complement accumulated knowledge[26] and provide a very useful consistency check on subjective judgments. Also, it was realized that the consolidated industry classification which expediency necessitated might considerably qualify the validity of our classification of certain industries.

[26] See, among others, Vining, R., 'Location of Industry and Regional Patterns of Business Cycle Behavior,' *Econometrica*, Vol. 14, January 1946; and Hoover, E. M., *The Location of Economic Activity*, New York, 1948, pp. 120-28.

CHART 4



OHIO: PERCENTAGE SURPLUSES AND DEFICITS BY INDUSTRIES

The problems at this stage are best demonstrated with the use of charts. First, what is a national commodity? If one state accounts for all of the output of an industry and, if, at the same time, a good many or all states are substantial consumers, the corresponding commodity can be classified as national (assuming national exports and imports are relatively unimportant). Motor vehicles comes close to meeting these criteria. Chart 5 shows that most of the production is concentrated in a few states, and that most of the states have very large percentage deficits, though this chart does not indicate the absolute levels of consumption in the various deficit states. However, the data in the table for motor vehicles do show that consumption in deficit states is substantial.

CHART 5

INDUSTRY 8: MOTOR VEHICLES

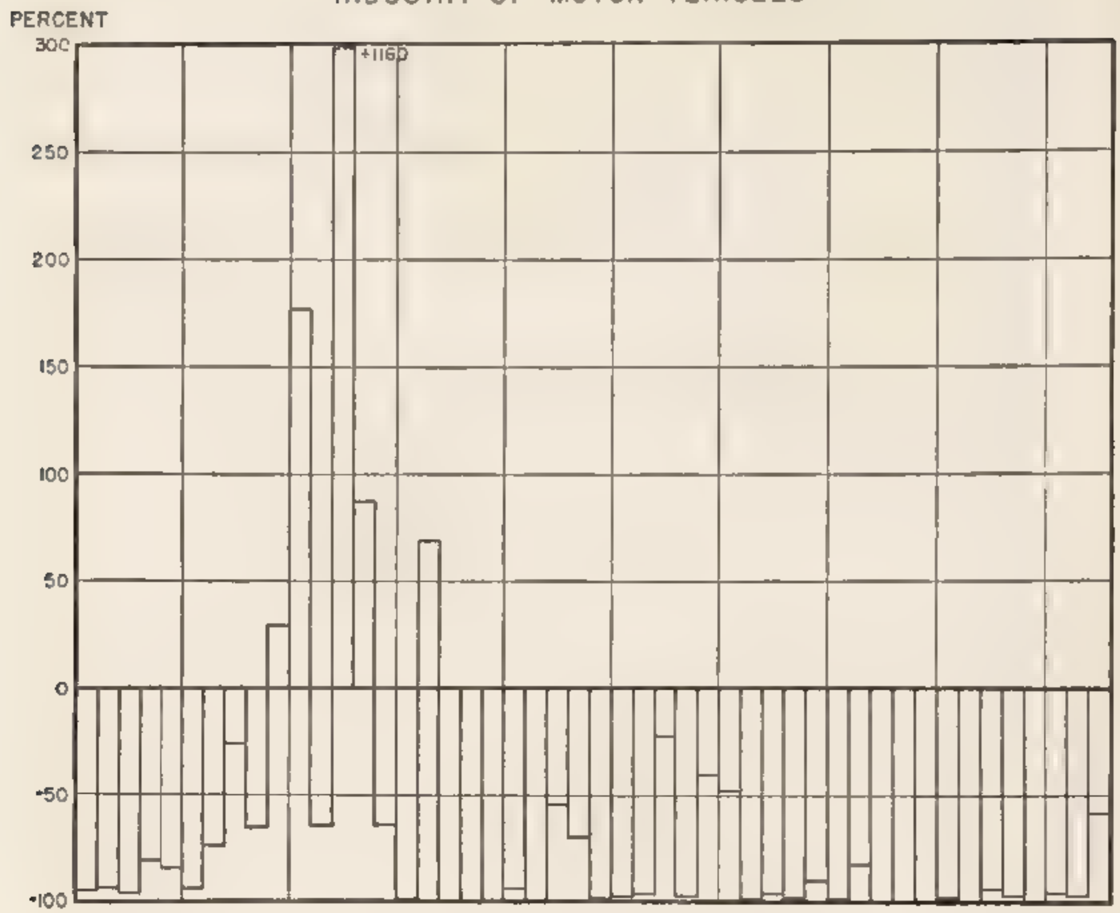
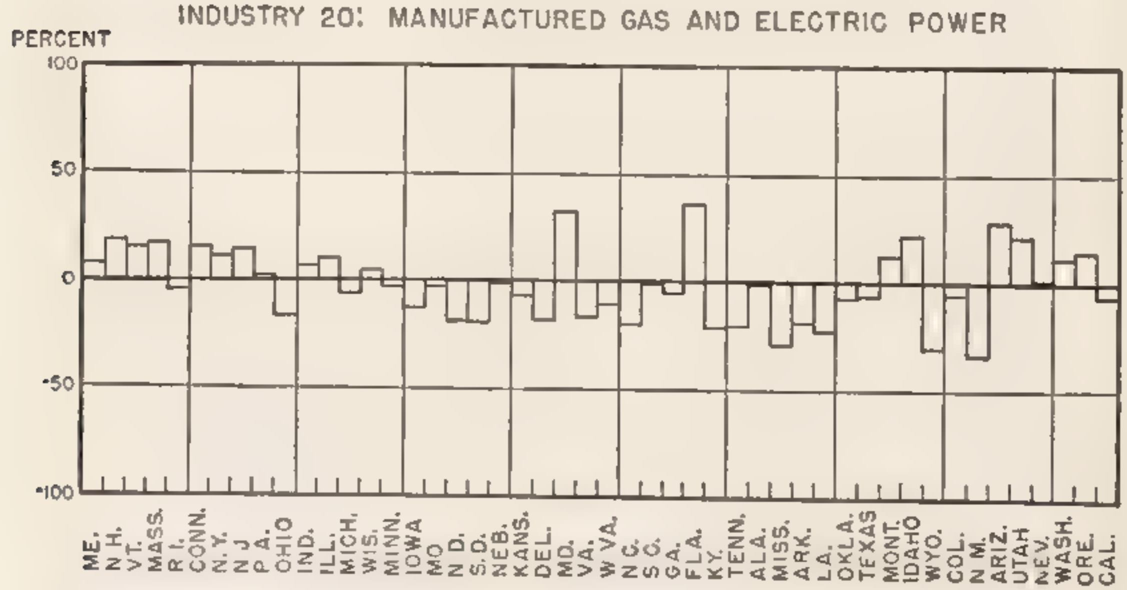


CHART 6

INDUSTRY 20: MANUFACTURED GAS AND ELECTRIC POWER



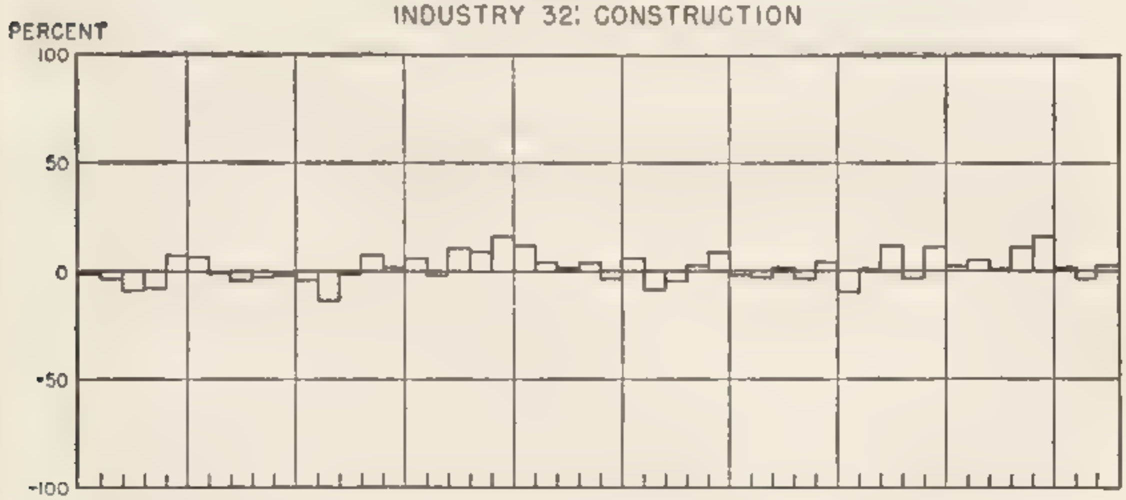
PERCENTAGE SURPLUSES AND DEFICITS BY STATES

(BASE = ESTIMATED STATE CONSUMPTION)

Likewise, agricultural machinery (Industry 6), aircraft (Industry 9), machine tools (Industry 12), and engines and turbines (Industry 7)

CHART 7

INDUSTRY 32: CONSTRUCTION

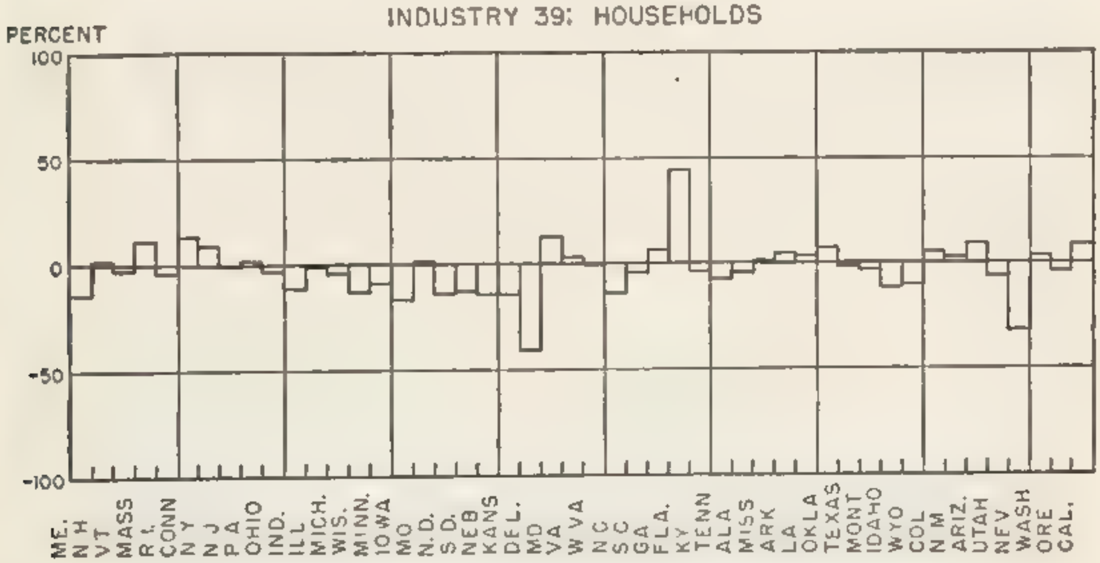


seem to meet these sufficiency conditions and may be classified as national commodities.

However, not all national commodities need to satisfy these conditions. A good many states may be surplus and a few deficit, or the deficits and

CHART 8

INDUSTRY 39: HOUSEHOLDS



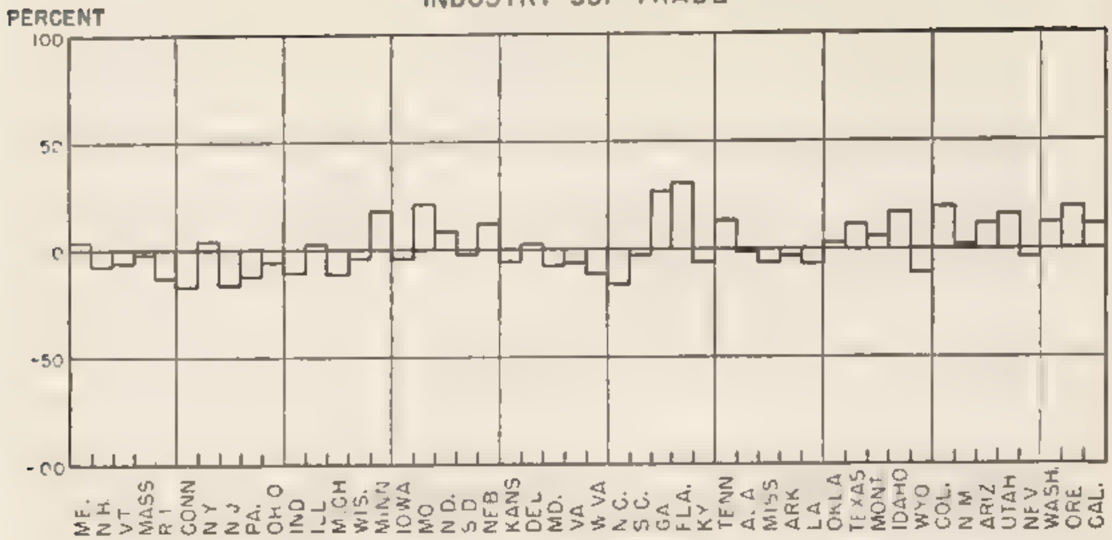
PERCENTAGE SURPLUSES AND DEFICITS BY STATES

(BASE = ESTIMATED STATE CONSUMPTION)

surpluses in the states may be on the average of minor extent, and so forth. In contrast to Chart 5, an industry may have a surplus-deficit pattern by states corresponding to that depicted for manufactured gas and electric power in Chart 6. So long as the product of this hypothetical

CHART 9

INDUSTRY 35: TRADE

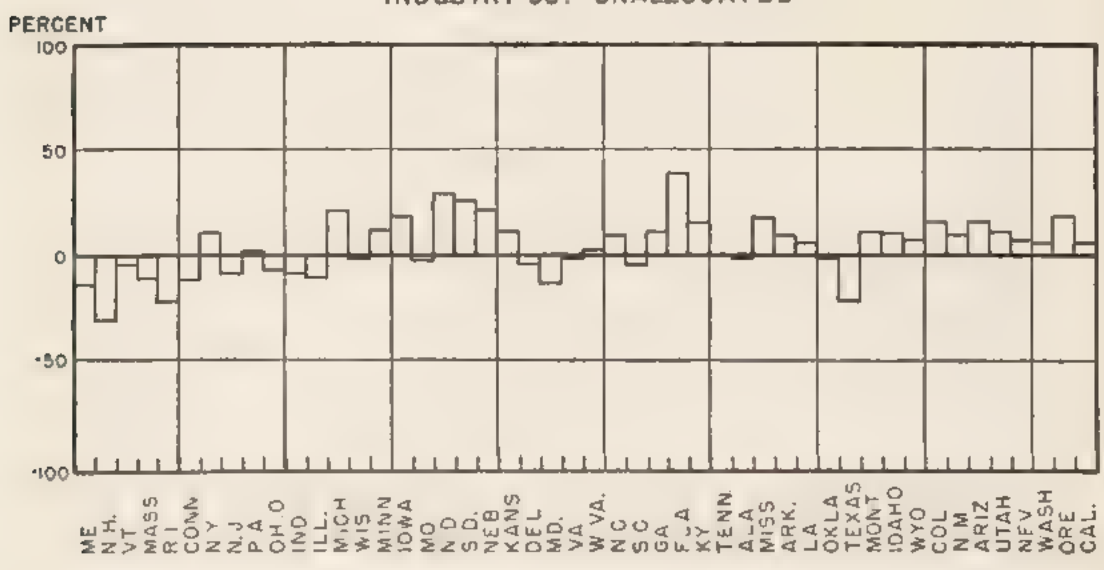


PERCENTAGE SURPLUSES AND DEFICITS BY STATES

(BASE = ESTIMATED STATE CONSUMPTION)

CHART 10

INDUSTRY 38: UNALLOCATED



PERCENTAGE SURPLUSES AND DEFICITS BY STATES

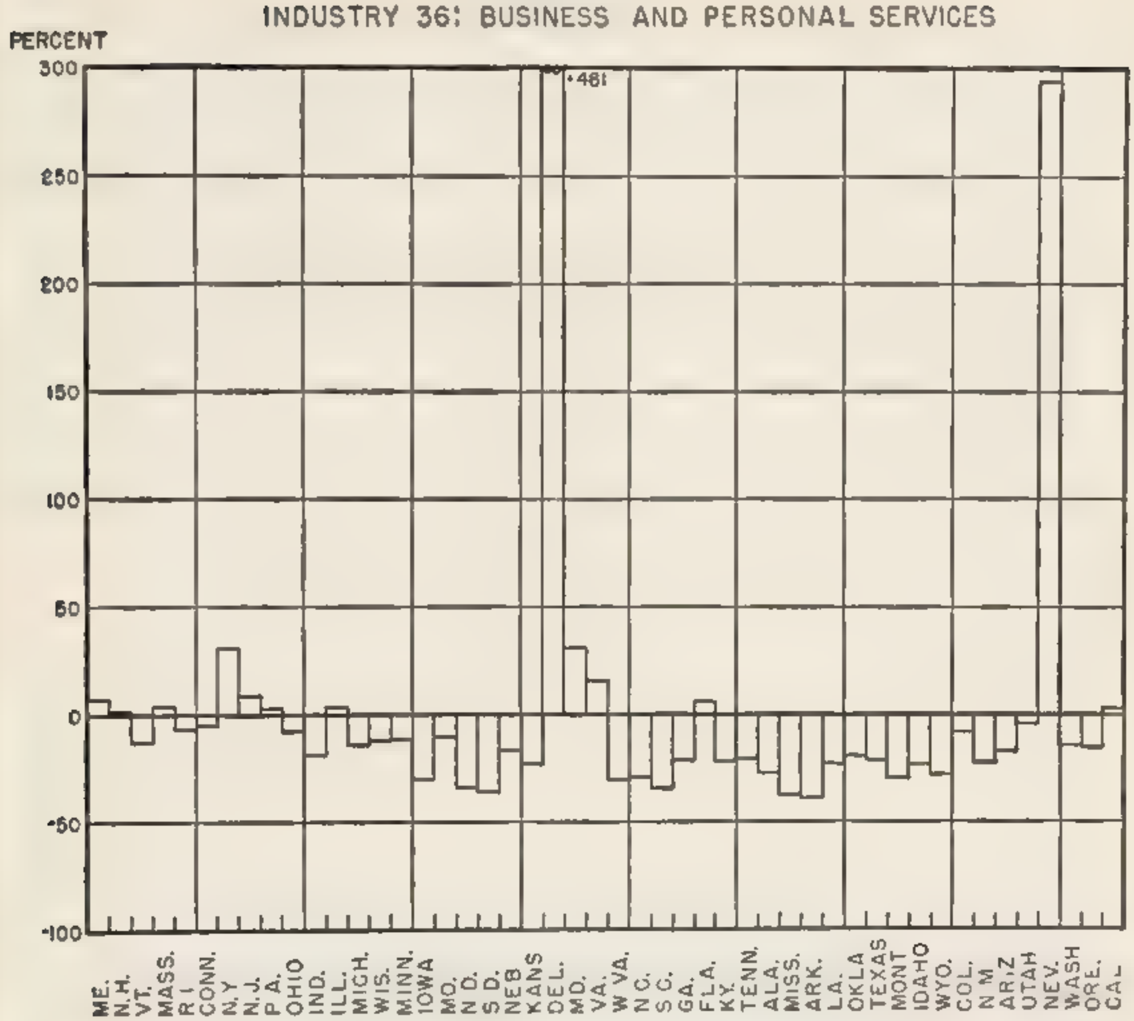
(BASE = ESTIMATED STATE CONSUMPTION)

industry in each of the various states is shipped both short and long distances to other states, the corresponding commodity could be national. This could be the case for certain nationally advertised items produced in many states and for which transport costs are negligible.

Before one determines which other commodities may be classified as national, it is best to go to the other extreme in order to classify those

CHART 11

INDUSTRY 36: BUSINESS AND PERSONAL SERVICES



PERCENTAGE SURPLUSES AND DEFICITS BY STATES

(BASE = ESTIMATED STATE CONSUMPTION)

commodities which are clearly local. We should expect that a local commodity would be one which is produced and consumed to approximately the same relative extent in each state and which moves only relatively short distances. Such a commodity is produced by the manufactured gas and electric power industry (Industry 20). Chart 6 shows rough balances of its production and consumption for each state. Likewise, the commodities corresponding to construction (Industry 32), Chart 7, and households (Industry 39), Chart 8, meet the criteria above, and in the light of theoretical and other accumulated empirical knowledge may be tentatively classified as local. In addition, trade (Industry 35) and unallocated (Industry 38) show a range of variation by states similar to that of Chart 6. See Charts 9 and 10 respectively. Because trade services (especially wholesale), in contrast to households and construction activities,

CHART 12

INDUSTRY 21: COMMUNICATIONS

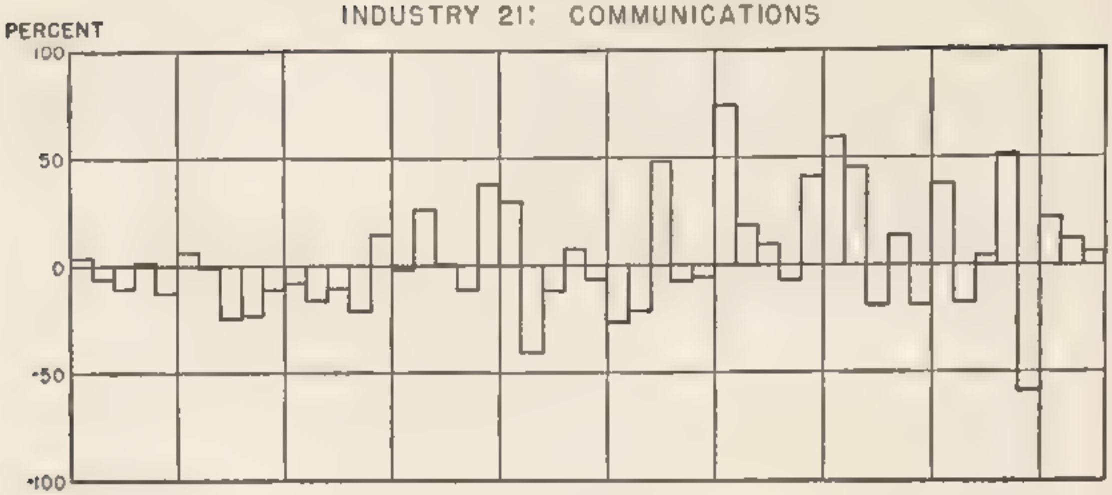
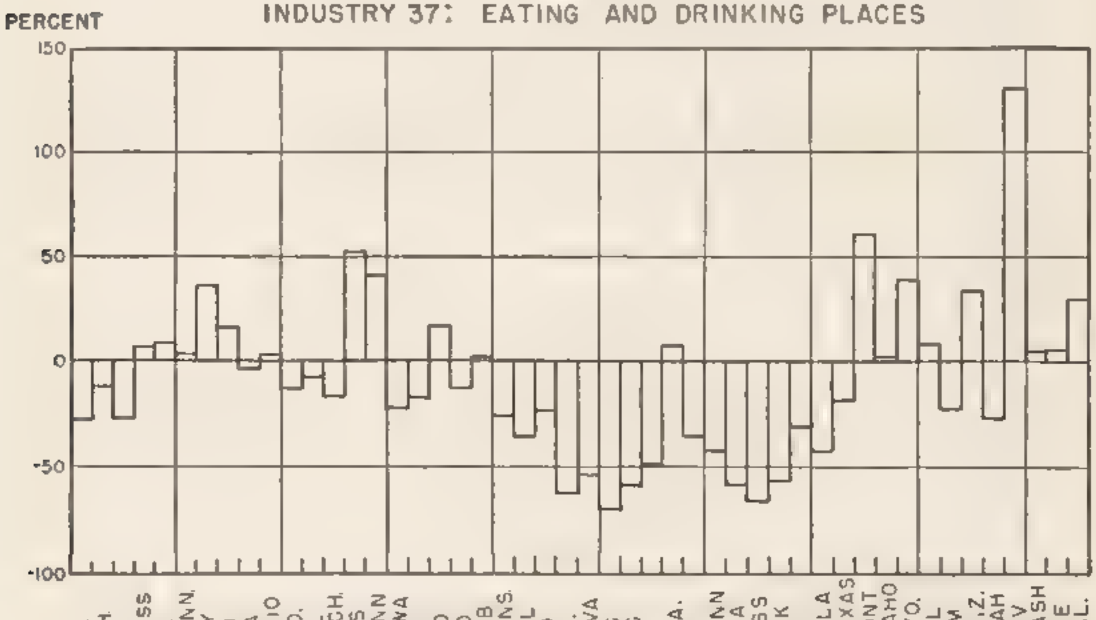


CHART 13

INDUSTRY 37: EATING AND DRINKING PLACES



PERCENTAGE SURPLUSES AND DEFICITS BY STATES

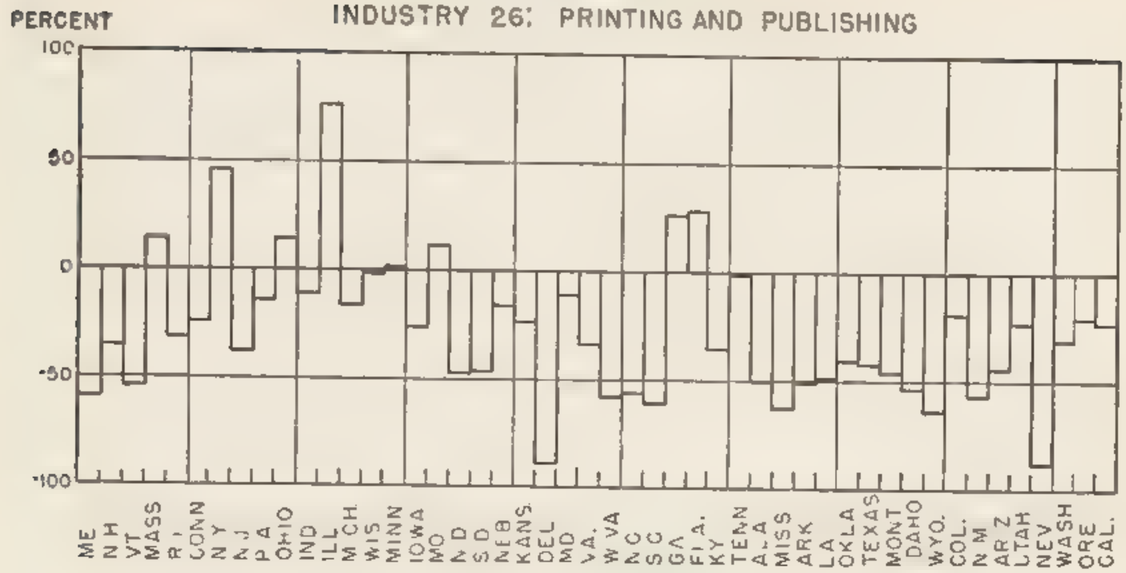
(BASE = ESTIMATED STATE CONSUMPTION)

can be provided at considerable distance from the point of purchase, the ground for denoting trade as local is subject to some question. No attempt was made at this time to classify 'unallocated.'

Other industries that might be considered for a local classification because of their production (service) and transport characteristics are business and personal services (Industry 36), communications (Industry 21), eating and drinking places (Industry 37), printing and publishing (Industry 26), steam railroad transportation (Industry 34), and perhaps non-

CHART 14

INDUSTRY 26: PRINTING AND PUBLISHING



PERCENTAGE SURPLUSES AND DEFICITS BY STATES
(BASE = ESTIMATED STATE CONSUMPTION)

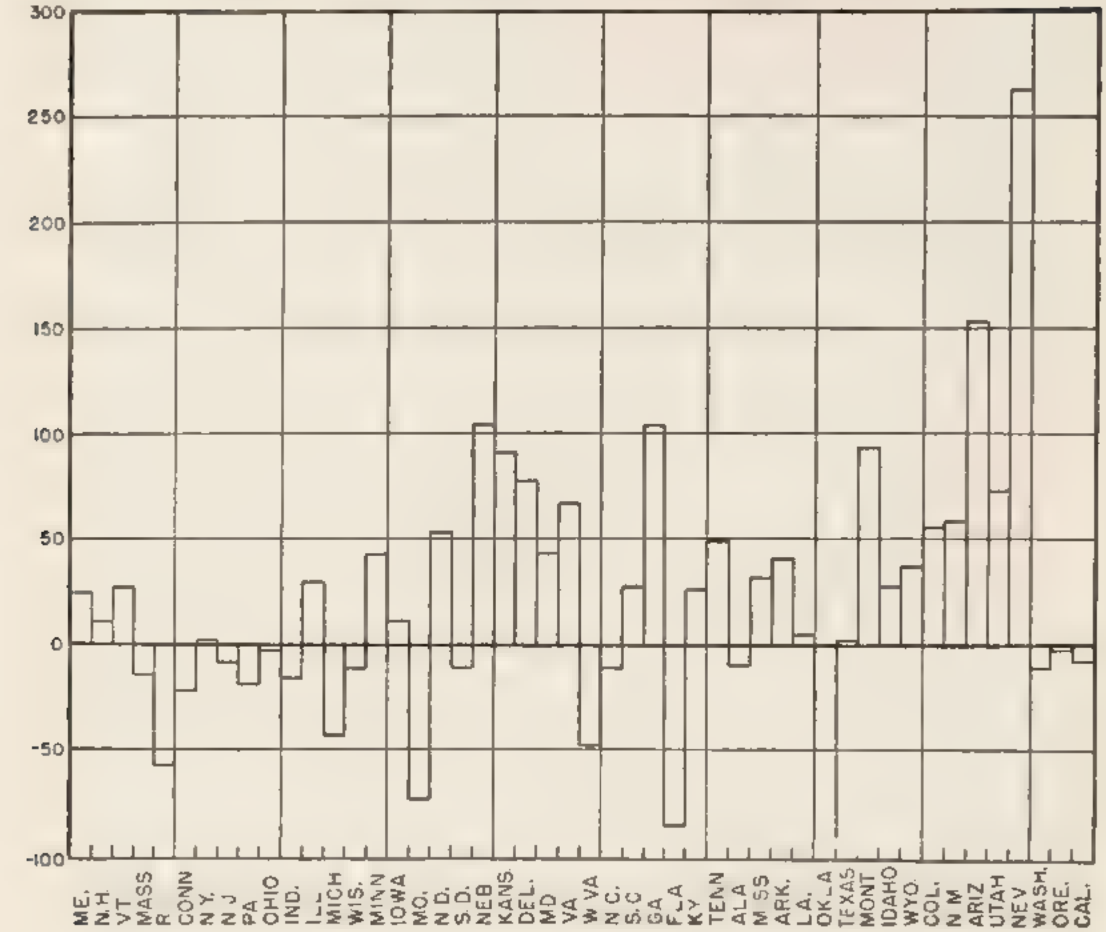
metallic minerals and their products (Industry 17). The percentage data on surpluses and deficits by states are presented for these industries in Charts 11 to 16. In order, they show an increasing range of variation in the percentage data.

But the range of variation is only one of many factors to be considered. The average percentage surplus or deficit is also important. Hence, the device of summing surpluses and deficits of states without regard to sign and expressing the total as a percentage of national consumption for each of the 39 consolidated industries was used. The resulting percentages arrayed in increasing order are shown in Chart 17.

This chart clearly indicates some of the industries which may be national. Those industries whose production is concentrated in a few states and whose consumption is substantial in many states are characterized by large percentage figures and tall bars toward the right of the distribution. Here are found agricultural machinery, aircraft, machine tools, engines and turbines, and motor vehicles, each already designated as national. To these should be added textile mill products (Industry 27), shipbuilding (Industry 5), ferrous metals (Industry 3), coal mining and manufactured solid fuels (Industry 19), and petroleum products and refining (Industry 18), each of which evidence about the same or a greater average percentage discrepancy. But not all national industries may be represented by large percentage figures in these charts. As noted above, state surpluses and deficits for a commodity may be small, yet such a commodity may characteristically be shipped both long and

CHART 15

INDUSTRY 34: STEAM RAILROAD TRANSPORTATION



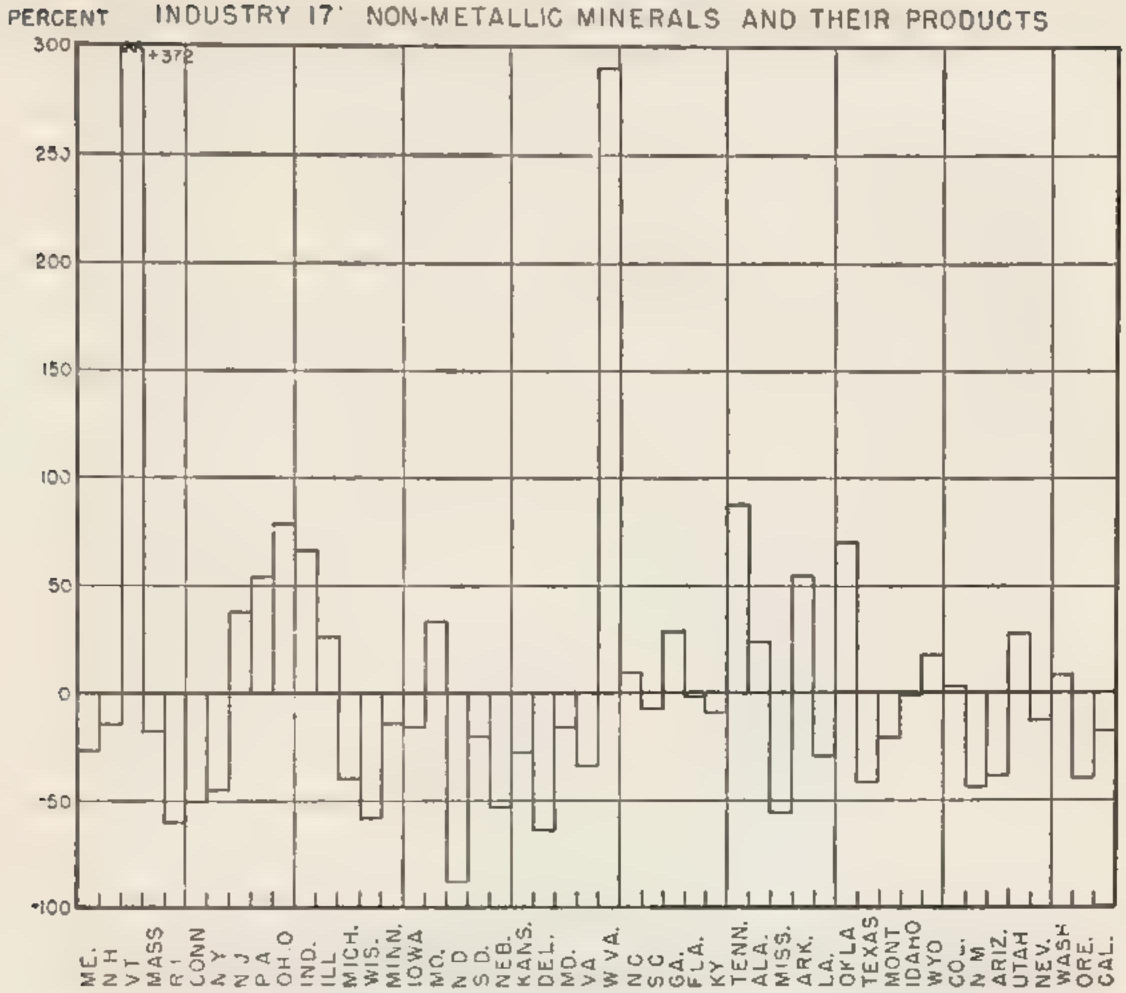
PERCENTAGE SURPLUSES AND DEFICITS BY STATES

(BASE = ESTIMATED STATE CONSUMPTION)

short distances. It is conceivable that a national industry may fall in a group along with local industries which tend to be at the extreme left of this chart.

A particular feature of Chart 17 lies in the breaks which occur and which tend to set apart groups of industries. The group of industries beginning with apparel and other finished textile products (Industry 28), at the left, and ending with agricultural machinery at the right appear to be national, although of this group those to the left of petroleum products and refining seem to be less national and more local in character than the remaining ones. This latter point seems true, too, of the group beginning with eating and drinking places (Industry 37) at the left and ending with industrial and heating equipment, n.e.c. (Industry 11) at the right, when compared to the group to its right. Finally, the group at the extreme left, beginning with construction (Industry 32) and ending

CHART 16



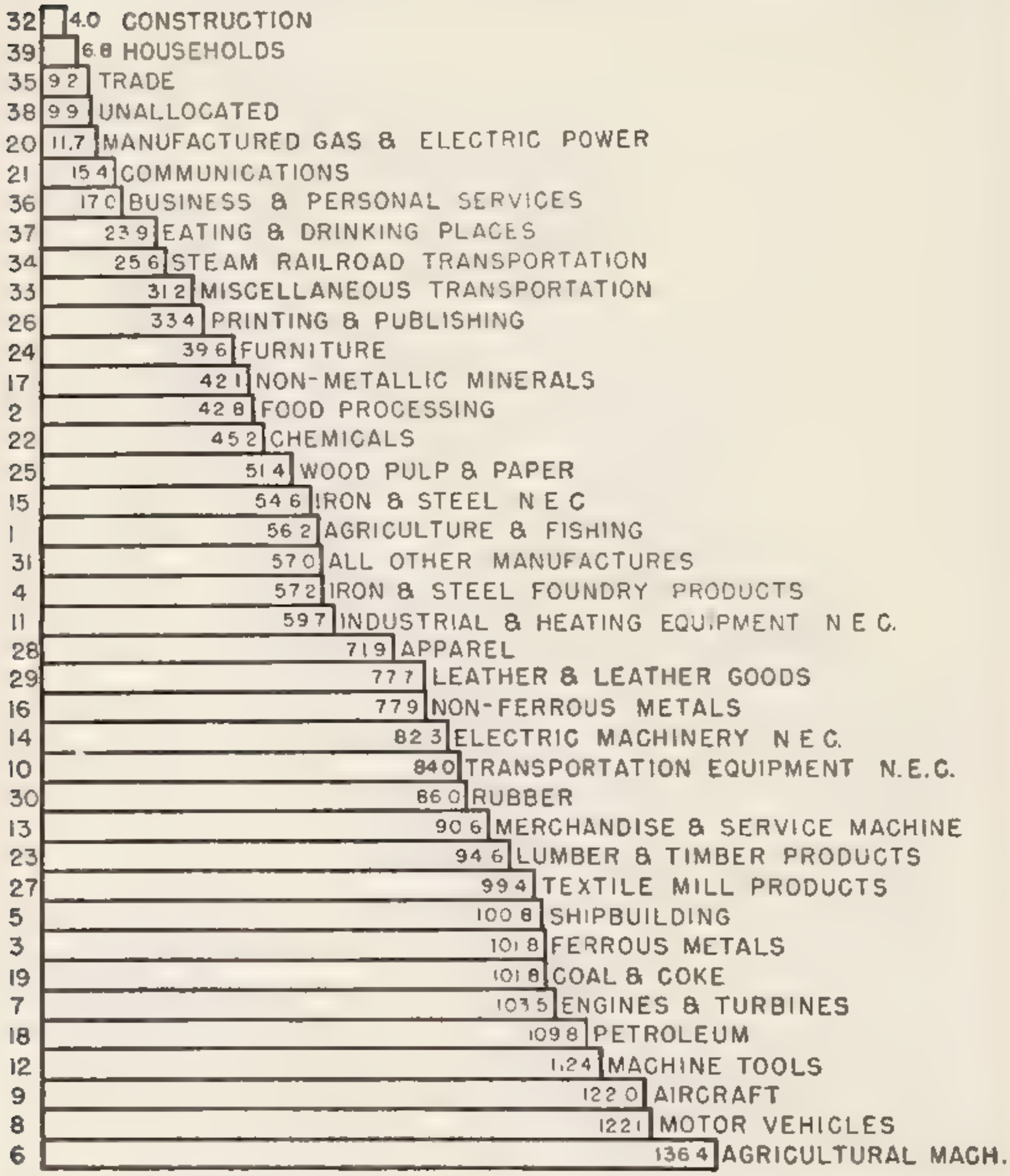
PERCENTAGE SURPLUSES AND DEFICITS BY STATES
(BASE = ESTIMATED STATE CONSUMPTION)

with business and personal services (Industry 36), appears to be least national of all, in fact entirely local. The reader may want to attribute significance to one break only, namely, that between industrial heating equipment, n.e.c., and apparel and other finished textile products. It should be mentioned, however, that in other charts of similar type (for example, see Chart 19 below), the basis for recognizing more than one break is stronger.

In short, these breaks suggest a tentative division of commodities into groups according to the degree to which they may be local or national. This in turn raises the problem, discussed in the previous chapter, of classifying commodities as national, regional of the first order, regional of the second order $\cdots$, regional of the $(n-1)$ order, and finally regional of the n^{th} order (or local). On Chart 17, for example, the break separating apparel and other textile products from industrial and heating equip-

CHART 17

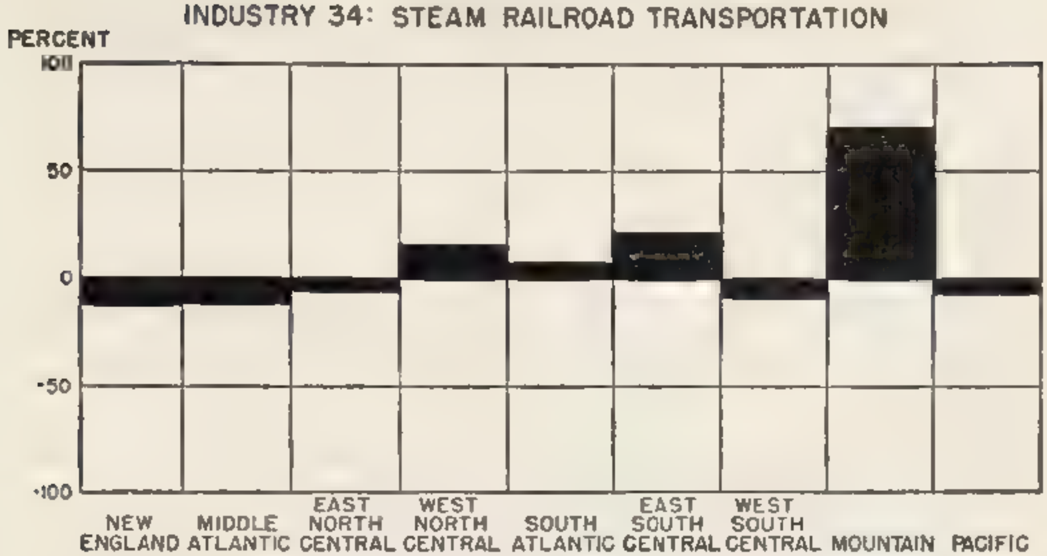
THE SUM (WITHOUT REGARD TO SIGN) OF SURPLUSES AND DEFICITS OF STATES EXPRESSED AS A PERCENTAGE OF NATIONAL CONSUMPTION, BY INDUSTRIES



ment, n.e.c., might appear as a logical dividing point between regional commodities of a given order and regional commodities of the next higher order.

However, this is not so. We have already mentioned the possibility that a national industry may conceivably fall in any part of the distribution. It is only sufficient, not necessary, that an industry fall in the group at the

CHART 18



PERCENTAGE SURPLUSES AND DEFICITS BY CENSUS REGIONS

(BASE = ESTIMATED CENSUS REGION CONSUMPTION)

extreme right in order to be classified as national. In the same way, regional industries of the first order may fall in any parts of the distribution which lie to the left of the national industries at the extreme right; and regional industries of the second order may fall in any parts of the distribution which lie to the left of regional industries of the first order which are clustered next to national ones at the extreme right. And so forth. Hence, such breaks are at most suggestive. They must be interpreted with extreme caution.

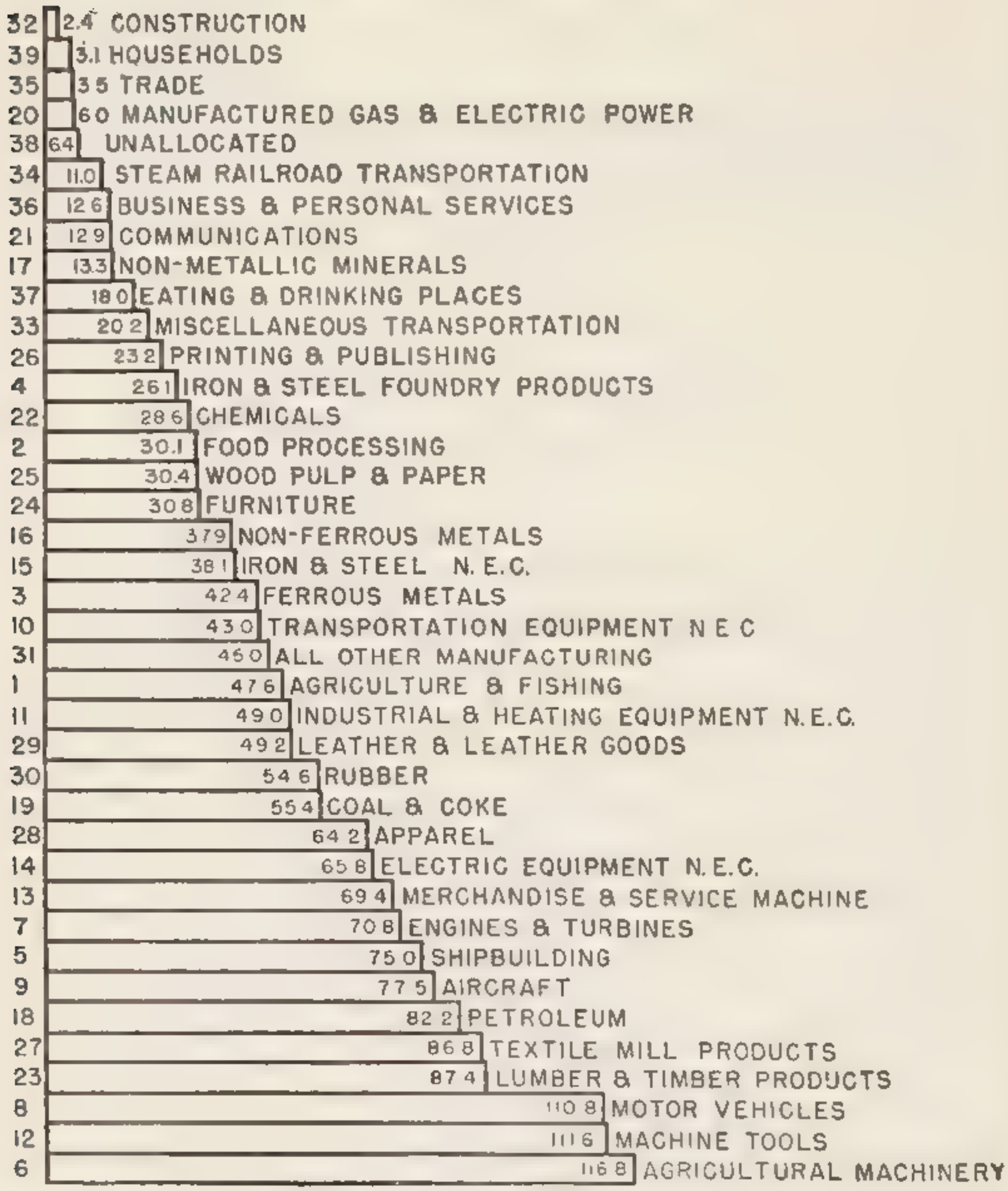
Aside from the industries which appear to be definitely local or national in character, there is as yet no clear-cut basis for classifying the industries which lie in between these extremes, and which form the bulk of the distribution in terms of numbers. Further, no clear-cut classification is possible until we can specify the number and type of regions to be considered. At the same time it is desirable to leave considerable flexibility in the orders of regions to be treated as well as in their demarcation in order to bring out to the fullest extent possible the inherent nature and structural characteristics of intra-national commodity flows.

At the start the data were considered in terms of the Census regions which historically have had significance.[26] As could be anticipated certain industries show a much better balance of production and consump-

[26] *A priori*, it was deemed more important to minimize surpluses and deficits in manufacturing, agriculture, and mining than in service industries. For that reason the historical Census regions rather than metropolitan regions seemed better for a starting point.

CHART 19

THE SUM (WITHOUT REGARD TO SIGN) OF SURPLUSES AND DEFICITS OF CENSUS REGIONS EXPRESSED AS A PERCENTAGE OF NATIONAL CONSUMPTION, BY INDUSTRIES



tion by Census regions than by states; none of course can show worse. For example, steam railroad transportation (Industry 34) evidences considerable imbalance by states (Chart 15): when put on a Census region basis (Chart 18), much of the imbalance disappears. The 8th Census region, the Mountain region, with a large surplus, exhibits least balance in Chart 18, but this region accounted for only 5 per cent of steam rail-

road transportation services in 1939.[27] Hence, it appears that with Census regions (and not states) as the smallest areal units, steam railroad transportation might be classified as local (regional of the n^{th} order); but with states as the smallest areal units, as a regional commodity of the $(n - 1)$ order if Census regions are used in the first aggregation of states.[28]

Chart 19 was constructed to shed light on how the balance of production and consumption for a particular industry changes relative to other industries as the order of region changes, in particular as we proceed from states (which for the moment we may take as regions of the n^{th} order) to Census regions (regions of the $n - 1$ order). Chart 19 presents in order of magnitude a percentage figure for each of the 39 consolidated industries. The figure for any industry is obtained by summing for that industry surpluses and deficits by Census regions without regard to sign and expressing the total as a percentage of national consumption of its product.

The usefulness of Chart 19 is clearly demonstrated when we examine nonmetallic minerals and their products (Industry 17). As already indicated, Chart 16, depicting percentage surpluses and deficits by states, strongly suggests that this is not a local industry when states are the local areal units. And further, Chart 17 shows that nonmetallic minerals and their products lies to the right of those industries we have already designated as local. Closer scrutiny of Chart 16 reveals that not all surplus states are adjacent to each other and that significant distances separate some of them. This is to be anticipated on the ground that the ratio of transport cost to value of product is relatively high in this industry, and that transport cost differentials are critical as a location factor for some of the component parts of this industry which covers, for example, stone, clay, and glass products. There is a presumption then that nonmetallic minerals and their products is neither local nor national, but regional of some order.

It is now relevant to compare this industry's rank in Charts 17 and 19. Whereas in Chart 17 it stands to the right of industries suggested as eligible for a local classification, in Chart 19 it stands among them. This relative improvement in rank (balance) achieved when Census regions

[27] In reading these charts, it must be borne in mind that only *percentage* surpluses and deficits are recorded. These charts give no indication of the relative per cent of production in each state or region, and thus the relative importance of each surplus and deficit. If a state shows a 1,000 per cent surplus, this may not be significant if the state accounts for only 1 per cent of production. Hence, the charts must be read in conjunction with the data and other charts to be discussed later.

[28] Chart 18 brings out one of the shortcomings of any standard classification procedure. With respect to most regions, and the major ones industrially speaking, steam railroad transportation seems to be a local commodity. But with respect to the mountain region, it has features of a national commodity. These are suppressed when steam railroad transportation is classified as local.

instead of states are used in deriving the desired percentages provides some basis for classifying this industry either as (1) regional of the $(n-1)$ order, that is, one which tends to balance generally within each Census region, but not within each state;[29] or (2) as local, if Census regions are designated local areal units. Chart 19 also shows the improvement in relative balance for steam railroad transportation (Industry 34). As already indicated this industry, too, might be classified in the same manner as nonmetallic minerals and their products.

Still we are not in a position to classify commodities, to specify orders of regions, or to delineate regions, despite the value of Charts 17, 19, and others derived by summing surpluses and deficits for different geographic divisions of the nation. Some experimentation with a rough pilot model is required, for example, to evaluate the general effect of any given percentage imbalance, and to attack the related problem of regional hierarchy. Here it is forcefully demonstrated how empirical work and the development of the conceptual framework must go hand in hand.

III. Initial Experiments with the Model

The first experiment involved dividing the industries roughly into two numerically equal groups. One included food processing (Industry 2), nonferrous metals and their products (Industry 16), transportation equipment, n.e.c. (Industry 10), agriculture and fishing (Industry 1), industrial and heating equipment, n.e.c. (Industry 11), leather and leather products (Industry 29), apparel and other finished textile products (Industry 28), and all other industries to the right of the last-mentioned industry on Chart 19. The 18 industries in this group were designated as national. The remaining 21 industries in the second group, which thus by definition are regional of various orders, were simply considered as non-national.

Simultaneously with this classification of industries, a division of the United States into major regions was undertaken. Two major regions were selected for analysis, Region I comprising the New England and Middle Atlantic regions, and Region II, which included all the other Census regions. In the light of the existing conceptual framework, the data, and charts, this design of two major regions with 18 national industries, rather than a model including more major regions or one with more or less national industries, or both, seemed best for our experiment. However, it cannot be stressed too much that a considerable degree of arbitrariness entered into the choice of the design which we

[29] The chart of surpluses and deficits for this industry by Census regions supports this judgment.

attempted to orient to existing location theory as well as to the empirical knowledge of flows and regional interrelations.

The arbitrariness involved can be illustrated by considering the problem of classifying printing and publishing (Industry 26). Chart 14 shows its state-by-state pattern of surpluses and deficits. Though printing and publishing seems to be regional of the n^{th} or the $(n - 1)$ order to a significant degree, certain parts of this industry produce for a national market. This is borne out in the chart by the percentage surpluses in New York and Illinois, and in our data by the large absolute surpluses in these two states. Printing and publishing also points up the problem of setting up appropriate regional divisions. Our knowledge suggests that the local elements of this industry are organized more along metropolitan than Census region lines. Pending improvements in industrial and regional breakdowns, this industry was tentatively designated as non-national. It was not classified as local since its state pattern of percentage surpluses and deficits, when contrasted with that of manufactured gas and electric power (Chart 4), seemed too unbalanced. At the same time the fact that Charts 17 and 19 do not indicate improvement in relative standing when Census regions rather than states are considered seems to strengthen the argument that Census regions are not the most appropriate areas for analyzing the spatial structural relations of this industry.

Ferrous metals (Chart 20) is another industry where no clear classification was possible, partially because of the non-homogeneity arising out of institutional factors affecting price and marketing arrangements. The basing-point pricing system which existed in 1939 allowed considerable cross-hauling which in turn increased the national character of the industry. On the other hand, comparative cost and locational studies suggest that by 1939 the iron and steel industry was inclining toward a strong regional orientation.[30] Chart 20 is not inconsistent with this suggestion. The industry was finally designated as non-national.

Limitations of space preclude further discussion of the problems of classifying industries for the preliminary experiment. Given the non-national industries and our regions, we constructed the matrices required and performed the necessary matrix multiplications and inversions.[31]

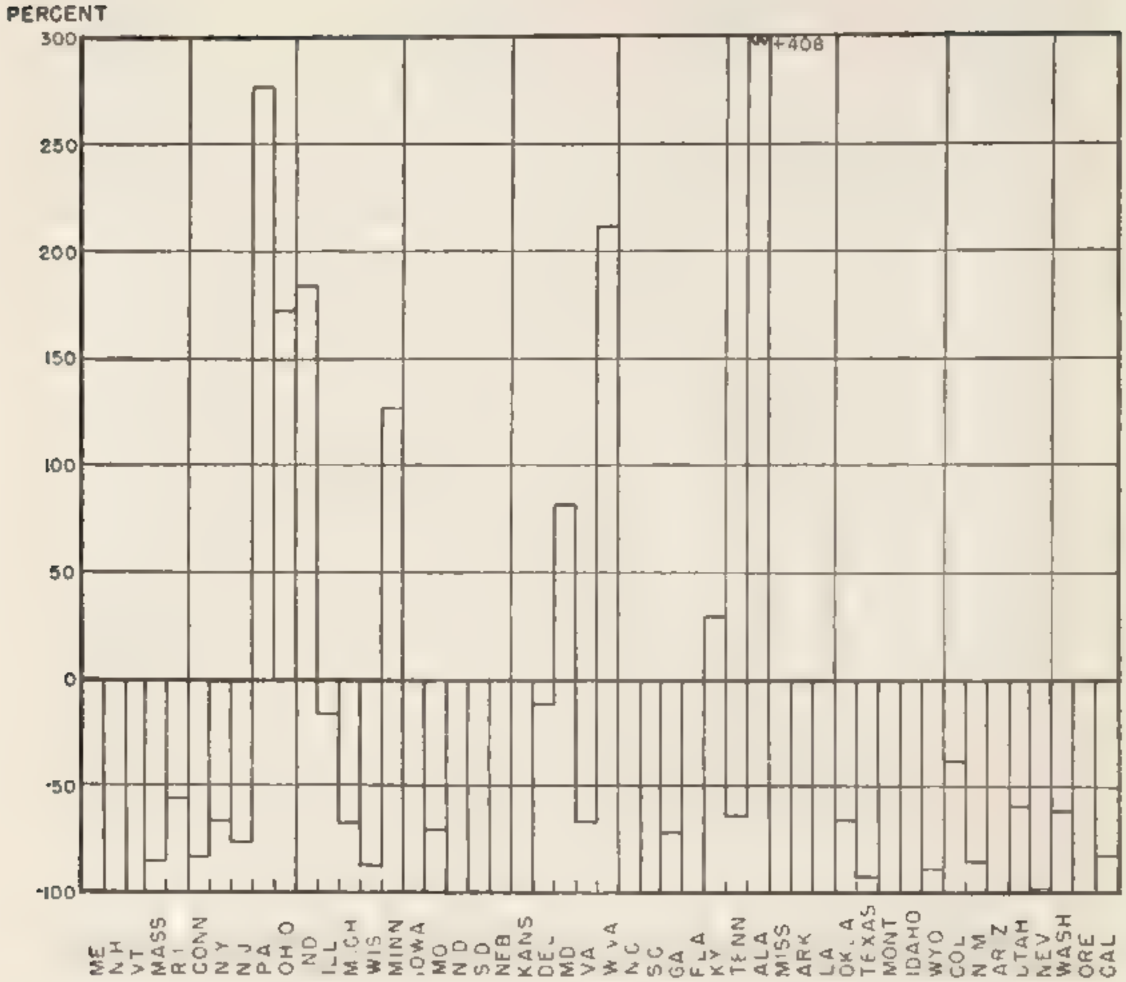
The product matrices finally derived permitted us to compute indirectly the output of any non-national industry in any given region, using the 1939 national bill of goods and the breakdown into regional bills of goods as described earlier. Hence, a comparison of the computed regional

[30] See, for example, Isard, W., and Capron, W. M., 'The Future Locational Pattern of Iron and Steel Production in the United States,' *Journal of Political Economy*, Vol LVII, No 2, April 1949; and U S. Transportation and Investigation Board, *The Economics of Iron and Steel Transportation*, Washington, 1945.

[31] For procedure, see the mathematical note to Chapter 4.

CHART 20

INDUSTRY 3: FERROUS METALS



PERCENTAGE SURPLUSES AND DEFICITS BY STATES

(BASE = ESTIMATED STATE CONSUMPTION)

outputs of non national industries with the actual regional outputs in 1939 could show up the distortion of actual empirical relationships implicitly involved in the crude 'twenty-one non-national industry, two major regional' design.

Such a comparison was made under various conditions for Major Region I. In general the discrepancies between indirectly computed and actual outputs ranged from over-estimates as high as 30 per cent of actual, to under-estimates of approximately 40 per cent of actual. Clearly, this first experimental design was inappropriate. Yet the experimentation carried on through making small changes in the bill of goods and other items permitted us to arrive at one important conclusion. *The designation of any industry as regional of any order (i.e. non-national) where the imbalance between production and consumption within the relevant order of*

regions is moderate may lead to marked discrepancies between predicted and actual output of various other industries. A 'moderate' imbalance may be tentatively taken as one which on the average exceeds, say, 10-15 *per cent as the lower limit.*

Immediately, if we take Census regions as regions of a certain order, then at most only the first nine industries at the left of Chart 19 can be taken to be industries of the same order, or as regional of higher order; and the remaining industries are necessarily regional of lower order or national. If we wish to use other areas as regions, charts similar to Charts 17 and 19 can be constructed and employed in the same manner in classifying industries.

IV. Major Region Analysis

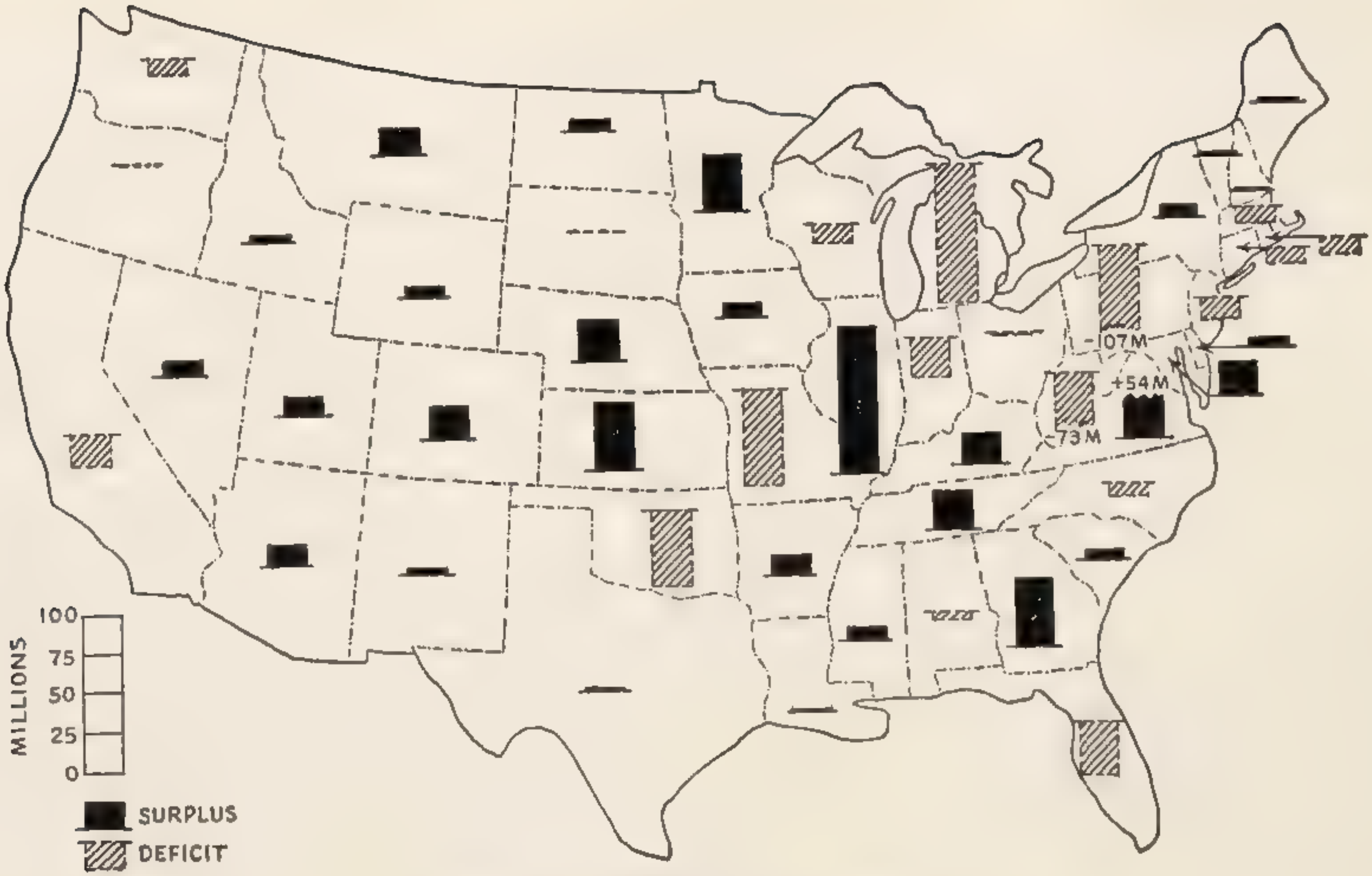
At this point another experiment was initiated. It seemed desirable to design a model in which the nation is divided into major regions, and each major region into sub-regions. Hence a classification of commodities (industries) into the following three categories was required:

(1) national (where production and consumption tend to balance within the nation only);
(2) regional (where production and consumption tend to balance within each major region as well as within the nation, but not within sub-regions); and,
(3) sub-regional (where production and consumption tend to balance within each sub-region, as well as within each major region and nation).[32]

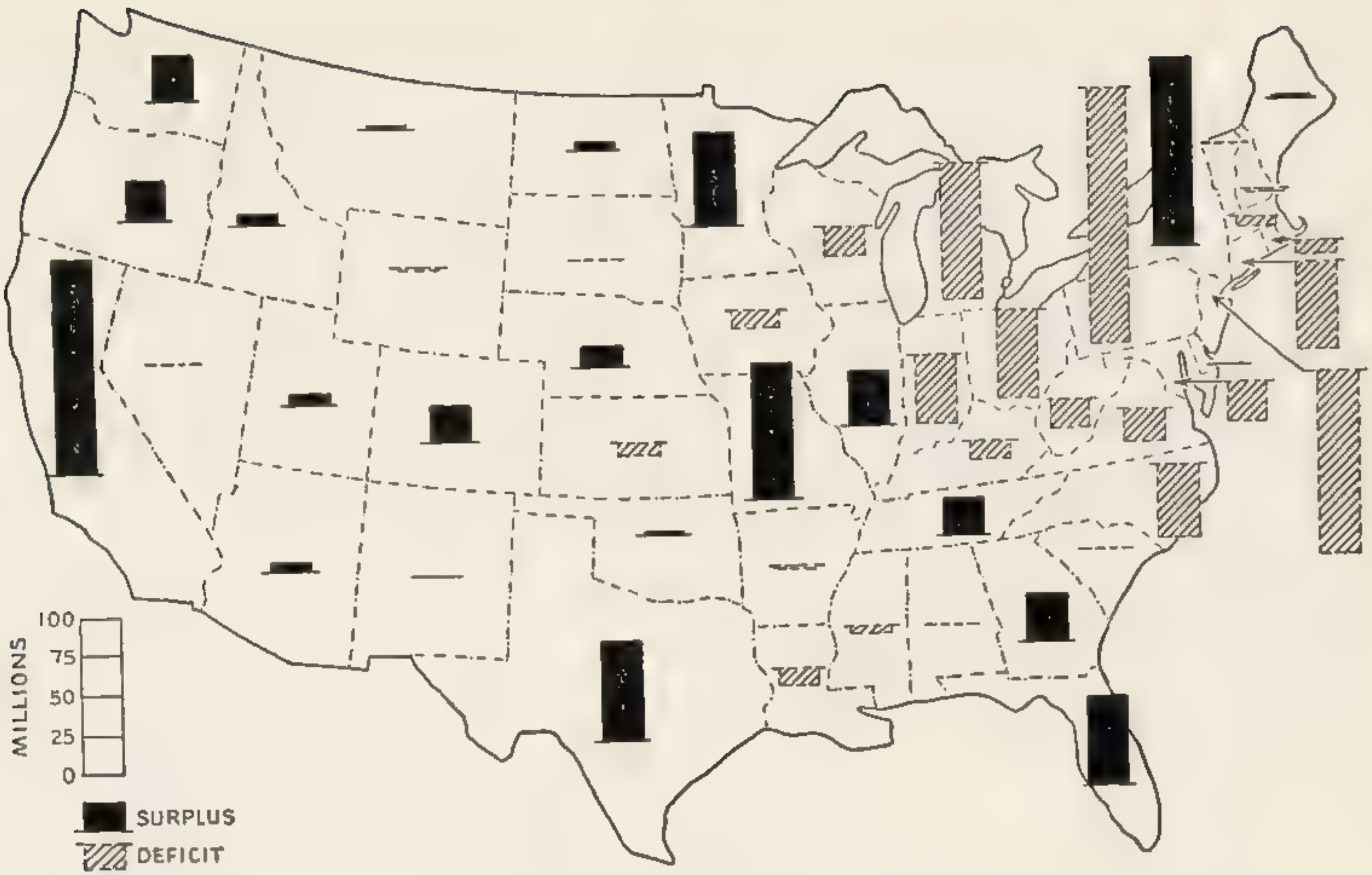
Also, it seemed best to consider classifying at most only 12 of the 39 consolidated industries as non-national. Non-national industries include both regional and sub-regional industries. These 12 comprised the first 10 listed on the left in Chart 19, printing and publishing, and iron and steel foundry products. Though a number of the remaining industries classified as national did evidence a smaller than 10 per cent imbalance for various sets of major regions considered, our theoretical and empirical knowledge suggested that this was so only because of large flows of their products in several directions across major regional boundaries. This, by definition, indicates a national industry.

Given the 12 industries eligible for non-national classification, what areas ought to be designated as major regions? It is clear that sub-regions cannot be demarcated until major ones are, since each major region must, according to the scheme of the previous chapter, contain an

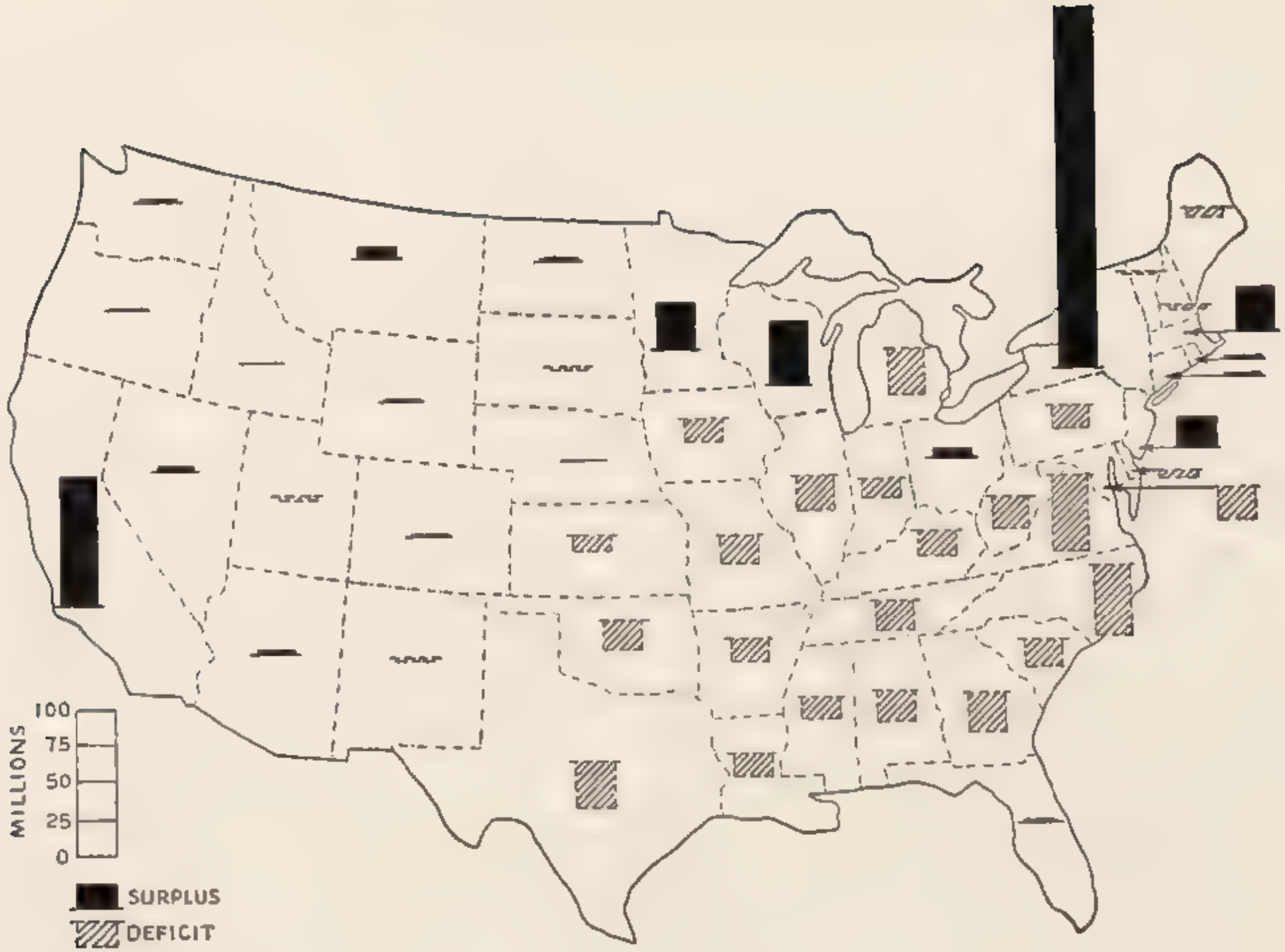
[32] The term local was reserved for later use in case sub-regions were to be split up into still smaller areas.



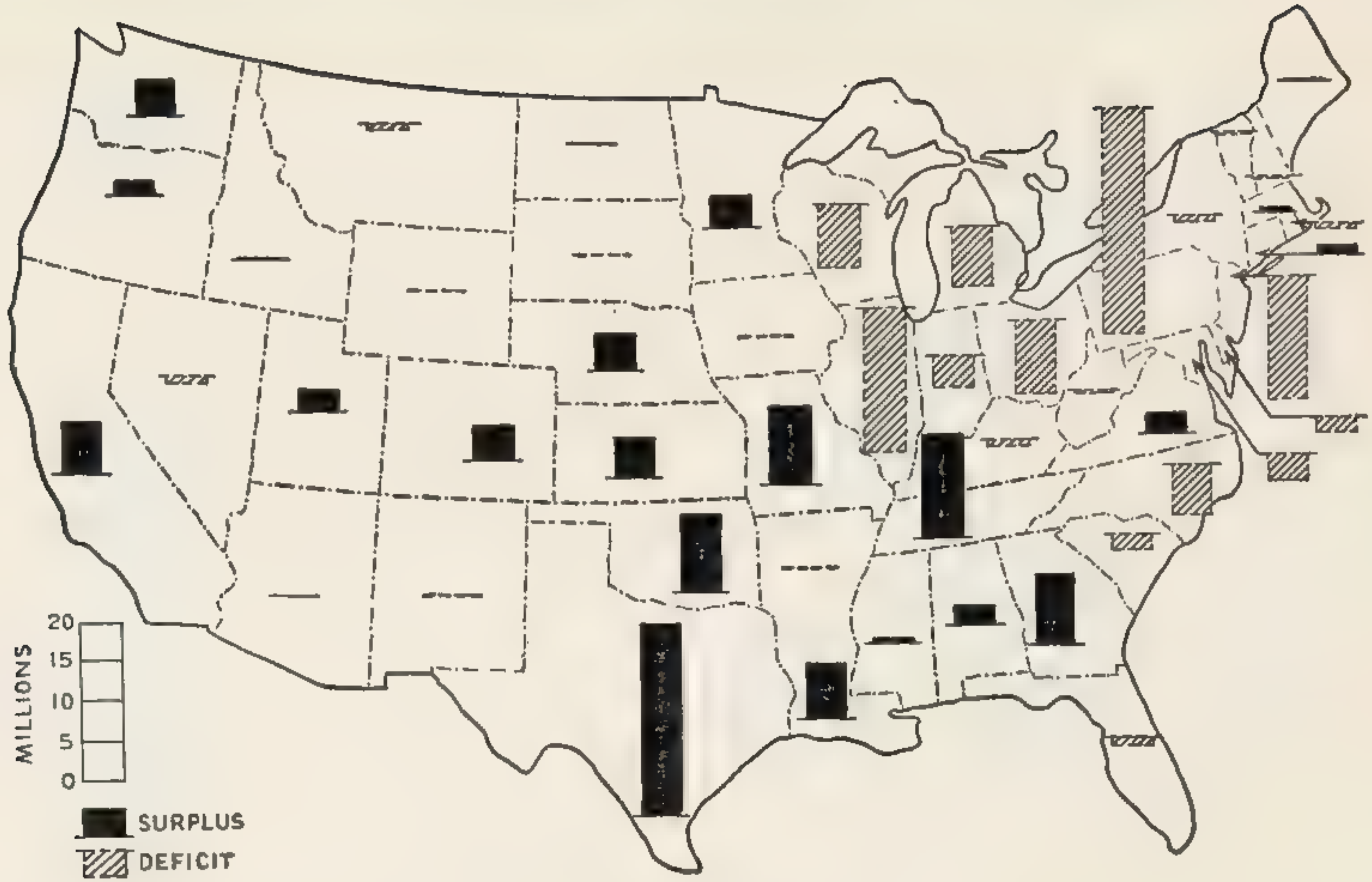
MAP 1
Industry 34: Steam Railroad Transportation
Absolute Surpluses and Deficits by States



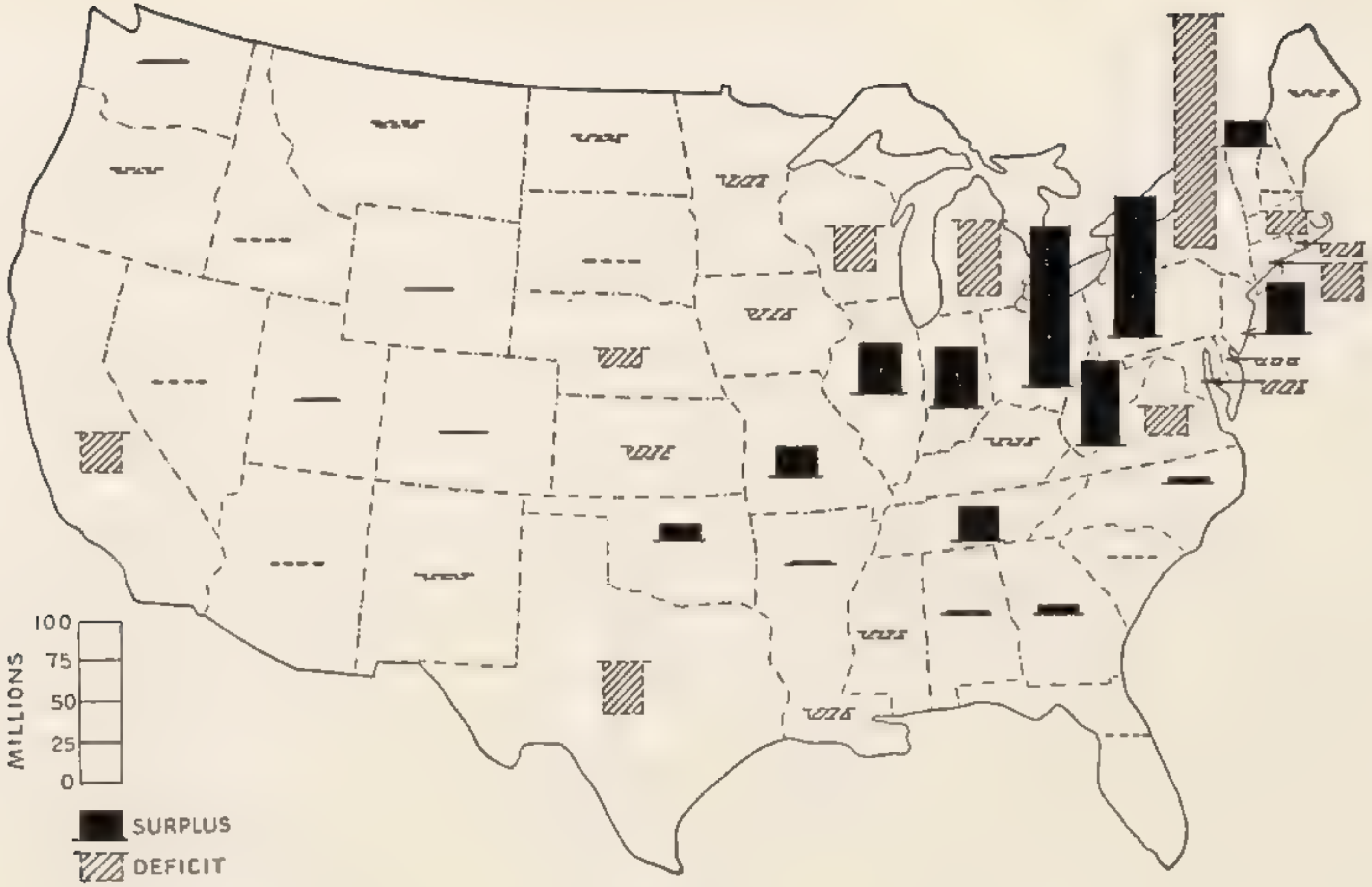
MAP 2
Absolute Surpluses and Deficits by States
Industry 35: Trade



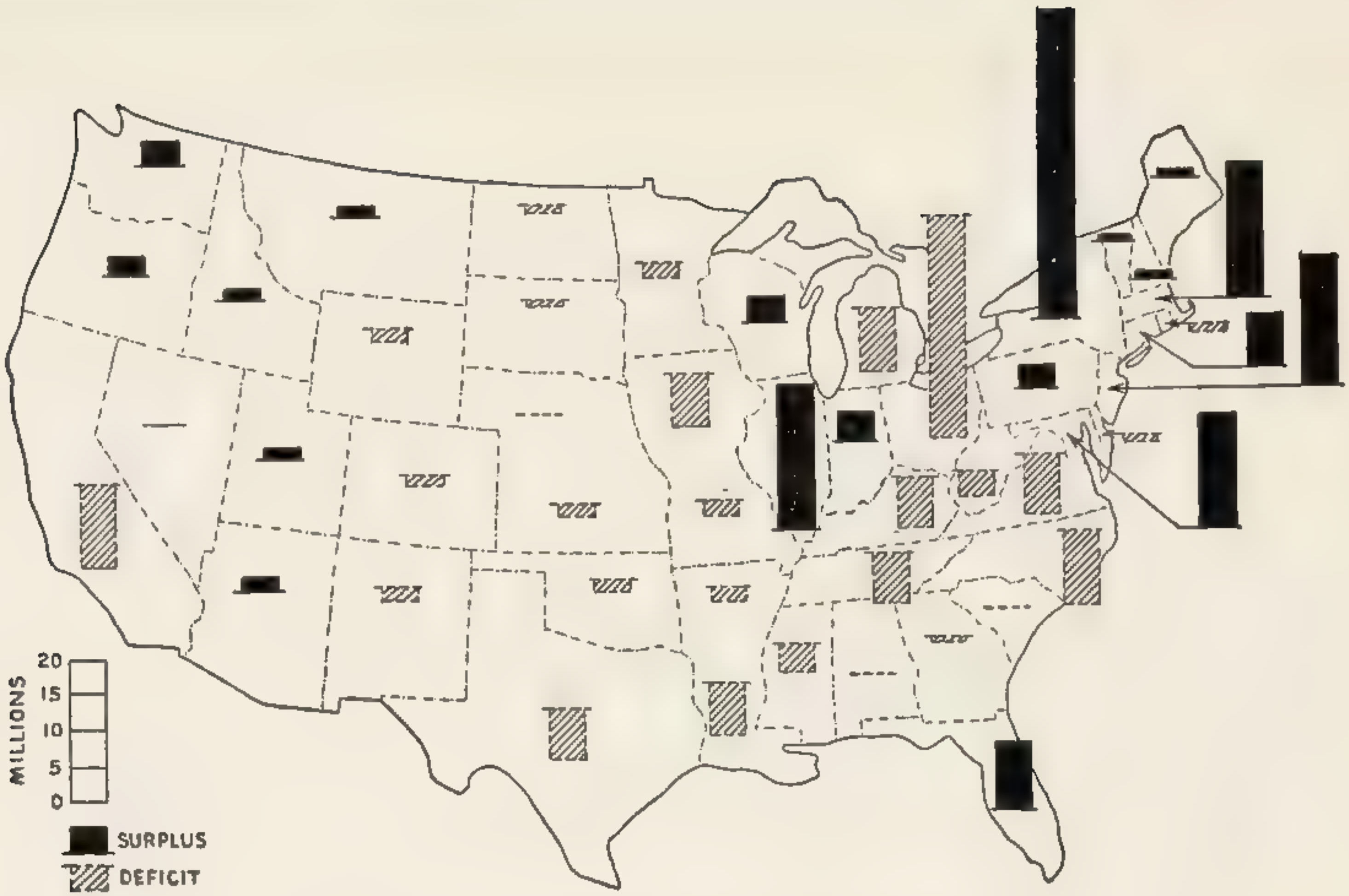
MAP 3
Industry 37: Eating and Drinking Places
Absolute Surpluses and Deficits by States



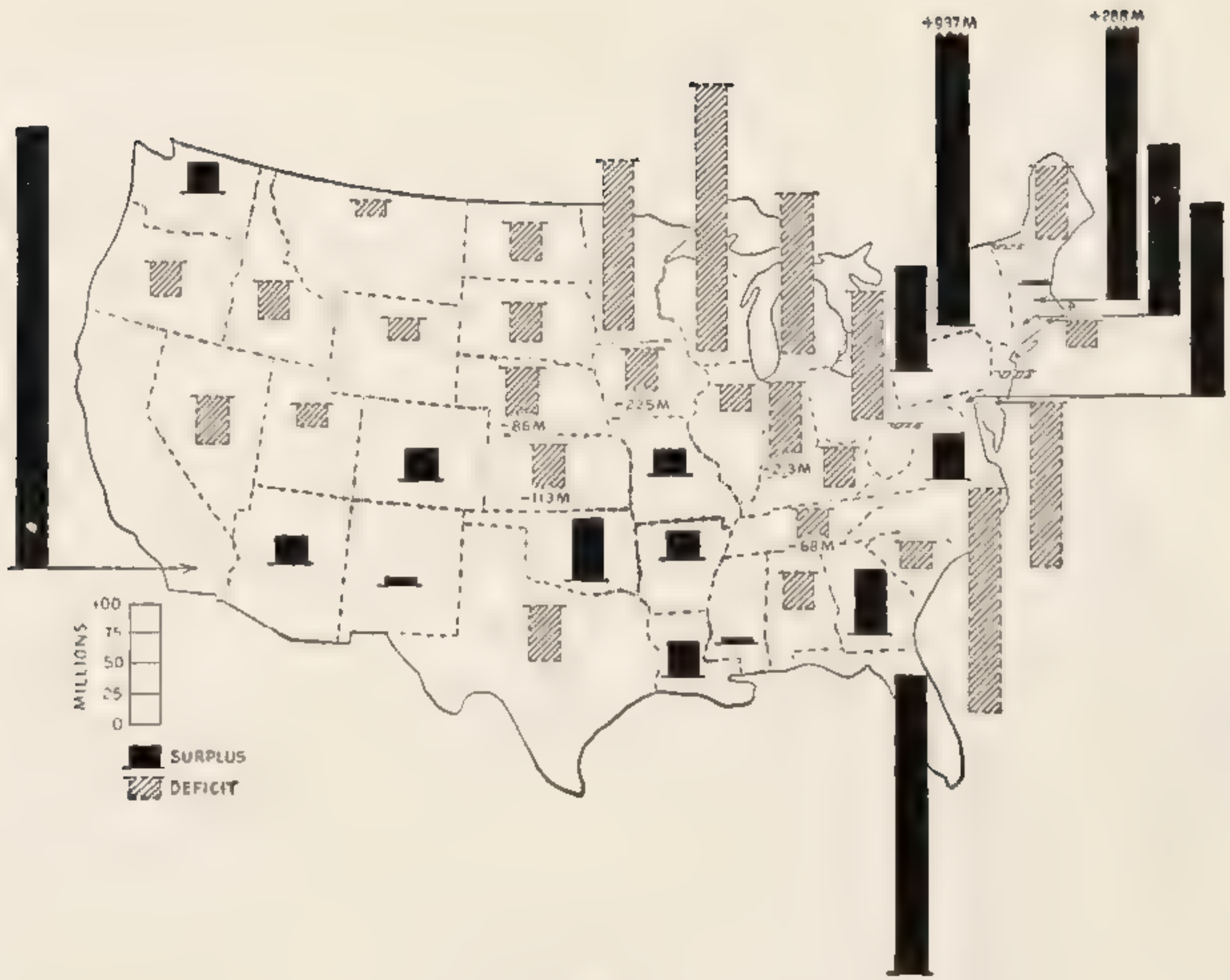
MAP 4
Industry 21. Communications
Absolute Surpluses and Deficits by States



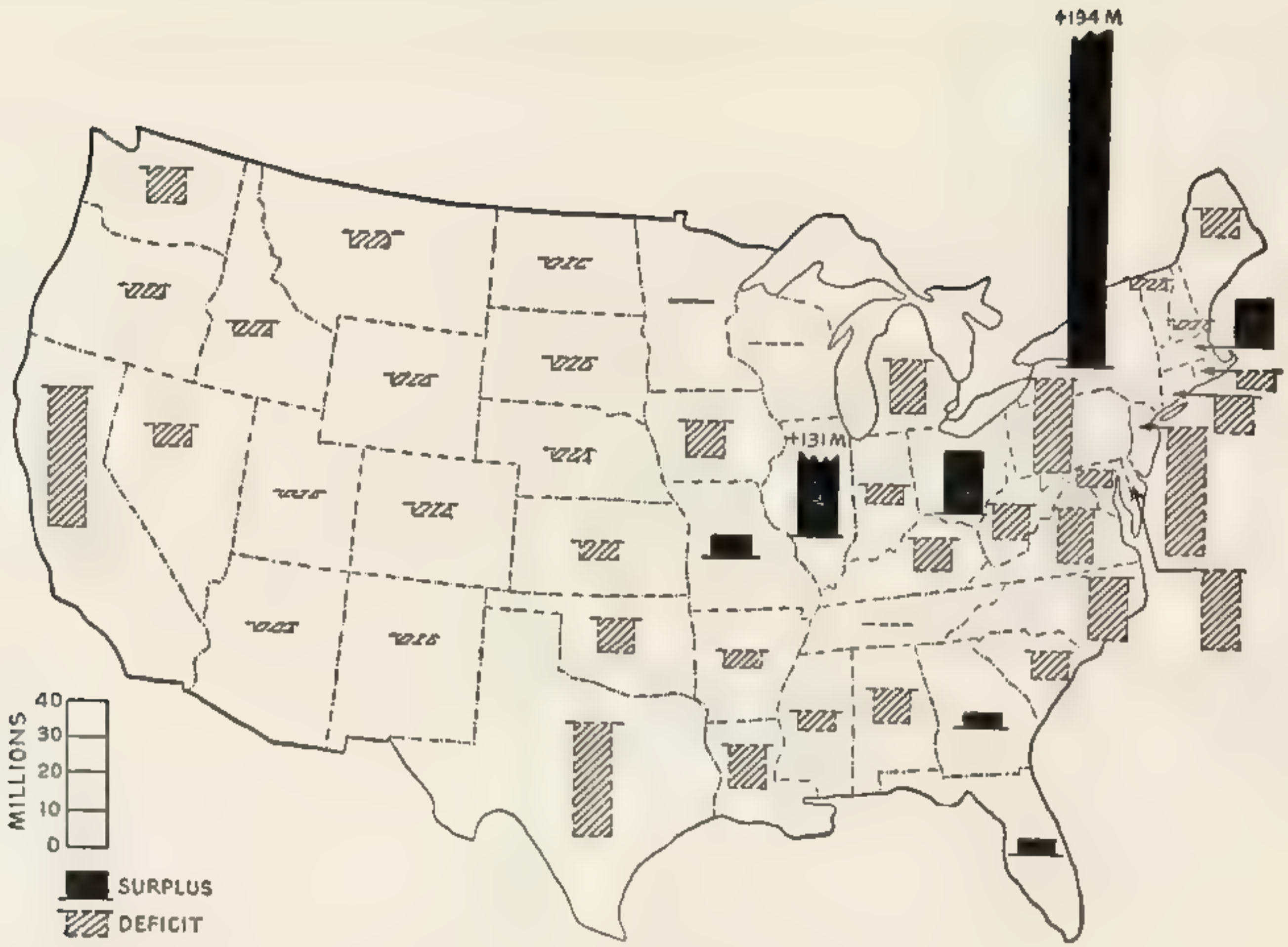
MAP 5
Industry 17: Nonmetallic Minerals and Their Products
Absolute Surpluses and Deficits by States



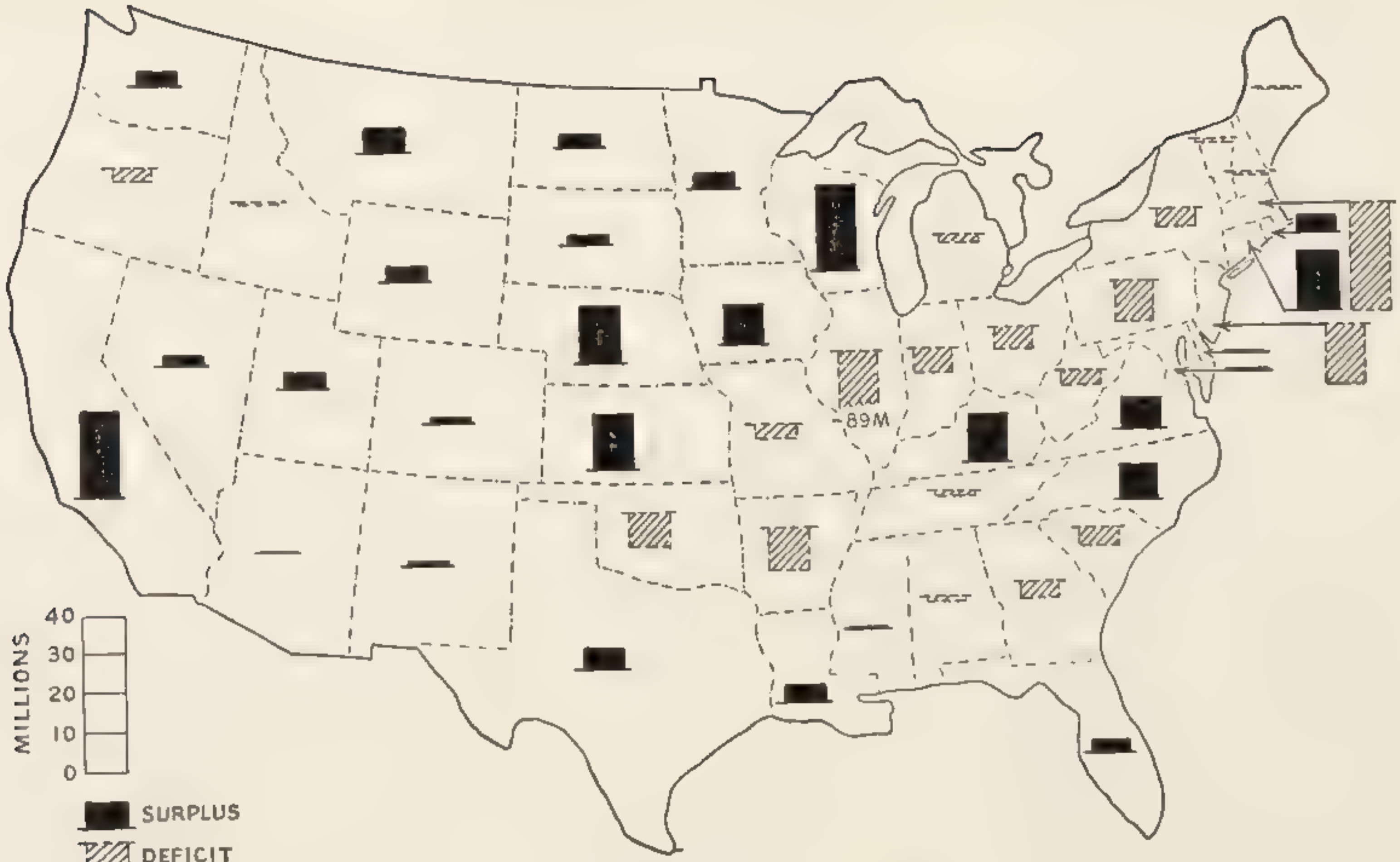
MAP 6
Industry 20. Manufactured Gas and Electric Power
Absolute Surpluses and Deficits by States



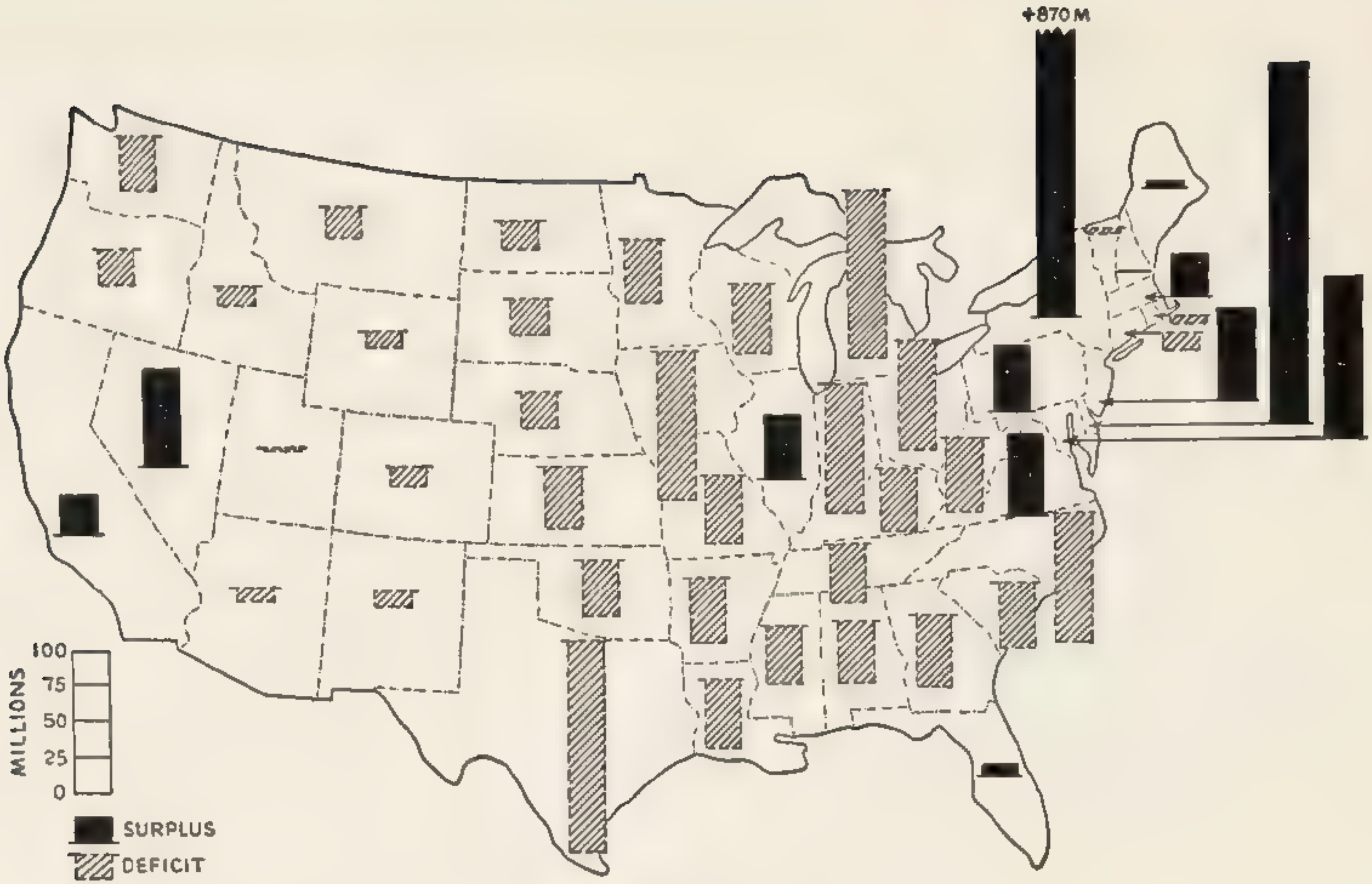
MAP 7
Industry 39: Households
Absolute Surpluses and Deficits by States



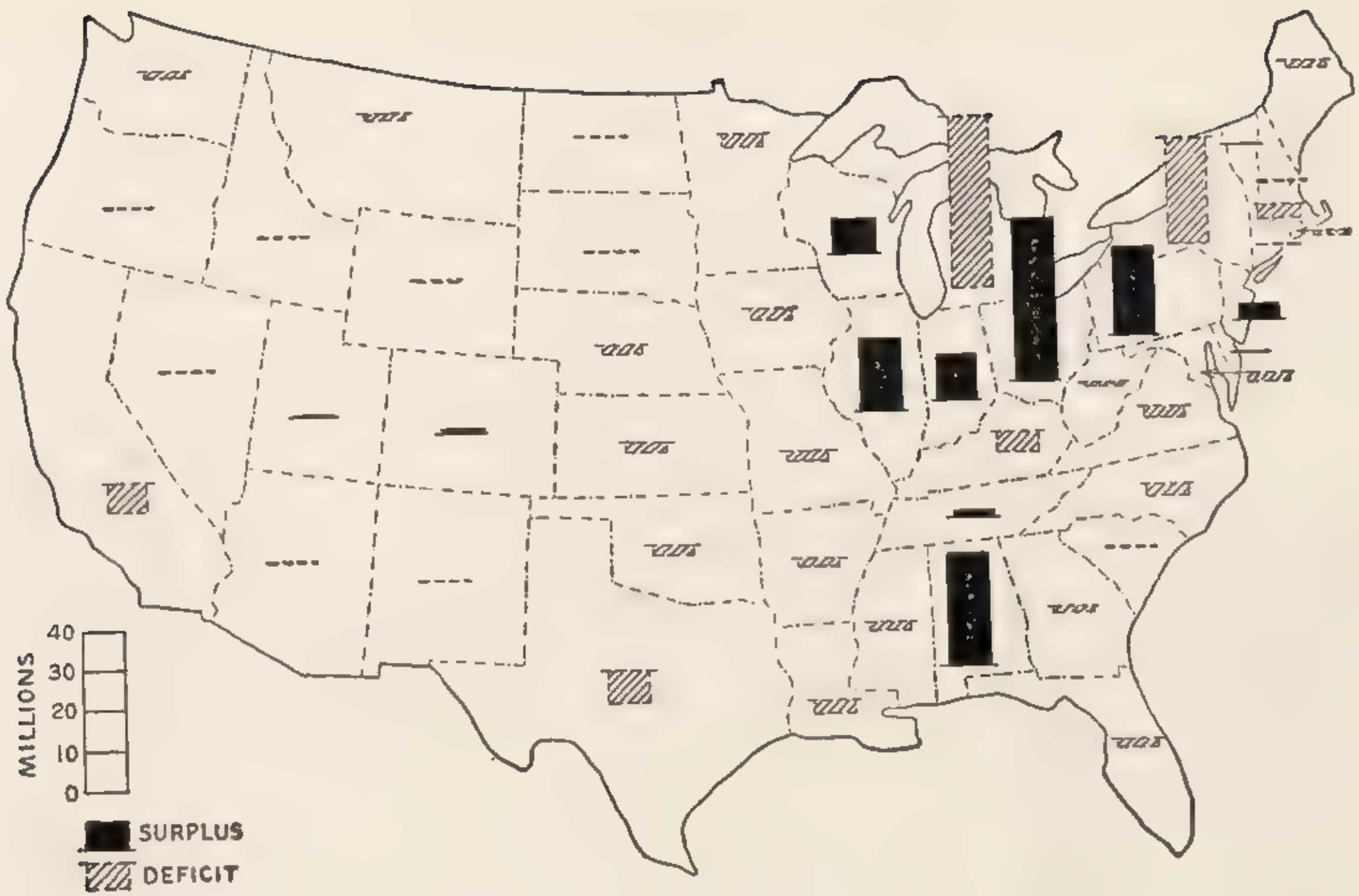
MAP 8
Industry 26: Printing and Publishing
Absolute Surpluses and Deficits by States



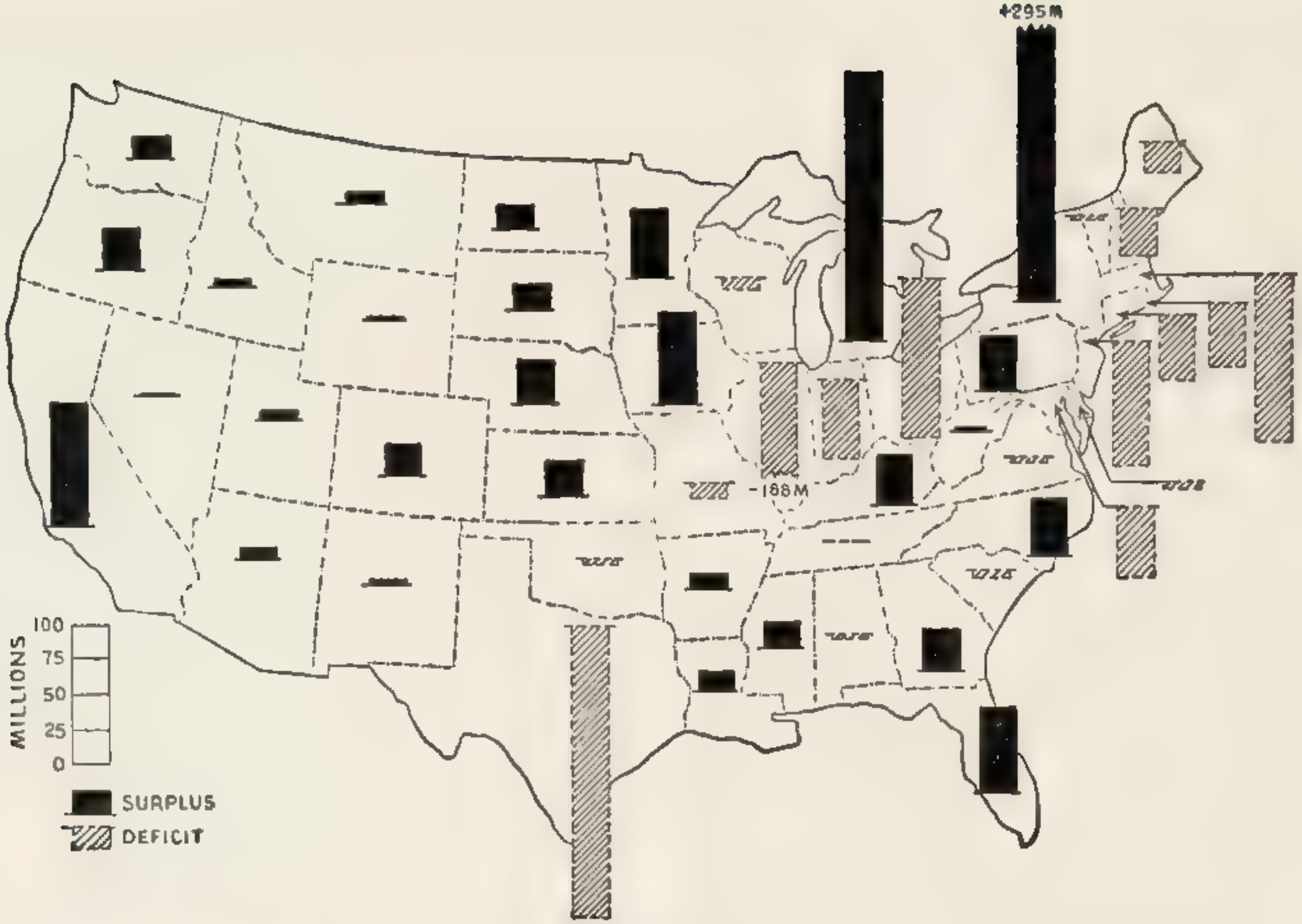
MAP 9
Industry 32: Construction
Absolute Surpluses and Deficits by States



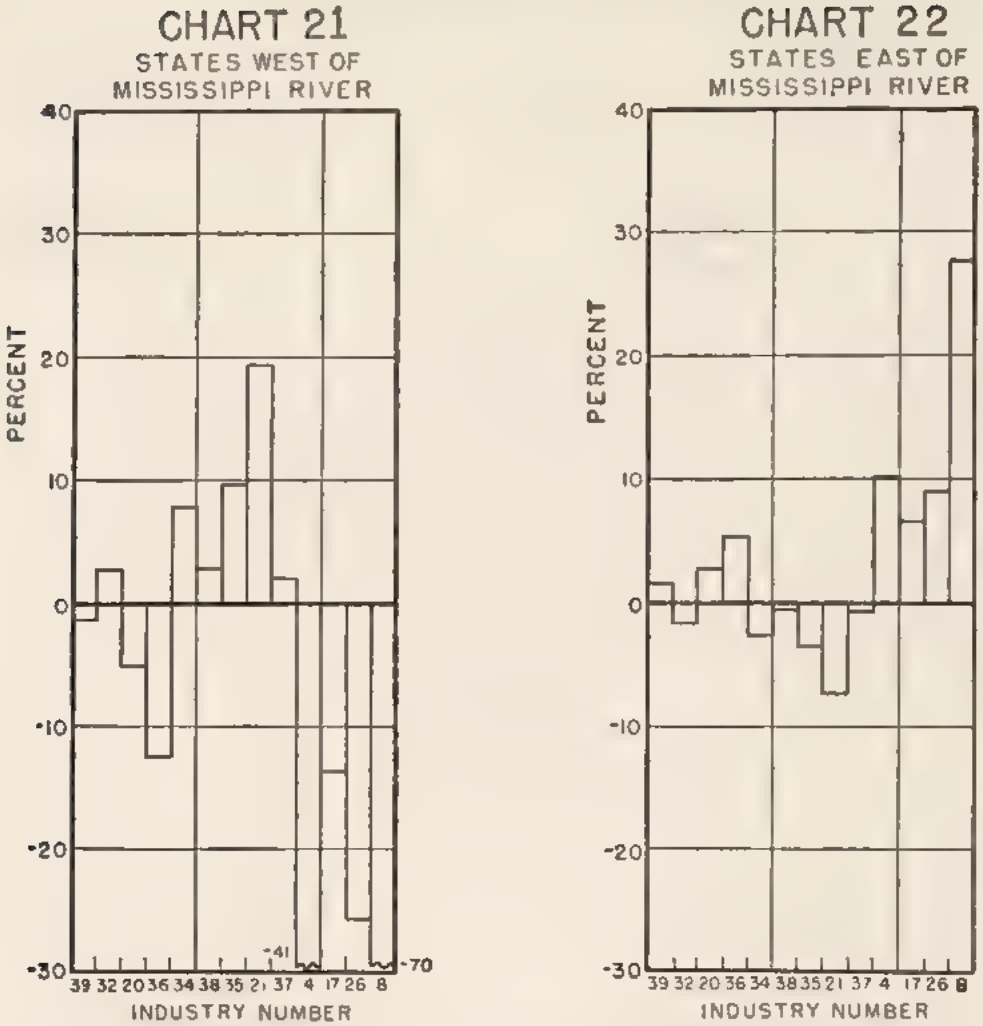
MAP 10
Industry 36: Business and Personal Services
Absolute Surpluses and Deficits by States



MAP 11
Industry 4: Iron and Steel Foundry Products
Absolute Surpluses and Deficits by States



MAP 12
Industry 38: Unallocated
Absolute Surpluses and Deficits by States



NET SURPLUSES AND DEFICITS AS A PERCENT
OF TOTAL CONSUMPTION BY SELECTED INDUSTRIES

integral number of sub-regions. Also, the choice of major regions in turn influences the industries which are finally classified as non-national.

Since there was no presumption at this stage of the analysis for considering three or four major regions rather than two—the study was not sufficiently advanced to do so—we took the simplest course, namely, setting up two major regions. In selecting the two major regions for analysis, Maps 1-12 depicting the *absolute* surpluses and deficits by states for each of the 12 industries were studied.[33] Such maps are invaluable in

[33] Percentage surplus and deficit maps are of greater value in ranking commodities according to the degree to which they are national, or regional of any order. Once having determined which commodities are non-national, we find that maps portraying absolute surpluses and deficits by states are then useful in determining those areas in which the production and consumption of these commodities balance. However, the reader is cautioned against attributing to maps of *absolute* surpluses and deficits any significance in classifying commodities For they cannot have any significance in this regard since they do not reveal anything about absolute consumption.

determining meaningful regions of various orders for each commodity. For our purposes, the most meaningful set of regions of any order for a given commodity is the one where production and consumption by regions tend to balance to the greatest extent. For example, cursory examination of Map 1 showing absolute surpluses and deficits by states for steam railroad transportation indicates that the seven western states bounded by Idaho, Utah, and Arizona on the east might make a meaningful region; that the remaining states west of the Mississippi might constitute a second region; that Illinois, Wisconsin, Michigan, Indiana, and Ohio, a third; that the New England states, New York, Pennsylvania, New Jersey, Maryland, and Delaware, a fourth; and the remaining states east of the Mississippi and south of the Ohio River a fifth.

However, since we desired to set up two major regions, specifically a set of major regions which represented the best compromise set, and since the best set is not the same for each commodity, we also utilized net percentage data such as those in Charts 21-24. Each of these charts relates to a possible major region. Chart 21 applies to the group of states west of the Mississippi and presents for motor vehicles and for each of the 12 industries which are candidates for non-national classification the net surplus or deficit as a per cent of total consumption of this group of states. Motor vehicles is included to bring out the contrast between national and possible non-national commodities.

The companion to Chart 21 is Chart 22, which applies to the group of states east of the Mississippi. Together they form a set of possible major regions. Charts 23 and 24, applying respectively to the New England states and New York, Pennsylvania, and New Jersey, and to the remaining area of the United States, refer to another set of possible major regions. Which of these two sets is the better?

The first set shows better balance with respect to households (39), manufactured gas and electric power (20), business and personal services (36), steam railroad transportation (34), and eating and drinking places (37); and poorer balance with respect to the other seven industries. *A priori,* it does not seem to be superior to the second set. Similarly, for other sets of possible major regions. In addition, since we had already processed some of the data with respect to the second set of possible major regions, we tentatively decided to use this set.

After re-examination of the various charts and reconsideration of the theoretical questions involved, only 10 of the 12 industries were classified as non-national. The two excluded were nonmetallic minerals and their products (17), and printing and publishing (26). Both these industries seemed to have too much of a national character to permit a non-national classification.

CHART 23

NEW ENGLAND NEW YORK
PENNSYLVANIA NEW JERSEY

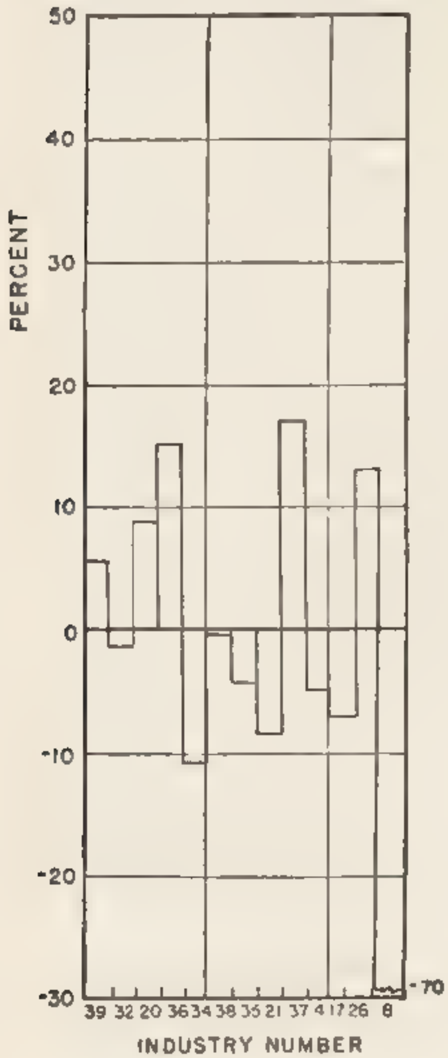
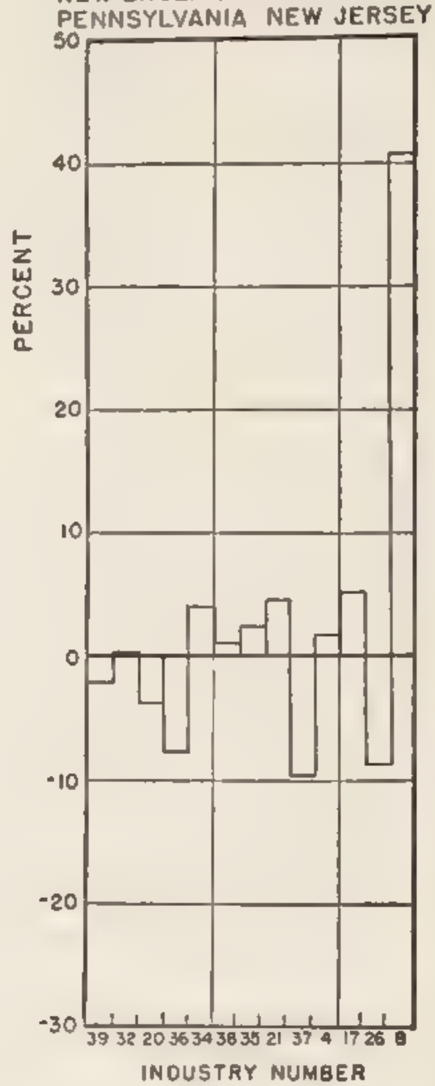


CHART 24

UNITED STATES EXCLUDING
NEW ENGLAND NEW YORK
PENNSYLVANIA NEW JERSEY



NET SURPLUSES AND DEFICITS AS A PERCENT
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Though only 10 industries, roughly one-quarter of the total number of industries, were classified as non-national, it should not be inferred that their relative importance is only of like order. Excluding households in order to avoid double counting, and unallocated for which income data are not available, 8 industries remain from which roughly 50 per cent of national income originated in 1939.[34] Especially since households is a major sector of the economy, the 50 per cent figure is a serious understatement of the relative importance of this group. Undoubtedly, when an industrial classification more appropriate for regional analysis is derived

[34] *National Income Supplement, Survey of Current Business,* July 1947, p. 26.

for future studies, the relative number of non-national industries will increase.

Given the two major regions and the 10 non-national industries, the 1939 national and regional bills of goods were inserted into the model of the previous chapter. The indirectly computed outputs for the 10 industries appear in Column 2 of Tables 3 and 4; the former refers to Major Region I and the latter to Major Region II. These computed outputs contrast with actual outputs in Column 1 of these tables. Column 3 presents the differences between actual and computed outputs, and Column 4, the differences as per cents of actual.

Percentage-wise, these differences are significant. They range from −9.2 per cent to +23.1 per cent for Major Region I, and from −17.1 per cent to +3.4 per cent for Major Region II. Why do such large discrepancies occur between computed and actual outputs?

The causes of these discrepancies have already been indicated. The more important of these are:

(1) industries combined into any single consolidated category may not be homogeneous as regards market areas;
(2) the set of regions chosen is a compromise set, one that tends to be appropriate for the 10 non-national commodities as a whole, and not one that is most desirable from the standpoint of each commodity alone;
(3) each non-national commodity is to some (though generally small) degree national; and,
(4) there exist differences in regional production practices and consumption patterns which are reflected in the actual data [35] but not in the computed.

These causes can be illustrated by referring to the data of Tables 3 and 4. Households, for example, shows a discrepancy of $2.45 billion in Major Region I and a corresponding one of different sign in Major Region II. This is consistent with the fact that certain household labor performs services of a national character, such as the administrative and other personnel in the central offices of many industries. More important, the major regional lines which have been drawn failed to yield a very good balance with respect to households. Charts 23 and 24 evidence a surplus of 5.8 per cent and a deficit of 2.1 per cent for Major Regions I and II respectively. Clearly, Major Region I exports household services to Major Region II.

Since households is a major sector of the economy, any imperfection in drawing regional boundaries with respect to this industry may result

[35] In this connection, see, for example, Neff, P., Baum, L. C., and Heilmann, G. E., *Production Cost Trends in Selected Industrial Areas*, Berkeley, California, 1948.

TABLE 3

Major Region I
Actual and Computed Production, and Absolute and Percentage Differences, by Non-National Industries
(in thousands of dollars)

		Actual Production	Computed Production	Difference Between Actual and Computed Production	Difference as a Per Cent of Actual Production	Adjusted Difference Between Actual and Computed Production	Adjusted Difference as a Per Cent of Actual Production
	Regional Industries	(1)	(2)	(3)	(4)	(5)	(6)
(1)	35 Trade	5707003	5453892	+253111	+ 4.44	−153276	− 2.69
(2)	21 Communications	503002	501568	+ 1424	+ 0.28	− 18286	− 3.63
(3)	37 Eating and drinking places	1770000	1361622	+408378	+23.07	+258773	+14.62
(4)	4 Iron and steel foundry products	134596	139842	− 5246	− 3.90	− 5246	− 3.90
(5)	38 Unallocated	7690000	7395001	+294999	+ 3.84	+294999	− 2.96
	Sub-regional Industries						
(6)	36 Business and personal services	7554668	5963237	+1591431	+ 21.07	+1061508	+14.05
(7)	32 Construction	3581600	3581600	0	0	0	0
(8)	34 Steam railroad transportation	1169113	1276117	− 107064	− 9.16	− 131890	−11.28
(9)	20 Manufactured gas and electric power	1110261	962668	+ 147593	+ 13.29	+ 110012	+ 9.91
(10)	39 Households	24806000	22351175	+2454825	+ 9.90	0	0

[1] Adjusted difference derived from assuming that households of Major Region I exports $2,454,825,000 of output to Major Region II (*see* text).

TABLE 4

Major Region II
Actual and Computed Production, and Absolute and Percentage Differences, by Non-National Industries
(in thousands of dollars)

	Regional Industries	Actual Production (1)	Computed Production (2)	Difference Between Actual and Computed Production (3)	Difference as a Per Cent of Actual Production (4)	Adjusted Difference Between Actual and Computed Production[1] (5)	Adjusted Difference as a Per Cent of Actual Production (6)
(1)	35 Trade	10863997	11117108	−253111	− 2.33	+153726	+ 1.42
(2)	21 Communications	1012998	1014432	− 1424	− 0.14	+ 18286	+ 1.81
(3)	37 Eating and drinking places	2382000	2790378	−408378	−17.14	−258773	−10.86
(4)	4 Iron and steel foundry products	358404	353158	+ 5246	+ 1.46	+ 5246	+ 1.46
(5)	38 Unallocated			−294999	− 2.03	−294999	− 2.03
	Sub-regional Industries						
(6)	36 Business and personal services	10970332	12561763	−1591431	−14.51	−1061500	− 9.68
(7)	32 Construction	6507400	6507400	0	0	0	0
(8)	34 Steam railroad transportation	3140887	3033823	+ 107064	+ 3.41	+ 131890	+ 4.20
(9)	20 Manufactured gas and electric power	1753759	1901332	− 147593	− 8.42	− 110012	− 6.27
(10)	39 Households	43323000	45777825	−2454825	− 5.67	0	0

[1] Adjusted difference derived from assuming that households of Major Region I exports $2,454,825,000 of output to Major Region II (*see* text).

in substantial discrepancies between computed and actual outputs of other industries feeding into households. Therefore, the effects of an adjustment for this imperfection should at least be examined; the more so, since households absorbs all the output of eating and drinking places (Industry 37), and a large part of business and personal services (Industry 36), both of which industries show the largest discrepancies between computed and actual output for both the major regions.

Accordingly, for experimental purposes, it was assumed that there are two basic sectors of the households industry, one which balances in major regions, the other only nationally. The exports of the national sector in Major Region I to Major Region II was taken at $2,454,825,000, the amount of the discrepancy between computed and actual output of the household sector for each of these regions. To produce these exports, various inputs of the non-national industries in Major Region I are required. Therefore, these exports and their required inputs must be added to total computed outputs in Major Region I (i.e. to Column 2 of Table 3) and subtracted from total computed outputs in Major Region II (i.e. from Column 2 of Table 4). The adjusted discrepancies in absolute terms appear in Column 5 of Tables 3 and 4, and in percentage terms in Column 6 of Tables 3 and 4. It is significant to note that the range of percentage discrepancies is considerably compressed for both regions. They now range from +14.6 per cent to −11.3 per cent for Major Region I, and from +4.2 per cent to −10.9 per cent for Major Region II.

With these adjustments, the discrepancies between actual and computed outputs are narrowed down sufficiently so that one might be inclined to consider the model as it now stands satisfactory for cautious and limited use. However, it is much wiser not to attach any significance to the empirical results at this stage. Better results (smaller discrepancies) can yet be obtained for those major regions. For example, we need to study thoroughly the export segment of the household industry as well as that of business and personal services and others. By doing so we can encompass much more precisely local activities in any given region, and thus point up much more sharply the inherent logic of the model. Further, this experiment already brings out certain additional considerations in demarcating major regions. In selecting a compromise set of regions, particular importance should be attached to balance within the household sector, since any imbalance with respect to this sector will to a large degree be responsible for an imbalance in at least two other non-national industries. Thus, the relative importance of the relations of each non-national industry to other non-national industries when placed side-by-side with its imbalances for various possible sets of major regions must be considered in demarcating the best compromise set of major regions, rather than imbalances alone. This new consideration, in fact,

strongly suggests that our *a priori* selection of major regions may have been a poor one.

One additional empirical result is of considerable interest. Even after adjustment for the so-called export sector of households, eating and drinking places (Industry 37) shows a discrepancy of +14.6 per cent and −10.9 per cent for Major Regions I and II, respectively. Since households is the only industry which absorbs the output of eating and drinking places, these discrepancies must reflect for the most part regional differences in consumption patterns. The states contained in Major Region I are more urbanized than those in Major Region II; hence one would anticipate that per dollar output of households more of the output of eating and drinking places would be absorbed in Major Region I than in Major Region II. To eliminate this discrepancy, research on regional differences in consumption patterns must be pursued.

But also there is a presumption that the above discrepancies reflect the fact that to some extent at least the output of eating and drinking places in Major Region I consists of services rendered to households of Major Region II, whose members may be journeying through, vacationing, or otherwise temporarily residing in Major Region I. To some extent, then, eating and drinking places is a national industry. This matter requires further investigation since the national aspect of eating and drinking places may be a crucial factor in the structural analysis of sub-regions or states in which the recreation industry plays a significant role.

V. Sub-Regional Analysis

It might seem logical to the reader that we should have proceeded to explore the merits of other possible sets of major regions and to select the best compromise set before pushing into analysis on a sub-regional level. We decided not to do so on two scores. First, it was felt that the results were already good enough so as not to distort seriously any data indirectly computed on a sub-regional level; and that consequently we could gain from whatever light exploratory sub-regional analysis might cast on the selection of proper major regions. Obviously, the selection of proper major regions and that of proper sub-regions are to some extent interdependent.

Second, it is always possible to eliminate in sub-regional analysis most of the distorting influence of discrepancies between actual and computed outputs in major regions. This can be done by assuming that computed outputs did turn out to be equal to actual, and thus by substituting actual for computed in any operations which involve computed outputs of major regional industries.

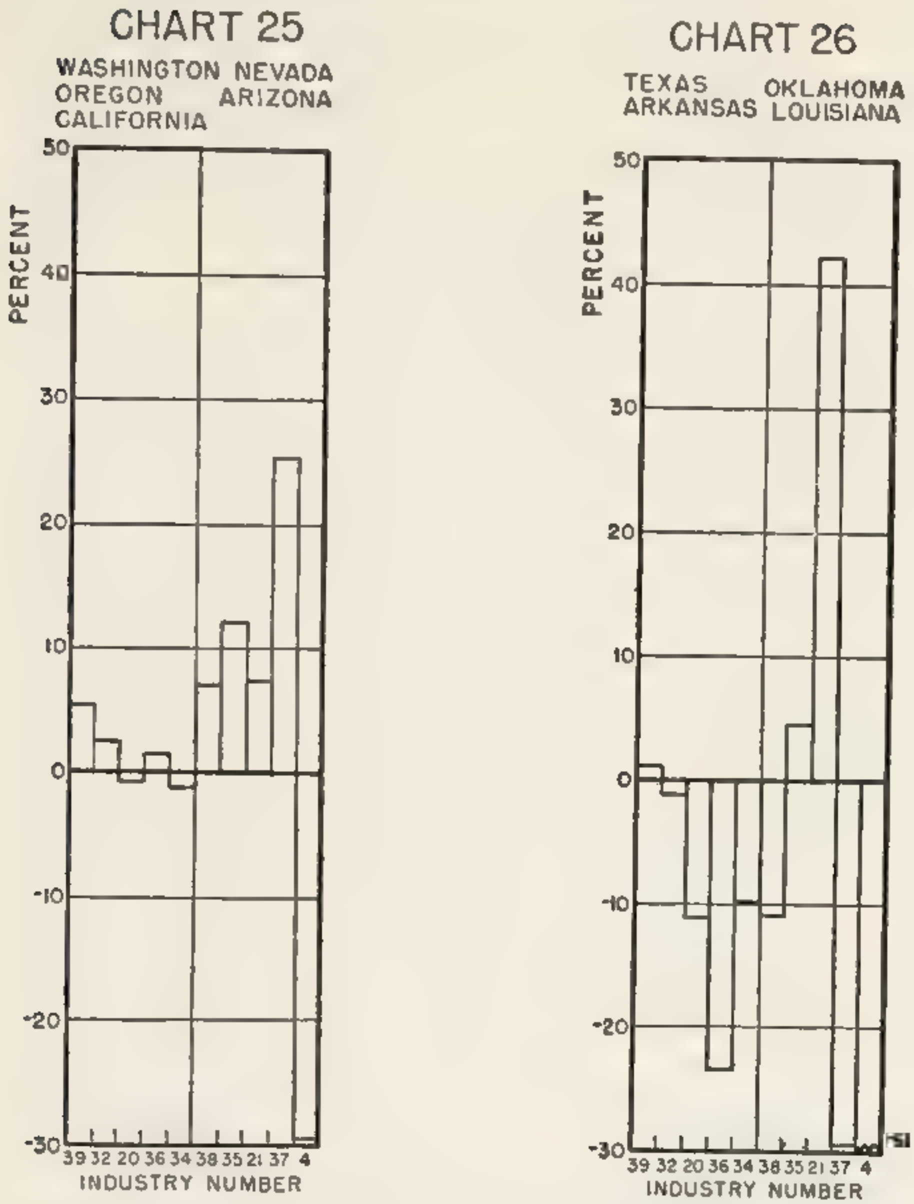
Accordingly, we proceeded to demarcate a sub-region for analysis. A number of possible ones were considered. For each Census region, balances of production and consumption for the 10 non-national industries were examined. Since there was no presumption that Census regions were more desirable than others, states were added and subtracted from these regions, or Census regions were combined, or both. Other regional classifications of economists, geographers, and sociologists were looked into.[36] For each of the more promising sub-regions, a chart showing the per cent surplus or deficit for each of the 10 non national commodities was constructed. See, for example, Chart 25.

In choosing a sub-region for analysis, previous experience in selecting major regions was extremely helpful. It was recognized that a major region's resource endowment, transportation facilities, urban-rural pattern, institutional structure, and so forth, can conceivably give rise to substantial interstate trade and intra-regional trade, with respect to non-national commodities, even among the most self-sufficient of the sub-regions. In such a case, division of a major region into two or more sub-regions would have little value for our purposes. Certainly, this point had to be kept in mind in considering the partition of our Major Region I composed of New England and three Middle Atlantic states.

The problem of selecting a sub-region for analysis could not be divorced from the problem of designating the commodities to be classified as sub-regional (i.e. commodities which show balance within the sub-region as well as the major region). Conceivably, because of regional differences in human and natural resources and settlement patterns, one set of commodities may be appropriately classified as sub-regional with respect to one major region, and another set as sub-regional with respect to another region.

The relative importance of the relations of any sub-regional industry with other sub-regional industries as well as its imbalance for any given sub-region had to be weighed both in selecting a sub-region and in designating sub-regional commodities. After much comparison and deliberation, a sub-region composed of the states of Washington, Oregon, California, Nevada, and Arizona was chosen for analysis, and the industries, households, construction, manufactured gas and electric power, business and personal services, and steam railroad transportation were designated sub-regional. Parenthetically, it should be noted that though only 5 of the 39 industries are taken as sub-regional, roughly 30 per cent of national

[36] See, for example, National Resources Committee, ***Regional Factors in National Planning***, Washington, 1935, ch. 15; Victor Roterus and Sterling March, ***Economic Development Atlas***, Department of Commerce, Washington, 1950; and R. E. Dickinson, ***City, Region and Regionalism***, London, 1947, ch. 12.

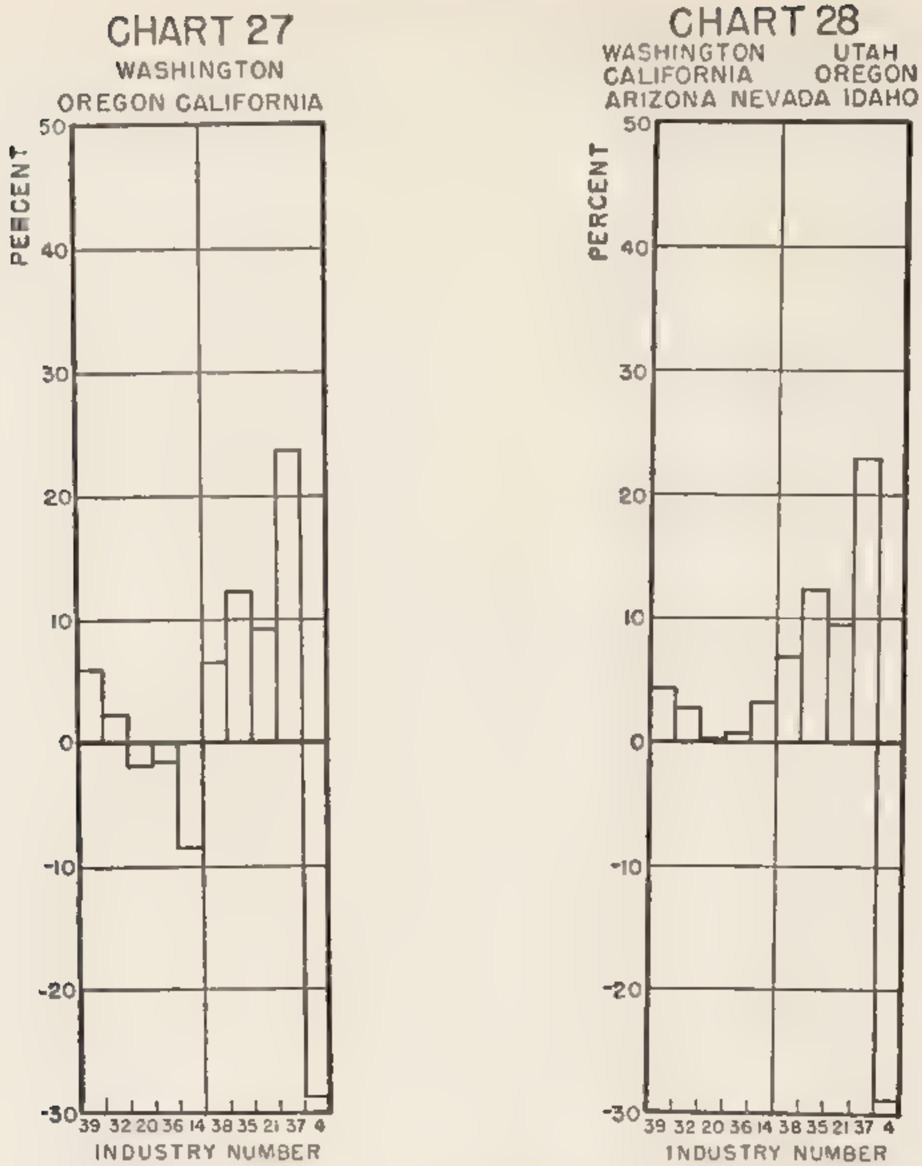


NET SURPLUSES AND DEFICITS AS A PERCENT OF TOTAL CONSUMPTION, BY NON-NATIONAL INDUSTRIES

income in 1939 originated in these sub-regional industries, excluding households.[37]

Some of the factors relevant to this specific decision can be illustrated with the use of charts. Charts 25 and 26 show percentage surpluses and deficits for each of the 10 non-national industries for the group of states already mentioned and for Texas, Oklahoma, Arkansas, and Louisiana, respectively. The latter group of states shows a better balance than the former with respect to households and construction, both basic industries and ones which on an *a priori* basis could well be classified as sub-regional.

[37] *National Income Supplement, Survey of Current Business,* July 1947, p. 26.



NET SURPLUSES AND DEFICITS AS A PERCENT OF TOTAL CONSUMPTION, BY NON-NATIONAL INDUSTRIES

On the other hand, only one other of the 10 non-national industries, namely trade, shows an imbalance considerably less than 10 per cent. All the others evidence an imbalance of approximately 10 per cent or more. Since previously we reached the tentative conclusion with respect to major regions that industries showing imbalances which on the average exceed, say, 10-15 per cent ought not to be classified non-national, and since our experience indicated that a still lower limit is to be desired if it can be achieved, we decided to fix the lower limit at 5-7 per cent for sub-regional analysis. By this criterion, the sub-region including Texas would have only three sub-regional industries, whereas the sub-region including California would have five. On this ground we chose the latter rather than the former, although it was realized that only after actual

experimentation with both sub-regions could a firm preference be stated. Also, it was recognized that the imbalances in the sub-region including Texas may reflect to a large extent major differences in production practices and consumption patterns between this sub-region and the nation as a whole, and that in fact this sub-region may be much more self-composed with respect to a set of basic sub-regional commodities than the sub-region including California. Once again more regional data and experimentation are required.

Another question which logically arises is why this particular set of five states was chosen. Why not a set of three states, Washington, Oregon, and California; or of seven, the selected sub-region plus Utah and Idaho? Both these sets seem to constitute meaningful geographic blocs, and evidence in Charts 27 and 28 respectively balances of roughly the same order as the selected sub-region of five states.

On the basis of knowledge available to us, especially in terms of metropolitan dominance since the geographic flows of some of the non-national commodities and services are strongly oriented to metropolitan-urban areas, the sub-region of five states seemed preferable to that of three. Large parts of Arizona and Nevada seem to be dominated by the two major metropolises of California, Los Angeles and San Francisco, respectively; and this may be responsible for the somewhat better balance that is evidenced by the sub-region of five states.

On the other hand, the sub-region of seven states seems to present a slightly better balance than that of five states. However, in terms of the hierarchical relations of a metropolitan regional economy,[38] the states of Idaho and Utah seem *a priori* to be considerably independent of the Pacific Coast states. Thus Idaho and Utah do not tend to form with Nevada and Arizona as integral a unit with the coastal states as Nevada and Arizona alone do.

Limitation of space precludes any further discussion of the numerous problems which beset us. Given the design of our sub-region, we were in a position to compute in accordance with the basic model outputs for each of the sub-regional industries and contrast these with actual outputs. In Table 5, the actual outputs and computed outputs are presented in Columns 1 and 2 respectively. In Column 3 the surpluses and deficits are registered and in Column 4 they are expressed as percentages of actual outputs. It is significant to note that these discrepancies are of a much smaller order than those for the major regions. They range from +5.04 per cent to −1.05 per cent.

[38] Refer, for example, to Bogue, Don J., *The Structure of the Metropolitan Community*, University of Michigan, 1949; Hawley, A., *Human Ecology*, New York, 1950; and to Isard, W., and Whitney, V., 'Metropolitan Site Selection,' *Social Forces*, Vol. 27, March 1949.

The causes of these discrepancies are in many respects similar to those enumerated in the discussion of discrepancies for major regions. There is, however, the possibility of one other major cause, namely, the fact that the requirements of sub-regional industries are according to the model geared to the computed and not the actual outputs of the five regional industries [39] in Major Region II. Thus discrepancies in these latter can lead to discrepancies with respect to sub-regional industries, and distort results on a sub-regional level. From another angle, we may ask to what extent the 'goodness' or 'poorness' of the results for our sub-region can be attributed to the cancellation or magnification of sub-regional imperfections by the imperfections on the major region level.

The answer to this question can in large part be gained by substituting actual for computed outputs of the five regional industries in our model, i.e. by assuming that no imperfections exist on a major region level. The new surpluses and deficits for the five sub-regional industries are registered in Column 5 of Table 5, and these surpluses and deficits as per cents of actual in Column 6. As a consequence of this operation, the percentage range of discrepancies increases, its limits now being +0.39 per cent and −0.57 per cent. Nonetheless, the discrepancies can be considered of a relatively small order. The results are good and they point to considerable potential significance in the operation of the model on a sub-regional level—such as is implied, for example, in Table 1 in the first section of this chapter.

VI. Further Experiments with Major Regions

Having obtained quite satisfactory results on a sub-regional level, we returned to the problem of analysis and experimentation on a major region level. It has already been noted that consideration of the relative importance of the relations of each non-national industry to other non-national industries strongly suggests that our *a priori* selection of major regions may have been a poor one. Why not, then, select a set of major regions with a view to this new consideration? In particular, why not select a set of major regions with better balances *à propos* households?

Our previous work indicated that a set of major regions consisting of states east and west [40] of the Mississippi River (henceforth designated Major Regions Ia and IIa) evidences better balances with respect to households. On the other hand, there is an inconsistency in such a division in that the role of the Mississippi River has been one of connecting

[39] The reader is reminded that *regional* industries constitute part of the *non-national* group, and that *sub-regional* industries constitute another part.

[40] West of the Mississippi River is taken to include Minnesota.

TABLE 5

Sub-Region A
Actual and Computed Production, and Absolute and Percentage Differences, by Sub-Regional Industries
(in thousands of dollars)

	Sub-Regional Industries	Actual Production (1)	Computed Production (2)	Difference Between Actual and Computed Production (3)	Difference as a Per Cent of Actual Production (4)	Adjusted[1] Difference Between Actual and Computed Production (5)	Adjusted Difference as a Per Cent of Actual Production (6)
(1)	36 Business and personal services	1838390	1749018	+ 89372	+ 4.86	+117387	+ 6.39
(2)	32 Construction	1055654	1031024	+ 24630	+ 2.33	24630	+ 2.33
(3)	34 Steam railroad transportation	357324	361064	– 3740	– 1.05	– 2052	– 0.57
(4)	20 Manufactured gas and electric power	264574	256119	+ 8455	+ 3.20	13496	+ 5.10
(5)	39 Households	6713505	6375183	+338322	+ 5.04	+411446	+ 6.13

[1]Adjusted difference obtained from substituting actual data on production of regional commodities in Major Region II for computed data in deriving computed production of sub-regional industries (*see* text).

rather than separating areas on either side, of intensifying rather than diminishing bonds between them. Nevertheless, it seemed worth while to experiment with Major Regions Ia and IIa.

Reconsideration of major regions compels reconsideration of industry classification. It was immediately apparent from maps and other charts and the data that iron and steel foundry products would not be a meaningful regional industry from the standpoint of Major Regions Ia and IIa. Likewise, for most sets of major regions which differ considerably from that consisting of Major Regions I and II. Imbalance would be too large. For that reason, iron and steel foundry products was reclassified as national, leaving 9 non-national industries, 5 of which are sub-regional. Other changes in classification did not seem to be warranted, though for Major Regions Ia and IIa it was questionable whether the communications industry ought to retain the label of non-national. This, because both major communication centers of the United States, Chicago and New York City, which are responsible for a large part of the national sector of this industry, are in Major Region Ia (east of the Mississippi River).

The computations were performed for Major Regions Ia and IIa. The resulting percentage discrepancies between computed and actual outputs for each non-national industry are recorded for Major Regions Ia and IIa in Columns 3 and 4, respectively, of Table 6. They contrast with those for Major Regions I and II in Columns 1 and 2, respectively. Note the considerable improvement in balance effected when the set of Major Regions Ia and IIa is employed. The range of discrepancies narrows down from -17.14 per cent at one extreme and $+23.07$ at the other to -16.77 and $+14.59$. And in general the entire set of discrepancies is reduced, only two items evidencing percentage discrepancies greater than 10 per cent. As might be expected, the communications industry in both regions shows the largest discrepancy. If Major Regions Ia and IIa should be employed as a desirable set of regions, serious consideration should be given to reclassifying the communications industry as national.

Major Regions I and II, and Ia and IIa form more or less two extremes as sets of major regions. Major Region I accounted for 36.4 per cent of national income in 1939, Major Region II for 63.6 per cent. In contrast, Major Region Ia accounted for 66.6 per cent of national income, while Major Region IIa, 33.4 per cent. It would not seem expedient, in general, to consider any set of major regions where the differences in terms of national income percentages accounted for by each major region were greater.

Hence, possible demarcation lines (1) east of the Mississippi River, and (2) west and/or south of Pennsylvania were examined. Three other sets of major regions were chosen for experimentation. The set of Major

Regions Ib and IIb comprise on the one hand the New England and Middle Atlantic census regions (as in Major Region I) plus the South Atlantic census region (Delaware, Maryland, Virginia, West Virginia, North Carolina, South Carolina, Georgia, and Florida), and on the other hand the remainder of the United States. The set of Major Regions Ic and IIc contain, respectively, the area corresponding to Major Region Ib plus the East South Central census region (Kentucky, Tennessee, Oklahoma, and Mississippi), and the remaining part of the United States. Finally, the set of Major Regions Id and IId were constructed identically with the set of Major Regions Ic and IIc save that Ohio was taken out of Major Region IIc leaving Major Region IId and put into Major Region Ic thus forming Major Region Id.

Space does not permit discussion of the various reasons for selecting these sets of major regions. Computations were carried through for each, and the resulting percentage discrepancies are listed in Table 6. Note that the improvement over Major Regions Ia and IIa achieved with the use, say, of Major Regions Ic and IIc or Id and IId is noticeable but not too significant. The set Id and IId seems to yield the best set of percentage discrepancies, although at this stage the choice of a best set to a large extent depends upon the nature of a study and the inclination of the researcher evaluating any particular discrepancy. More important is the fact that business and personal services shows up with large percentage discrepancies in each set of major regions. Certainly with respect to this industry, research must be pushed forward along the lines already mentioned above in order to yield a more effective technique of analysis.

VII. Conclusions

A few concluding remarks are in order. Little if any significance should be attached *per se* to the empirical results which can be derived from the operation of the experimental model as it now stands. Rather, the value of these results lies in the insight gained into the methods and techniques for improving the existing design and in pointing out critical areas for regional research.

Clearly, a consolidation of industries more appropriate to regional analysis is required. Component industries of future consolidations should be more homogeneous with respect to market areas. This will be possible with the finer industrial breakdown used in the input-output table for 1947. Connected with this line of improvement is the need to separate the national and non-national segments of any industrial category, as, for example, in the case of households and eating and drinking places. In view of the increasing supply of regional data, research along these lines will be facilitated, as well as the research required to show the differences

TABLE 6

Percentage Differences between Actual and Computed Production for Various Sets of Major Regions, by Regional and Subregional Industries

	Listing of Industries	Major Region I	Major Region II	Major Region Ia	Major Region IIa	Major Region Ib	Major Region IIb	Major Region Ic	Major Region IIc	Major Region Id	Major Region IId
	Regional	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1)	35 Trade	+ 4.44	− 2.33	+3.56	− 8.48	− 2.12	+ 1.52	− 1.25	+ 0.98	+ 0.58	− 0.90
(2)	21 Communications	+ 0.28	− 0.14	+8.45	−16.77	+ 1.62	− 1.00	+ 0.11	+ 0.12	+ 2.83	− 2.93
(3)	37 Eating and drinking places	+23.07	−17.14	+0.11	− 0.88	−10.97	+ 9.90	− 5.76	+ 5.50	− 4.23	+ 5.06
(4)	4 Iron and steel foundry products	− 3.90	+ 1.46	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]
(5)	38 Unallocated	+ 3.84	− 2.03	+1.39	− 3.91	− 2.57	+ 1.83	− 2.24	+ 1.87	− 0.36	+ 0.18
	Sub-regional										
(6)	36 Business and personal services	+21.07	−14.51	−4.73	+14.59	−17.43	+18.39	−13.80	+16.47	−10.76	+16.08
(7)	32 Construction	0	0	+1.23	− 3.01	+ 0.76	− 0.65	+ 0.50	− 0.51	+ 0.56	− 0.72
(8)	34 Steam railroad transportation	− 9.16	+ 3.41	+3.01	− 7.62	+ 4.75	− 3.26	+ 1.52	− 1.56	+ 1.79	− 2.30
(9)	20 Manufactured gas and electric power	+13.29	− 8.42	−2.20	+ 6.01	−10.05	+ 9.16	− 7.52	+ 7.77	− 3.88	+ 4.81
(10)	39 Households	+ 9.90	− 5.67	−0.56	+ 1.15	− 7.50	+ 6.34	− 5.95	+ 5.87	− 4.15	+ 5.14

[1] No figure derivable since in this set of major regions iron and steel foundry products is classified as national and not regional.

among regions in production practice and consumption patterns. Allowances for these differences, especially on the sub-regional level, will increase the reliability of empirical results. Simultaneously, more experiments with various combinations of regions of various orders and associated industries are of utmost importance in order to determine the best compromise sets of regions and industries, and to minimize unavoidable imperfections in the model.

Concomitant with improvement of existing design, some basic refinements need to be incorporated into the theoretical scheme. As already indicated, it is desirable to introduce both pure interregional and dynamic elements into the design: the former, by linking the output of any industry in a given region to inputs from the same and other specified regions; the latter, by expressing regional production and consumption as a function of time.

If such improvements and refinements can be attained, a general equilibrium system with both a space and a time axis will result. A major step will have been made toward describing and anticipating the dynamic operation of a space-economy, a process frequently visualized by earlier writers such as Weigmann, Ohlin, and Lösch,[41] but hitherto never empirically depicted.

[41] Weigmann, H., 'Ideen zu einer theorie der Raumwirtschaft,' *Weltwirtschaftliches Archiv*, Bd XXXIV, 1931; Ohlin, B., *Interregional and International Trade*, Cambridge, Massachusetts, 1933; and Lösch, A., *Die räumliche Ordnung der Wirtschaft*, Jena, 1944. For others, *see* Isard, W., 'The General Theory of Location and Space-Economy,' *Quarterly Journal of Economics*, Vol. LXIII, November 1949

PART III

THE CAPITAL STRUCTURE OF THE AMERICAN ECONOMY

Chapter 6

THE STRUCTURE OF CAPITAL

Robert N. Grosse [1]

I. Introduction

Analyses of dynamic models of general interdependence such as those discussed in Chapter 3 require knowledge of capital-output (or capital-capacity) ratios which are analogous to the input-output ratios used in static models of general interdependence. The relation between ton miles of freight hauled per year by railroads and railroad consumption of coal is of the latter type. The relation between hauling capacity, measured in ton miles per year and number of locomotives *on hand,* illustrates the capital-output relationship.

The input-output systems previously published have included only relations between current flows and current outputs. This has been so not because of any lack of recognition of the importance of stock-flow relationships, but because of lack of data. This chapter presents for the first time a table of capital requirements per unit of capacity (capacity being measured in terms of rates of output per year) for each industrial group by industry of origin. The data constitute another input-output table which supplements the flow type of input-output table previously presented. The dimensions of the new table are different from those of the current flow table since each coefficient is the ratio of an absolute amount to a time rate of output.

The concept of a capital coefficient, i.e. the quantity of capital required per unit of capacity in an industry, though superficially simple, is actually rather complicated. Consider the production of any particular item by a given technique. With given stocks of various types of equipment, buildings, and inventories, a certain optimum rate of production can be achieved with the technique in question. In an analysis in which each product and each type of capital is dealt with separately, capital coefficients would be the ratios of the number of units of a given type of capital to the maximum output mentioned above. The proportions between the different types of capital would be fixed by the technique employed. It

[1] James S. Duesenberry and Elizabeth W. Gilboy worked with the author in the various stages of revising the initial manuscript.

can be seen at once that if capital coefficients defined in that way are used in an analysis of dynamic processes which purports to be realistic, a number of assumptions have to be made. First, economies or diseconomies of scale may cause variations in the optimum output per unit of capital even with a constant technique. Second, it may be necessary to allow for substitution between techniques. Third, the question of aggregation arises when we estimate capital coefficients for industries rather than for particular products.

The first two questions can be dealt with simultaneously. Theoretical considerations indicate that in general the increments in capital stocks of various types resulting from increments in various outputs will depend on (1) the relative prices of different types of capital and labor; and (2) the scale of operation of the firms in the industries in question.

A great deal of emphasis has been placed upon the role of relative prices in determining the method of production employed by a firm. In fact, however, the relative prices of capital equipment and labor can vary only within a limited range. That is so because labor cost is by far the largest element in the cost of producing capital goods. Provided then that relative wages in capital-producing and capital-using industries vary only within narrow limits, there is likely to be at any one time a single 'best practice' in any production process. That 'best practice' technique will usually be the one which uses the minimum amount of labor per unit of output, taking into account both direct labor and labor 'congealed' in the construction of equipment. Substitution occurs chiefly when there are technical improvements in capital-goods production which result in a fall in the ratio of the price of capital goods to the price of labor. But with a given technique, there will be relatively little price substitution. There are, of course, exceptions, most of which result from changes in wage rates in different industries. The outstanding example is the switch from steam to diesel power resulting from increased wage rates in the coal industry. But it is our view that exceptional cases of that sort do not occur frequently enough to destroy the usefulness of the fixed capital coefficient concept.

Technique may also vary with the scale of output. In this connection we have to consider changes in output in the firm and in the industry. As to the first it seems reasonable to suppose that competitive forces will control the size of firms in such a way as to eliminate firms too small to take full advantage of economies of scale. An exception must be made in the case of new and rapidly expanding industries, but that case is not likely to be important in the total industrial picture. Putting aside the case of new industries, it is unlikely that external economies will produce very serious changes in technique.

The aggregation problem has been discussed elsewhere in connection with the current flow input-output table (see Chapters 1 and 9).

Once the concept of fixed capital coefficients is accepted, the problem of obtaining empirical estimates of those coefficients must be faced. It should be noted that for purposes of dynamic models such as those discussed in Chapter 3, incremental rather than average capital coefficients are needed. If technique were constant the two would be the same. But in fact the capital owned by a firm at any one time is the result of accumulation over a number of years and therefore represents various states of technique. The actual ratio of capital to capacity for a firm will differ considerably from the ratio of increments in capital to increments in capacity. How much difference there will be depends on the rate of change of technique in the industry in question.

True incremental coefficients can be obtained either from data on the construction of new plants or directly from engineering sources. Some of the capital coefficients presented here were obtained by the first method. Studies of input requirements based on engineering data are discussed in Chapters 8, 10, and 11. Because of lack of information, however, most of these coefficients represent average rather than incremental coefficients. This reduces their usefulness, but does not vitiate it since the incremental and average coefficients are for the most part of the same order of magnitude.[2]

It should be noted that the capital coefficients are expressed in terms of capacity units, and not, as in the case of the flow coefficients, in terms of output units. An industry seldom operates continuously at maximum short-run capacity, due to marketing conditions, differences in shifts, necessity for repairs, and, in some cases, the maintenance of a 'normal' spare capacity.[3] Consequently, capital equipment is usually maintained on the basis of capacity output rather than actual output at any one time. Assuming that the capital stock unused at any period bears the same relation to its potential output as the capital in use to actual output, capacity appears to be more closely related to total capital on hand than output. For that reason the capital coefficients computed here are capital-capacity ratios rather than capital-output ratios (see Table 1, folded into pocket at back of book).

In addition to the over-all capital coefficients, it has been possible to estimate capital coefficients by industry of origin. As in the case of technical flow coefficients based on current inputs, these coefficients permit

[2] See Table 3, p. 209.

[3] See Chapter 7 on this practice in the telephone industry. Regional problems are also important in connection with capacity, which may be adequate on a national basis, but inadequate to meet the demands of specific localities (see Part II).

the splitting up of the total stock of capital used by any sector of the economy by its industrial origin. It thus becomes possible to trace the interaction of capital flows throughout the complex of interindustrial relationships, as well as to follow the flows of current inputs. We know, for example, not only the total amount of fixed capital used per unit of capacity in the shipbuilding industry, but also how much of this capital equipment was supplied to it by specified industries.

The amounts of raw materials and goods-in-process held by the various industries and held 'for them' by producers have been estimated. Since speculation may play a significant role in determining actual inventory holdings, they have been adjusted where possible to eliminate stocks exceeding what might be called the normal inventory stocks. The inventory coefficients, presented later in this chapter, are the ratio of inventories held in 1939 to output in 1939, i.e. they give the amount of inventories per unit of output utilized by industries in the American economy. Inventory coefficients by industry of origin have also been computed, thus permitting the examination of inventory flows, as well as of fixed capital and current input flows.

The sources of the data and the methods of computation used in the derivation of these coefficients are described in the next section. It may be stated here, however, that, while the inevitable statistical inaccuracies are well recognized, the internal checks on the methods of estimation and the comparisons with other estimates (presented in Section III) indicate that this preliminary investigation of the capital structure of American industry gives results which are reasonable in the light of what is otherwise known about the economy.

The primary significance of the capital and inventory coefficients as mentioned previously lies in their importance for dynamic analysis. When input-output information is restricted to current flows, only a static analysis is possible. What would happen to the economic system, for example, if the capacity of the clothing industry were expanding? The capital coefficient tells us that for each additional unit of capacity a certain amount of additional capital will be needed; similarly the inventory coefficients indicate the necessary increases in working capital. What is more, the capital coefficients by industry of origin show upon what industries the clothing industry depends for its capital and to what extent. Such changes in capacity lead the way to a change in the structure of the economy.

To illustrate the use of the fixed capital coefficients in the solution of a specific problem, a hypothetical example of a 10 per cent increase in the output of the clothing industry, Industry 61, has been assumed (see Table 2). How will this increase affect the purchases of the industry?

To determine changes in inventory requirements and in the flow of goods for current use, both of which are considered to have a fixed relation to changes in output, the inventory and current flow coefficients are multiplied by the desired output, raising the inventories and current flows by 10 per cent.

The determination of the expenditures on fixed capital goods is somewhat different, being complicated by the problems of replacement and capacity utilization. Replacement requirements to maintain capacity are estimated by multiplying the stock of each type of capital good by its appropriate depreciation rate. If the desired output is greater than the capacity of the industry in the preceding year, the additions to the capital stock can be calculated by multiplying the increment to capacity by the capital coefficient for each type of capital good. In our example, using 1939 data, the output and capacity of the clothing industry were:

1939 output	$3,824 483,000
1939 capacity	4,153,101,000
'1940' output	4,206,931,300
increase in capacity	53,830,000

It is interesting to note that, even with a 10 per cent rise in the output of the clothing industry, the purchases of capital goods would be lower in 1940 than in 1939. This is, of course, an example of the operation of the 'acceleration principle'; the clothing industry is (hypothetically) expanding at a lower rate than it had in 1939, and while current inputs of current items continue to rise, in this case by 10 per cent, capital-goods purchases fall by about 60 per cent.

To trace the further effects of the change in the clothing industry, similar calculations based on the changes in the inputs of the clothing industry should be made for each supplying industry, and, in turn, the effects of the changes in their input requirements can be calculated. The mathematical technique for calculating the consistent and simultaneous effects on every sector of the economy is described in Chapter 2.

Here the actual empirically determined capital and inventory coefficients of American industries for the year 1939 are presented on the basis of the 96-industry classification, which has, however, been consolidated to 68 industries for this purpose. Table 1 gives the main data on capital coefficients. Tables in Appendix 1 present the inventory coefficients and a summary of the capital stock data. The remainder of this chapter will deal with a description and explanation of the sources and statistical methods used, internal checks on the estimates, comparisons with other estimates, and some tentative interpretation of the results in the light of their economic significance.

TABLE 2

Computation of the Direct Expenditures on Capital Goods by the Clothing Industry, Assuming a 10 Per cent Rise in its Output
(dollars are in millions)

Industry of Origin	Existing Capital Stock (1939)	Depreciation Rate	Replacement Requirements (1) x (2)	Increment to Annual Capacity (1940)	Capital Coefficient	Additions to Capital Stock 1940 (4) x (5)	Replacement Requirements and Additions to Stock, 1940 (3) + (6)	Actual Purchases, 1939
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) Engines and turbines	$ 1,661	.050	$ 84	$53,830	.0004	$ 22	$ 106	$ 7
(2) Industrial equipment, n.e.c.	115,100	.067	7,684	53,830	.0278	1,500	9,184	26,305
(3) Merchandising and service machines	13,380	.067	891	53,830	.0032	172	1,063	3,034
(4) Iron and steel, n.e.c.	25,350	.100	2,533	53,830	.0061	328	2,861	6,184
(5) Construction	135,250	.020	2,708	53,830	.0326	1,753	4,461	7,672

II. Derivation of the Estimates [4]

A. SUMMARY OF METHODS

Incremental over-all capital coefficients, i.e. the stock of capital goods of all kinds required per unit of capacity increase in each industry, were computed from data on newly constructed plants or from direct engineering information. When neither of these types of information was available, average coefficients were calculated from estimates of total stock and capacity. For 13 industries incremental coefficients were computed; for two more, coefficients of certain subgroups were derived on an incremental basis. Average coefficients, computed on both an undepreciated and depreciated capital stock basis, were calculated for the remaining industries.

The calculation of capital coefficients by industry of origin was essentially an apportionment of the over-all coefficients, based partly on accounting information submitted by firms to a governmental regulatory agency, and partly on a calculation involving the observed annual flows of capital goods into an industry, its capacity, depreciation rates, and rate of growth during 1939.

B. DERIVATION OF OVER-ALL CAPITAL COEFFICIENTS

1. *Incremental capital coefficients*

Estimates of the over-all capital coefficients were derived as far as possible from either engineering estimates of 'best practice' or from wartime data on recently built new plants or additions to existing facilities, and corresponding additions to capacity. Estimates derived from these sources have two major limitations: wartime prices were very high, and the capacity data were undoubtedly based on wartime practices. The facilities and capacity-output figures (when given in monetary terms) have both been deflated to 1939 prices through the use of available price indices. Every effort has been made to calculate individual coefficients from as large a sample as possible, and to utilize information on new plants rather than additions to old ones. Where the coefficients varied considerably from plant to plant in the same industrial category, those at the extremes of the range have been eliminated before calculating a weighted over-all coefficient for the industry.

[4] The basic statistical research from which these estimates were derived was carried on under the supervision of A. Benjamin Handler, now Associate Professor at the University of Michigan, and Robert Solow, now Assistant Professor of Economics at the Massachusetts Institute of Technology. Additional research on the derivation of capital coefficients on an incremental basis for the post-war period is now being carried on by a number of government agencies and universities under the guidance of the Division of Statistical Standards, Bureau of the Budget.

The industries whose capital coefficients were obtained from records of expansion or engineering estimates are listed below:

INDUSTRY	SOURCES
23 Blast furnaces	Surplus Property Administration, *Report on Disposal of Government Iron and Steel Plants and Facilities*, 1945, p. 7; War Production Board, *War Industrial Facilities Authorized by Company and Plant Location*, p. 135.
24 Steel works and rolling mills	Surplus Property Administration, op. cit. pp. 7-8, 25-30.
25 Iron and steel foundry products: iron castings; steel castings—electric furnace	Ibid.; War Production Board, *War Manufacturing Facilities Authorized Through October* 1944, *by General Type of Product of Operation in* 1939, 1945, Vol. I, Table II.
41 Aluminum products	Surplus Property Board, *Report to Congress on Aluminum Plants and Facilities*, 1945, p. 59.
49 Coke and manufactured solid fuel	Surplus Property Administration, *Report on Disposal of Government Iron and Steel Plants and Facilities*, 1945, p. 7.
20 Edible fats and oils, n.e.c.	War Production Board, *War Manufacturing Facilities Authorized Through October* 1944.
21 Other food products	Ibid.
27 Firearms	Ibid.
28 Munitions	Ibid.
45 Petroleum and natural gas: natural gasoline; natural gas	Ibid.; *Census of Mineral Industries*, 1939, p. 134.
58 Cotton yarn and cloth	Grosse, Anne, 'Technical Production Function for Carded Cotton Textiles.'
72 Transoceanic transportation	*Statistical Abstract of the United States*, 1941, pp. 513-14; Maritime Commission, *The Postwar Outlook for American Shipping*, p. 106.
68-69 Construction	Bureau of Labor Statistics, Bulletin #779, *Postwar Capacity and Characteristics of the Construction Industry*, pp. 9-10.
40 Smelting and refining of nonferrous metals	*Minerals Yearbook*, Reviews of 1940 and 1945; *Mining Industry During* 1941, p. 129; Surplus Property Board, *Report to Congress on Aluminum Plants and Facilities*, 1945, p. 59; War Production Board, *War Manufacturing Facilities Authorized Through October* 1944.
39 Nonferrous metal mining: other than gold and silver	*Minerals Yearbook*, Review of 1940, p. 74; 1941, p. 630; *Mining and Metallurgy*, Vol. 25, 1944, p. 429. Vol. 24, 1943; Vol. 23, 1942, pp. 33, 261; *The Mining Industry During* 1940, p. 158; 1941, pp. 146, 154; Bureau of Mines, Bulletin #433, 1941, p. 6.

2. *Average over-all capital coefficients*

All other over-all capital coefficients were derived from estimates of the total fixed capital stock of the individual industries and their corresponding capacities for the year 1939. Unpublished data of the Bureau of Internal Revenue of the kind contained in *Statistics of Income*, Part 2, but on a finer industrial breakdown, were the main sources of primary information. *Moody's Manual of Industrials*, trade-association publications, and special compilations of statistics of individual industries were also used.

The data from the Bureau of Internal Revenue were utilized in two ways. First, whenever a published estimate of the capacity of an industry in 1939 was available, the ratio of capacity to 1939 output was applied to the gross sales of firms covered. The capital coefficient was then computed as the ratio of capital stock to the corrected sales. Second, where no independent estimate of capacity could be found, the procedure was as follows: comparable data on gross sales were obtained for the years 1939-41 (sometimes 1937-42), deflated to 1939 prices with an appropriate price index, and then used for the calculation of the corresponding capital stock-sales ratios. The year in which the industry used the smallest quantity of capital per dollar of sales was assumed to be the year of capacity operation, and the capital stock-sales ratio of that year was taken as the average over-all capital coefficient.

Industries whose coefficients were derived from the Bureau of Internal Revenue data and independent capacity estimates are noted below.

	INDUSTRY	CAPACITY SOURCES
10	Flour and grist mill products	Department of Commerce, 1947 *Statistical Supplement to the Survey of Current Business*, p. 131.
11	Canning and preserving	Office of Price Administration, *Industrial Capacity in the United States.*
66	Rubber products: tires and tubes	Ibid.
19	Manufactured dairy products	Ibid.
22	Iron mining	*Minerals Yearbook*, Reviews of 1939 and 1942.
31	Automobiles	*Automotive Industries Journal*, 1 January 1940, p. 3.
47	Anthracite coal	*Minerals Yearbook*, 1940, 1941.
48	Bituminous coal	Ibid. 1945.
55	Furniture and other manufactures of wood	Department of Commerce, op. cit. p. 10.
56	Wood pulp, paper, and paper products: basic products	*Barrell's Paper Annual*, 1945-6; 1946-7.
57	Printing and publishing	Department of Labor, *Economic Factors Bearing on Minimum Wages in the Printing and Publishing Industry*, p. 57.

INDUSTRY	CAPACITY SOURCES
86 Hotels	National Industrial Conference Board, *Economic Almanac*, 1945-6, p. 293.
60 Woolen and worsted manufactures	Department of Commerce, *Facts for Industry*, 'Wool Manufactures, Machinery Activity Reports,' 1945-9.
54 Lumber and timber products: sawmills	*Monthly Labor Review*, December 1942, p. 1127.

Coefficients for the following industries were derived by computing both capital stock and capacity from Bureau of Internal Revenue data:

12 Bread and bakery products
15 Alcoholic beverages
16 Nonalcoholic beverages
26 Shipbuilding
32 Aircraft
29 Agricultural machinery
34 Industrial and household equipment, n.e.c.
35 Machine tools and metalworking machinery
44 Nonmetallic mineral manufactures
53 Chemicals
56 Wood pulp, paper, and paper products: converted products
61 Clothing
81 Automotive repair and services
87 Laundry, etc.
88 Personal services
89, 90, 91 Professional entertainment, motion picture theaters, amusement places
37 Electrical equipment, n.e.c.
38 Iron and steel, n.e.c.
43 Nonmetallic mineral mining
66 Rubber products: other than tires and tubes
67 Industries, n.e.c.
39 Nonferrous metal mining: gold and silver
76 Banking
77 Insurance
62 Other textile products
63 Leather
64 Leather shoes (capital stock includes United Shoe Machinery Corporation's value of machinery out on lease in United States)
65 Leather products, n.e.c.
74 Trade
78 Business services
54 Lumber and timber products: planing mills

Moody's Manual was used for 8 industries. In the case of 5, both capital stock and capacity data were available for a small number of companies, and the same sampling and weighting procedures have been used as with the wartime expansion data. In 3 industries capacity data were taken from another source, and the same procedure was used as with data from the Bureau of Internal Revenue. These industries are noted below:

INDUSTRY	CAPACITY SOURCES
14 Starch and glucose products	*Moody's Manual of Industrials*, 1940.
17 Tobacco manufactures	Ibid
18 Slaughtering and meat packing	American Meat Institute, *Meat*, p. 11.
30 Engines and turbines	*Moody's Manual of Industrials*, 1940.
33 Transportation equipment, n.e.c.	Office of Price Administration, *Industrial Capacity in the United States*.
42 Nonferrous metal manufactures and alloys	Ibid.
50 Manufactured gas	*Moody's Manual of Industrials*.
59 Silk and rayon products	Ibid.

In the 10 industries listed next, information on capital stock was obtained from special compilations of statistics of the individual industries. Independent estimates of capacity were available for sugar refining and petroleum refining. The method of computing the capacity of the electric public utilities industry is described below, page 204. The capacities of the other 7 industries were estimated by using the output of the year of highest output reasonably close to 1939 as a measure of capacity.

INDUSTRY	CAPITAL SOURCES	CAPACITY SOURCES
1-8 Agriculture	Department of Agriculture, *Balance Sheet of Agriculture.*	Bureau of Agricultural Economics, Department of Agriculture, *Agricultural Statistics,* 1948, Table 672, p. 590.
9 Fishing	Fish and Wildlife Service, *Fishery Statistics of the United States,* 1939.	Ibid.
15 Sugar refining	Farr and Company, *Manual of Sugar Companies; Moody's Manual of Industrials;* Department of Agriculture, *Agricultural Statistics;* Lynsky, M., *Sugar Economic Statistics.*	Office of Price Administration, *Industrial Capacity in the United States,* p. 2.
45 Petroleum and natural gas: crude petroleum	Pogue and Coqueron, *Sources, Disposition and Characteristics of Capital Employed by 30 Oil Companies.*	*Minerals Yearbook,* 1939, 1940, 1941.
46 Petroleum refining	Pogue and Coqueron, op. cit.	Department of Commerce, 1947 *Statistical Supplement to the Survey of Current Business,* p. 164.
51 Communications	Federal Communications Commission, *Statistics of the Communications Industry in the United States,* 1942, Tables 4 and 24, 1944, Table 25; ibid. *Public Service Responsibility of Broadcast Licensees,* 1946, Tables 1, 5, and 11.	Ibid.
52 Electric public utilities	Federal Power Commission, *Statistics of Electrical Utilities.*	Edison Electric Institute, *Statistical Bulletin,* 1947, p. 19.
71 Coastwise and inland water transportation	Interstate Commerce Commission, *Selected Financial and Operating Statistics from Annual Reports of Carriers by Inland and Coastal Waterways.*	Ibid.

INDUSTRY	CAPITAL SOURCES	CAPACITY SOURCES
70 Transportation, n.e.c.	Ibid. *Statistics of Railroads; Report of Federal Co-ordinator of Transport*, Vol. 1, part 2, p. 162; Civil Aeronautics Board, Senate Document #206, 77th Congress, 2d Session, 1942; Interstate Commerce Commission, *Statistics of Motor Carriers; Census of Electrical Industries, Street Railways and Trolley-Bus and Motorbus Operations*, 1937.	Ibid.
73 Steam railways	Ibid. *Statistics of Railways in the United States.*	Ibid.

In the cases of 2 industries, 36, merchandising and service machines, and 80, services allied to transportation, no data were available. It was assumed that the coefficient for Industry 36 was the same as that of 34, industrial and household equipment, n.e.c., and that the coefficient for Industry 80 was the same as that for the subgroup, wholesale trade of Industry 74, trade.

C. CAPITAL COEFFICIENTS BY INDUSTRY OF ORIGIN

Capital coefficients by industry of origin were derived in a number of ways:

1. The stock of capital goods produced by Industries 68-69, construction, held by each industry was derived as follows:

Estimates of the ratio of the value of structures to that of the total capital stock are obtainable, in most industries, from a study of the information submitted to the Securities and Exchange Commission by listed firms in each industry, and from reports of the War Production Board. Multiplying the over-all capital coefficient by this ratio, the capital coefficient for construction in each industry was derived.

2. In some industries, the stock of furniture was also available from the Securities and Exchange Commission.

3. The stock of engines and turbines held by each manufacturing industry was computed indirectly in the following manner:

For manufacturing industries, installed horsepower of engines and turbines is obtainable from the *Census of Manufactures*, 1939. Estimates of the value per horsepower of each type of engine and turbine are available in the commodity flow tables prepared by the Bureau of Labor Statistics for use in preparing the 1939 input-output table. The value of the stock of engines and turbines in each industry was determined by

multiplying the total installed horsepower by the value per horsepower in each industry. The latter figure divided by the capacity of the industry gives the capital coefficient for engines and turbines for the particular industry.

4. Agriculture. An analysis of the capital stock of agriculture was obtainable from the Conference on Research in Income and Wealth, *The Agricultural Segment of the National Balance Sheet*, and Department of Agriculture, *Balance Sheet of Agriculture*, *Agricultural Statistics*, *Livestock Market News*, and statistical releases. From this, data on the proportion of the capital stock of agriculture, consisting of structures other than dwellings, livestock, and automobiles and trucks, could be computed. Multiplying the ratio of each of these items to the total stock by the capital coefficient for agriculture, we obtained the capital coefficients in this industry for stocks of fixed capital produced by Industries 68-69, 31, and 8, which accounted for over 80 per cent of the net assets of agriculture.

5. In the railroad industry, breakdowns accounting for over 90 per cent of the capital stock of the industry were secured from reports submitted by the railroads to the Interstate Commerce Commission.

6. In the communications industry, breakdowns for the total capital stock of the industry were secured from reports submitted to the Federal Communications Commission by firms in the industry.

7. Capital coefficients by industry of origin for the equipment of all other industries, apart from engines and turbines, could not be determined as directly as for the industries listed above. The commodity flow sheets of the Bureau of Labor Statistics were a basic source of information for the industries of origin of the equipment stocks of the remaining industries. These worksheets contain estimates of the flows of the products of each industry to every other industry in 1939, given by detailed commodity breakdowns. These flows were examined, and those items which were capital goods from the point of view of the purchasing industry were selected as a first approximation of the capital goods flows. The appropriate data from either the *Census of Manufactures*, 1939, or the *Census of Mineral Industries*, 1939, were also examined for each industry to determine the aggregate expenditures of each industry on construction and equipment. The worksheet data, the first approximations of the capital flows, were then adjusted to bring about agreement with the Census information on total capital goods purchases. The Census figures on construction were given preference over the worksheet estimate whenever the two sources did not agree, except in those instances in which the Census coverage is known to be incomplete. When Census estimates of equipment purchases in any industry were higher than the estimates of the worksheets, the latter figures were raised by

transferring capital goods produced from the 'undistributed' category. The undistributed capital goods were taken from either Industry 34, industrial and household equipment, n.e.c., or Industry 35, machine tools and metalworking machinery, whichever seemed most appropriate, as both these industries had large undistributed amounts. In a few industries, the Census equipment expenditures were smaller than those recorded on the worksheets. In these cases, purchases from Industries 34 or 35 were reduced to reconcile the capital flow figures with Census information. These two industries were selected for this reduction because the inputs originating in them were large. Had the deduction been made from any other industry, it would, in many cases, have reduced the inputs charged from that industry to negative magnitudes.

a. The first, and simplest, use of the revised capital-flow data was the separation of those industries whose capital goods came from only one industry in addition to those whose capital coefficients had been computed by the methods described above. In these cases, the remaining coefficient was determined by subtracting the already computed ratios from the industry's over-all capital coefficient.[5]

b. Apart from the industries mentioned above, the coefficients for capital purchased from industries other than construction and engines and turbines (in the case of manufacturing industries) have been inferred from the 1939 capital flows, capacities, depreciation rates, and over-all capital coefficients through indirect computations based on the following general relationships:

Let

Y_{ij} = the annual dollar expenditure by industry j on capital goods produced by industry i,

S_{ij} = dollar value in current prices of the stock of capital goods produced by industry i which are used by industry j,

d_{ij} = the annual rate of depreciation of capital goods produced in industry i which are used by industry j,

$\overline{X}_j$ = the capacity of industry j, expressed as value of capacity output at current prices,

[5] This was done in the following industries:

10 Flour and grist mill products
11 Canning and preserving
12 Bread and bakery products
13 Sugar refining
14 Starch and glucose products
16 Nonalcoholic beverages
17 Tobacco manufactures
18 Slaughtering and meat packing
23 Blast furnaces
28 Munitions
40 Smelting and refining of nonferrous metals
49 Coke and manufactured solid fuel
71 Coastwise and inland water transportation
72 Transoceanic transportation
76 Banking
77 Insurance
78-79 Business services
80 Services allied to transportation

R_j = the annual rate of change in the capacity of industry j, and,
B_j = the over-all capital coefficient of industry j.

Accordingly,

Replacement expenditures by industry j on commodity i	$= S_{ij}d_{ij}$
Expenditures for additions to its capital stock by industry j on commodity i	$= S_{ij}R_{ij}$
The capital coefficient of industry j with respect to commodity i, i.e. b_{ij}	$= \dfrac{S_{ij}}{\bar{X}_j}$
The over-all capital coefficient, B_j	$= \Sigma \dfrac{S_{ij}}{\bar{X}_j}$

In general,

$$Y_{ij} = S_{ij}d_{ij} + S_{ij}R_j = S_{ij}(d_{ij} + R_j) \tag{6, 1}$$

Since

$$S_{ij} = b_{ij}\bar{X}_j \tag{6, 2}$$

Equation (6, 1) can be rewritten as

$$Y_{ij} = b_{ij}\bar{X}_j(d_{ij} + R_j) \tag{6, 3}$$

Solving for b_{ij}

$$b_{ij} = \frac{Y_{ij}}{\bar{X}_j(d_{ij} + R_j)} \tag{6, 4}$$

Furthermore,

$$\Sigma b_{ij} = B_j \tag{6, 5}$$

Given Y_{ij}, d_{ij}, and $\bar{X}_j$, equations (6, 4) and (6, 5) can be solved for b_{ij} and R_j. The capital flows, Y_{ij}, were described above. The depreciation rates, d_{ij}, used in these computations are taken from the Bureau of Internal Revenue, Bulletin 'F,' *Income Tax Depreciation and Obsolescence, Estimated Useful Lives and Depreciation Rates,* Revised January 1942. The estimates of $\bar{X}_j$, the capacities, were derived in a number of ways; the methods and sources are noted below.

Computation

Since equations (6, 4) are quadratic, a successive approximation method was used for their solution. First, an arbitrary value of R_j was assumed in each case and the magnitudes of the corresponding b_{ij}'s computed by inserting it in equation (6, 4). Next Σb_{ij} was compared with B_j. If Σb_{ij}

is less than B_j, then the true R_j must be less than the assumed R_j. If Σb_{ij} is greater than B_j, then the true value of R_j must be higher than the assumed one. The initially assumed R_j was reduced or increased depending on whether Σb_{ij} was greater or less than B_j, until an R_j was found for which Σb_{ij} nearly equalled B_j. The correct values of the b_{ij}'s are those corresponding to the final value of R_j.

As more than one b_{ij} had to be determined by use of equation (6, 4), more than one rate of expansion, R_j, could be found to satisfy equation (6, 5). However, only one R_j yields b_{ij}'s which are all positive. As a negative b_{ij} is without economic meaning, the value of R_j which yielded positive b_{ij}'s has been accepted. The R_j's which yielded one or more negative b_{ij}'s were those in which one or more of the $(R_j + d_{ij})$'s were negative, which means that an industry's capacity is declining at a greater rate than its depreciation rate.

D. DERIVATION OF CAPACITY FIGURES

Estimates of the capacity of each industry were required in order to use the indirect method of computing capital coefficients by industry of origin, and also, in a number of cases, to determine the over-all capital coefficients when only the capital stock of the industry was known. The capacities were estimated in a number of ways: (1) the use of independent estimates; (2) the selection of a year or month of peak output reasonably close to 1939; (3) the determination of a capacity output by finding a year of maximum utilization of capital; and (4) by specific analysis of the industry involved.

1. For the following industries estimates of the 1939 capacity were directly available in publications of the government or business service agencies:

	INDUSTRY	SOURCES
11	Canning and preserving	Office of Price Administration, *Industrial Capacity in the United States*, p. 2.
13	Sugar refining	Ibid.
14	Starch and glucose products	Ibid.
19	Manufactured dairy products	Ibid.
23	Blast furnaces	Ibid.
24	Steel works and rolling mills	Department of Commerce, 1947 *Statistical Supplement to the Survey of Current Business*, p. 147.
25	Iron and steel foundry products	Ibid. 1940 *Supplement*, pp. 132-3.
28	Munitions	Office of Price Administration, op. cit.
33	Transportation equipment, n.e.c.	Ibid.
35	Machine tools and metalworking machinery	National Machine Tool Builders Association's Index of Capital Utilization.
39	Nonferrous metal mining: copper	*Mining and Metallurgy*, 1944, p. 429.

	INDUSTRY	SOURCES
40	Nonferrous smelting and refining: primary copper, zinc, primary aluminum, magnesium	Office of Price Administration, op. cit.; Surplus Property Administration, *Aluminum Plants and Facilities*, p. 21; ibid. *Magnesium Plants and Facilities*, p. 10.
42	Nonferrous metal manufactures and alloys	Office of Price Administration, op. cit.
46	Petroleum refining	Department of Commerce, 1947 *Statistical Supplement to the Survey of Current Business*, p. 164.
47	Anthracite coal	*Minerals Yearbook*, 1940, p. 821.
48	Bituminous coal	Ibid. 1945, p. 852.
49	Coke and manufactured solid fuel	Ibid. 1939, p. 885.
53	Chemicals: paint, animal and vegetable oils, rayon, industrial chemicals	Office of Price Administration, op. cit.
54	Lumber and timber products: sawmills	*Monthly Labor Review*, December 1942.
56	Wood pulp, paper, and paper products: basic products	*Barrell's Paper Annual*, 1945-6, pp. 21-2; 1946-7, pp. 16 f.
57	Printing and publishing	Department of Labor, *Economic Factors Bearing on Minimum Wages in the Printing and Publishing Industry*, p. 57.
59	Silk and rayon products	*Rayon Organon*, November 1939.
66	Rubber: tires	Office of Price Administration, op. cit.
80	Services allied to transportation	Department of Commerce, op. cit. p. 37.
86	Hotels	National Industrial Conference Board, *Economic Almanac*, 1945-6, p. 293.

2. In the following instances the year of highest output reasonably close to 1939 was taken as a measure of capacity, either because no other information was available, or because other estimates of capacity were considered inadequate.

	INDUSTRY	PERIOD OF HIGHEST OUTPUT	SOURCES
1-8	Agriculture	1947	Bureau of Agricultural Economics, Department of Agriculture, *Agricultural Statistics*, 1948, Table 672, p. 590.
9	Fishing	1941	Fish and Wildlife Service, *Fishery Statistics of the United States.*
10	Flour and grist mill products	Feb. 1947	Department of Commerce, 1947 *Statistical Supplement to the Survey of Current Business*, p. 131.
17	Tobacco manufactures:		*Statistical Abstract of the United States*, 1947, Table 931, p. 845.
	tobacco	1938	
	cigars	1942	
	snuff	1943	
18	Slaughtering and meat packing	1940	American Meat Institute, *Meat*, p. 11.
20	Edible fats and oils, n.e.c.	1937	Department of Commerce, op. cit. p. 115.

	INDUSTRY	PERIOD OF HIGHEST OUTPUT	SOURCES
22	Iron mining	1942	*Minerals Yearbook*, Reviews of 1939 and 1942.
27	Firearms	1937	*Census of Manufactures*, 1931-9.
29	Agricultural machinery	1941	Bureau of Internal Revenue, *Statistics of Income*.
31	Automobiles	1937	*Automotive Industries Journal*, Jan. 1, 1940.
39	Nonferrous metal mining:		*Minerals Yearbook*, 1941, pp. 51, 629.
	gold and silver	1940	
	molybdenum	1939	
	zinc	1940	
	manganese	1939	
40	Nonferrous smelting and refining:		Ibid. 1940; ibid. 1941, p. 669.
	secondary copper	1939	
	secondary aluminum	1937	
	lead	1941	
	leaded zinc oxide	1941	
	nickel	1941	
51	Communications:		Federal Communications Commission, *Statistics of the Communications Industry*, 1942, 1944; ibid. *Public Service Responsibility of Broadcast Licensees*, Tables 1, 5, and 11.
	wire telegraph	1944	
	other	1939	
53	Chemicals: drugs and plastics	1939	Bureau of Internal Revenue, op. cit.
55	Furniture and other manufactures of wood	1941	Department of Commerce, op. cit. p. 10.
58	Cotton yarn and cloth	1947	Bureau of the Census, *Cotton Production and Distribution*.
60	Woolen and worsted manufactures	Feb. 1948	Department of Commerce, *Facts for Industry*, 'Wool Manufactures, Machinery Activity Reports.'
64	Leather shoes	1941	*Survey of Current Business*.
70	Transportation, n.e.c.:		Interstate Commerce Commission, *Statistics of Motor Carriers*.
	pipe lines	1943	
	motor carriers	1943	
	electric railways		
71	Coastwise and inland water transportation	1940	Ibid. *Selected Financial and Operating Statistics from Annual Reports of Carriers by Water*.
73	Railroads:		Ibid. *Statistics of Railways*.
	passengers per car	1942	
	passenger miles per day	1944	
	ton miles per day	1940	
	freight-car days	1944	
76	Service industries: advertising	1937	Department of Commerce, 1947 *Statistical Supplement to the Survey of Current Business*, p. 35.

3. The third method involved the examination of the 'capital stock-sales ratios' (in 1939 prices) of an industry over a period of years (see above, p. 193). The year of highest utilization of capital was assumed

to be a capacity year. The 1939 domestic production of the industry was multiplied by the relation of the capital-sales ratio in the capacity year thus defined to the corresponding ratio in 1939 to arrive at 1939 capacity. Data on gross sales and capital stock were taken from the source book of *Statistics of Income* for all the industries except for Industry 12, in which the data are from *Moody's Manual of Industrials* and the Securities and Exchange Commission's *Survey of American Listed Corporations*.

INDUSTRY

12 Bread and bakery products
15 Alcoholic beverages
16 Nonalcoholic beverages
26 Shipbuilding
32 Aircraft
34 Industrial and household equipment, n.e.c.
37 Electrical equipment, n.e.c.:
  radio
  other
38 Iron and steel, n.e.c.
43 Nonmetallic mining
44 Nonmetallic mineral manufactures
53 Chemicals:
  soap
  fertilizers
  hardwood distillation
  miscellaneous
54 Lumber and timber products: planing mills
56 Wood pulp, paper, and paper products: converted products
61 Clothing:
  knit goods
  hats
  millinery
  women's clothing
  other
63 Leather
65 Leather products, n.e.c.
66 Rubber: other than tires
67 Industries, n.e.c.
  ice
  jewelry
  motion picture production
  fur
  clocks
74 Trade
76 Service industries: banking
77 Insurance
87 Laundry, etc.
88 Personal services
89, 90, 91 Motion pictures, theaters, amusement places
81 Auto repair and services

Industries 41, aluminum products; 45, petroleum and natural gas; and the cigarette subgroup of industry 17, tobacco manufactures, were expanding in 1939 and therefore 1939 was taken as capacity.

In the case of Industry 30, engines and turbines, although production rose greatly during the war due to increased shifts, 1939 output had to be taken as capacity since no reliable information on shifts could be obtained to use in deflating wartime production.

In the case of two industries, it was considered best, in the absence of other data, to apply the capacity-utilization rates of similar industries.

INDUSTRY	CAPACITY-UTILIZATION ESTIMATE DERIVED FROM INDUSTRY
21 Other food products	12 Bread and bakery products, and 14 Starch and glucose products
36 Merchandising and service machines	34 Industrial and household equipment, n.e.c.

4. Other methods:

50. Manufactured gas. *Gas Facts*, 1945, 1946, p. 20, indicates 1945 daily capacity as 1,838 million cubic feet As this industry is subject to seasonal variation this was corrected by taking monthly 1939 sales figures (*Survey of Current Business,* February 1940, p. 80), finding what per cent sales were for each month of the highest month (February) and averaging them—81.8 per cent. On this basis 81.8 per cent (301.64 days) of rated annual manufacturing capacity is assumed to be full capacity; thus 1945 annual capacity was 554,414 million cubic feet. Miles of main in 1945 were 88,500. Thus, 554,414/88,500 = 6.265 million cubic feet per mile of main. This relationship, applied to the number of miles of mains in 1939 (85,400), was used to obtain 1939 capacity (535,458 mcf.) which was 150 per cent of actual production.

52. Electric public utilities. The capacity of the industry was 176 billion kilowatt hours or 164.3 per cent of distributed output (113 billion kwh). Capacity was estimated as follows: kilowatts of installed capacity, 31 December 1939[6] (38,863,000) × estimated ratio of net assured to installed capacity[7] (86 per cent) × estimated load factor at capacity[8] (60 per cent) × 8760 hours in a year = 176 billions of kilowatt hours of estimated output at capacity.

68-69. Building construction and construction other than building. The 1939 capital investment, $1,450,000,000 (Bureau of Labor Statistics, *Bulletin* #779, 1944), was divided by the capital coefficient, .0755, to obtain capacity. (The source above gave separate capital coefficients for building and non-building construction, derived from engineering estimates. These were weighted by output and combined to arrive at the above coefficient.)

39. Nonferrous metal mining. Lead—*Census of Mineral Industries.* 1939 output = 240 days (actually worked) × 310 days (assumed to have been number of days the industry would have worked at capacity).

70. Transportation, n.e.c. Airlines As this was a fast-growing industry it was assumed to be operating at capacity in 1939, except for passenger load. Load factor was boosted from 58.6 per cent to 65 per cent (Report of Federal Coordinator of Transport, Vol. 1, p. 340).

72. Transoceanic transportation. 1939 earnings were multiplied by 1.21,

[6] Federal Power Commission, reported in Edison Electric Institute, *Statistical Bulletin,* 1947, p. 19.

[7] The Federal Power Commission reports net assured capacity only for Class I Systems. As of 31 December 1947 this ratio was 89 per cent. From 1940 to 1947 reported required reserves of Class I Systems fell from 15.4 per cent to 9.3 per cent of dependable capacity. Considering the decline and to allow for actual change in physical plant, 86 per cent instead of 89 per cent is used in 1939.

[8] Wartime peak was 64.2 per cent (1945). In 1947 during high level of peacetime output the factor was 58.9 per cent. As a round figure for peacetime 60 per cent seems reasonable.

the relation between net tonnage cleared and net tonnage cleared with cargo in 1939 (*Statistical Abstract of the United States*, 1941, pp. 513-14).

E. DERIVATION OF REPLACEMENT REQUIREMENTS

The rate of input of capital good i into industry j necessary to maintain the capacity of the industry is here designated M_{ij}. It is assumed to be equal to the stock of fixed capital goods, S_{ij}, multiplied by the annual depreciation rate, d_{ij}. The stock is obtained by multiplying capacity, $\bar{X}_j$, by the capital coefficient, b_{ij}.

Thus,

$$M_{ij} = b_{ij}X_j d_{ij}$$

In making the replacement estimates in the accompanying tables, capital coefficients based on gross rather than net investment figures (see p. 191 above) have been used. This has been done because the depreciation rates refer to the original capital costs, i.e. gross capital stock.

As already mentioned, the source of the annual depreciation rates is the Bureau of Internal Revenue, Bulletin 'F,' *Income Tax Depreciation and Obsolescence, Estimated Useful Lives and Depreciation Rates,* Revised January 1942. Where more than one type of equipment was purchased by an industry from a single capital-producing industry, an average depreciation rate weighted by the value of each type of equipment purchased from the industry has been calculated.

F. INVENTORY COEFFICIENTS [9]

The inventory coefficient is an estimate of the total stocks of an input which must be held in the economy per unit of output. The explanation of the proportions in which the total inventory holdings are divided between producers and consumers of the particular commodity does not interest us here for its own sake, and in so far as, in the first approximation at least, this apportioning can be considered as being determined independently of the determination of the total inventory requirements, it can be neglected.

The inventory stocks held which are considered in relation to an industry's output to determine the coefficient are the stocks of raw materials and supplies held by that industry and the stocks of finished goods of other industries which are held for sale in the industry.

It is important to note, in considering the inventory figures presented, that the use of stocks 'held for' an industry is a signal departure from the usual way of describing the inventories of an industry. Most information on inventories considers the finished goods held by an industry as part of that industry's inventories. For the purposes of input-output analysis, finished-goods inventories are associated with the industry for which

[9] This section is partly based on the work of Mathilda Holzman and Richard Rosenthal.

they are being held, that is, the consuming industry, rather than the producing industry. Thus the inventory coefficients of an industry are based on stock figures which combine for each kind of commodity the stocks of finished goods held *for* that industry and the stocks of supplies, raw materials, and goods-in-process held *by* the industry. This procedure has been used here in the belief that the quantity of normal inventories held is a function of the output and, therefore, input requirements of the industry which will utilize the inventories.

For most commodities we have had to use an estimate of the stocks held at the end of 1939 for and by the consuming industry divided by the industry's 1939 output as the inventory coefficient. In a few industries we were able to secure time series of stocks held either by or for or both. In some of these last, smaller inventory-output ratios than the 1939 ratio existed in other years. Where this was true, the lowest ratio of a year reasonably close to 1939 was used as the inventory coefficient. This was done because the inventory coefficients are estimates of the stocks necessary to produce a given output. At some times, particularly where the supply of a commodity is inelastic, stocks may be higher than technically required in the production of the output. We are concerned with requirements, rather than with predicting the actual amount of the inventories. In much the same sense that an industry may be operating under capacity because of excess fixed capital, its inventories may permit outputs higher than the actual production. The source of the time series of stocks was the *Survey of Current Business*.[10]

We used data from the Survey in the following commodities and industries:

Cotton held by cotton yarn and cloth
Rayon yarn and filament held for silk and rayon products
Tobacco held by and for tobacco manufactures
Raw sugar held by sugar refining
Bituminous coal held by steel works and rolling mills
Bituminous coal held by coke and manufactured solid fuel
Crude petroleum held by and for petroleum refining
Wool held by woolen and worsted manufactures
Crude rubber held by rubber products
Bituminous coal held by electric public utilities

The other major source of inventory data used in this study, the *Census of Manufactures*, 1939, supplies two figures for each industry, one showing the aggregate dollar value of the industry's holding of its raw materials and goods-in-process, and the other the inventory of its finished prod-

[10] Department of Commerce, *Supplement to the Survey of Current Business*, 1940, pp. 88-157; ibid. 1947 *Statistical Supplement*, pp. 110-77.

ucts. These data were used as control totals. Where it was possible to use *Survey of Current Business* information, the Survey figures for the end of December 1939 were subtracted from the Census figures for the end of December. Lacking specific information on the components of the remaining stocks, it was necessary to adopt a number of special assumptions, or rather conventions, for estimating them. In the case of an industry's holding of raw materials and goods-in-process it is assumed that inventories of the various kinds of storable goods are proportional to their inputs in that particular industry. In the case of finished goods that the industry is holding for its various consumers, the assumption is made that it is holding inventories in proportion to the distribution of that industry's outputs to the corresponding consuming industries.[11] In so far as the input from one industry to another bears a technical relation to the latter's output,[12] it follows that so does the stock of inventories held for each input. This assumption is not made for final demand industries, households, exports, and government, as we do not assume any systematic relation between stocks of goods held for these industries and their outputs.

Inventories of finished capital equipment held by manufacturers and trade are not regarded as bearing any direct and simple relation to output of the equipment-using industries. They exist to supply replacements and additions to the capital stock of the industry for which they are destined. No relationship of these inventories with output is assumed, and no input-output ratios were computed for them.[13]

Stocks held by Industry 74, trade, for other industries, including households, were analyzed by industry of origin. The sources of data were the *Census of Wholesale Trade*, 1939, and *Census of Retail Trade*, 1939. The Census of Business Wholesale and Retail Trade classifications, as used in Table I of the *Census of Wholesale Trade* and Table 2A of the *Census of Retail Trade*, differ from the 96-industry input-output classification in that the data relative to the 'specialty lines dealers' are less aggregative than the corresponding input-output industries while the 'general lines dealers' and certain 'specialty dealers' are more aggregative. The inventories of the specialty lines could thus, on the whole, simply be aggregated to give input-output industry inventories. The inventory figures given for the general lines, however, have had to be broken down in order to be assigned to input-output industries. This breakdown is made by using the Commodity Sales, by Kinds of Business Tables (Column B of Table 8 in the *Census of Wholesale Trade*, and Column B of Table 18 in the *Census*

[11] Finished goods were raised to purchasers' value to be comparable to raw material inventories.

[12] This is a basic proposition of input-output analysis.

[13] Inventories of durable goods amounted to 4.4 per cent of the total purchases of fixed capital in 1939.

of Retail Trade), which give the percentage composition of dealers' sales by type of commodity. This breakdown is assumed to apply proportionately to inventories and is thereby used to distribute the inventory aggregates among the input-output industries. The groups distributed in this way are groceries (general line), fruits and vegetables (fresh), and shoes and other footwear, from Wholesale Trade; and general stores (with food), general merchandise stores, delicatessen stores, other food stores, drinking places, drug stores, and eating places, from Retail Trade.

No usable information is available on the holdings by agriculture of materials and supplies. Estimates of holdings of the agricultural industries of agricultural products have been arrived at by multiplying the physical stock figures furnished in *Agricultural Statistics*, 1939, and the *Survey of Current Business, Supplement*, 1939, by average prices obtained from the basic worksheets used in the compilation of the 1939 input-output table or, when these prices were not available, by average prices from the stock sources above. The stocks thus obtained are aggregated into the 96-industry classification. In the case of field crops, total stocks in the economy of the particular crops have been used, from which have been subtracted all known holdings of these crops as given in the *Census of Manufactures* and the *Census of Business*. The residual is distributed as farm holdings of its finished goods.

Since the outputs of utilities and transportation are services, the only inventory holdings by these industries are of raw materials and supplies. These inventories are obtained from *Electrical Statistics*, Federal Power Commission, 1939; *Statistics of Railways*, Interstate Commerce Commission, 1939; *Statistics of Oil Pipe Line Corporations*, Interstate Commerce Commission, 1939; *Statistics of Water Carriers*, Interstate Commerce Commission, 1939; and *Annual Airline Statistics*, Civil Aeronautics Board, 1939.

Stocks held by mining and service industries were available in data supplied by the Bureau of Internal Revenue.[14] The stocks reported for each industry were adjusted to approximate complete coverage by multiplying them by the ratio of the output of the industry in 1939 to the gross sales reported by the Bureau of Internal Revenue. There were no data indicating the proportions of raw materials, supplies, and finished goods in these stocks. As a first approximation, it was assumed that all stocks held by mining were finished goods, and all stocks held by service industries were supplies.

[14] The substantial inventories of materials and supplies held by the communications industry (telephone subgroup) have been omitted, as these are principally for construction and maintenance purposes. See Federal Communications Commission, *Statistics of the Communications Industry in the United States*, Year Ending 31 December 1939, p. 42, lines 28-9.

Transportation and trade margins were taken from Harvard Economic Research Project worksheets forming the basis of the 96-industry input-output table for 1939, except for manufacturer to wholesaler transportation charges on inventories held by wholesale trade, which are from P. W. Stewart and J. F. Dewhurst, *Docs Distribution Cost Too Much?*, Table 10, p. 62.

III. Internal Checks and Comparisons

A. AVERAGE AND INCREMENTAL COEFFICIENTS

Table 3 compares over-all capital coefficients derived from accounting data and representing the total capital stock in the industry with incremental coefficients derived from data on newly built plants. As is to be expected, each method yields different results, since the accounting data refer to average (old and new) plants, while the incremental coefficients are related only to modern plants. In food, cotton yarn and cloth, and leather, the coefficient derived from undepreciated accounting data is similar to the incremental coefficient. The other manufacturing industries show the incremental coefficients to be higher than the undepreciated average coefficient. To the extent to which this is true of other industries, the average capital coefficients would underestimate the amount of fixed capital required to increase capacity by any given amount. It must, however, be noted that most of the incremental coefficients are based on data relating to wartime expansion of facilities, and that there is a strong possibility that the deflators used to reduce the figures to a 1939 price level inadequately represent the rise in costs during the war.

The fact that the average coefficients are based on historical costs of capital makes it exceedingly difficult to use the average coefficients as sub-

TABLE 3

COMPARISON OF INCREMENTAL AND AVERAGE OVER-ALL CAPITAL COEFFICIENTS

INDUSTRY	INCREMENTAL	AVERAGE	
		Undepreciated	Depreciated
	(1)	(2)	(3)
(1) Leather	.20	.20	.09
(2) Edible fats and oils, n.e.c., and Other food products	.23	.25	.13
(3) Smelting and refining of nonferrous metals, and Aluminum products	.60	.41	.15
(4) Silk and rayon products	.70	.48	.29
(5) Cotton yarn and cloth	.82	.72	.33
(6) Blast furnaces, Steel works and rolling mills, Iron and steel foundry products, Firearms	1.61	.74	.39
(7) Construction	.08	.12	.06
(8) Nonferrous metal mining	1.07	2.32	
(9) Transoceanic transportation	2.58	1.02	.52

stitutes for incremental coefficients. While indices of changes in the cost of capital goods have been constructed, they cannot be applied for the purpose of deflating the costs of capital accumulated over long periods of time, as no inventory listing the years of purchase is available. In addition to this price problem, changes in technology, in the sense of changes in the incremental coefficients over time, have probably taken place to some extent in most industries.

B. CAPITAL COEFFICIENTS BY INDUSTRIES OF ORIGIN

The capital coefficients by industries of origin were derived in two ways: first, from accounting information on the structure of the existing capital stock; and second, from capital expenditures in 1939 on the basis of estimated capacity and depreciation rates and inferred rates of industrial growth. Since the accounting estimates were deemed to be the more reliable as being the result of an actual observation rather than a calculation from the flows of a single year, they were used wherever possible.

In 18 industries the indirect, flow-based method of estimating capital coefficients by industry of origin was not used at all. Either the whole capital-stock structure was computed from accounting data, or all but one coefficient was derived from accounting data, with the coefficient for the remaining industry of origin the residuum. In 19 other industries more than 95 per cent, and in 16 other industries more than 90 per cent of the capital stock was allocated to industries derived from accounting data plus one indirect coefficient. As this additional coefficient (usually from either industrial and household equipment, n.e.c., or machine tools and metalworking machinery) must have accounted for the bulk of the capital stock other than that coming from the accounting-based industries of origin, the percentage error in its coefficient is probably small. On the other hand, the remaining small coefficients may contain large percentage errors.

In 53 industries, then, the role of the indirect method can be considered as of little or no importance. In 20 industries,[15] however, more substantial portions of the capital stock were allocated to industries of origin with the use of the indirect method.

[15] Fishing
Alcoholic beverages
Edible fats and oils, n.e.c.
Steel works and rolling mills
Shipbuilding
Merchandising and service machines
Electrical equipment, n.e.c.
Aluminum products
Industries, n.e.c.
Trade
Agricultural machinery
Nonferrous metal manufactures and alloys
Coke and manufactured solid fuel
Electric public utilities
Woolen and worsted manufactures
Clothing
Other textile products
Leather shoes
Construction
Service industries (part)

The accounting estimates were considered more reliable than the indirect method, and were used wherever possible. By the same token they may be used as the standard by which to judge the validity of estimates derived from the other method. As described above, page 196, the capital coefficient in each industry representing the stocks of construction products was derived from accounting data and, since it usually constituted a substantial portion of the total capital stock, this exercised a marked influence on the distribution of capital coefficients by industry of origin. In Table 4, the coefficient for construction is shown for industries which had three industries of origin, as derived from accounting data, and as derived via the indirect method of inferring coefficients from flows of capital goods during 1939. This table indicates that inference from capital-flow data can and does in most cases lead to very different results when substituted for accounting data of construction stocks. Of the sample of 12 industries, the different methods yield similar results in only the first four. In the remaining industries shown, the flow-based method always yields a higher figure for construction, consequently lower figures for the other industries of origin, than does the derivation of construction from accounting information.

Actually, it is to be expected that the estimates based on construction stocks will differ appreciably from those based on flows. Because of the great durability and indivisibility of construction capital, capital supplied

TABLE 4

Comparison of Allocation of Capital Coefficient to Construction by Two Different Methods, for Industries with Three Industries of Origin

Industry	Coefficient for Construction: Derived from Accounting Information	Coefficient for Construction: Derived from Flow Data
	(1)	(2)
(1) Bread and bakery products	.107	.108
(2) Sugar refining	.046	.080
(3) Nonalcoholic beverages	.133	.133
(4) Tobacco manufactures	.030	.025
(5) Iron mining	.061	.725
(6) Blast furnaces	.414	1.390
(7) Munitions	1.020	1.918
(8) Nonferrous metal mining	.171	.541
(9) Smelting and refining of nonferrous metals	.270	.539
(10) Anthracite coal	.206	.707
(11) Bituminous coal	.115	.325
(12) Coke and manufactured solid fuel	.432	1.103

by this industry is, of all types of capital, least likely to behave in accordance with the scheme described by equations (6, 4) and (6, 5). (This is one reason why a special effort was made to get stock information for construction.)

C. RATE OF GROWTH IN CAPACITY

A further check on the reliability of the capital coefficients derived from the capital flows can be made by comparing a by-product of the method, the rate of growth in capacity, with independent estimates of increases in capacity. In determining the capital coefficients by industry of origin, a value representing the rate of growth for each industry was arrived at by successive approximations.[16]

Table 5 presents these computed rates for those industries where data on changes in capacity over the relevant time period were available. In those industries where the rate of growth was large, aircraft, machine

TABLE 5

Comparison of Rates of Growth, 1939

Industry	Percentage Increase In Capacity, 1939	
	By-product of Capital Coefficient Computation	Independent Estimate[1]
	(1)	(2)
(1) Aircraft	34.5	38.7
(2) Machine tools	16.0	14.9
(3) Furniture and other manufactures of wood	8.4	10.8
(4) Petroleum refining	3.1	6.5
(5) Flour and grist mill products	− 0.2	− 5.3
(6) Woolen and worsted manufactures	− 0.5	− 3.6
(7) Cotton yarn and cloth	− 1.1	− 4.1
(8) Bituminous coal	− 2.0	2.8
(9) Electric public utilities	− 2.4	3.7
(10) Iron and steel foundry products	− 2.7	− 0.1
(11) Anthracite coal	− 3.3	− 1.2
(12) Steel works and rolling mills	− 3.3	− 3.4

[1] The independent estimates of growth were from December 1938 to December 1939, except for the coal industries which were from the middle of 1938 to the middle of 1940, and the aircraft industry which was from January 1, 1939, to January 1, 1940.

Sources: *Survey of Current Business Supplement*, 1940; *Minerals Yearbook; Historical Statistics of the United States; Aviation Facts and Figures;* Releases of the National Association of Machine Tool Builders; Census Bulletin, *Machinery Activity Reports in Wool Manufactures*, 1939, 1940; ibid. *Cotton Production and Distribution*, 1939, 1940.

[16] See above, pp. 198-200.

tools, and furniture, the similarity in the orders of magnitude indicates that the error in the capital coefficients must have been small. Examination of the effect of changes in the capital coefficients on the by-product rates of growth indicate that a 1.0 per cent increase in the largest capital coefficient would bring about a decrease in the by-product rate of growth of 1.0 per cent in aircraft, 1.8 per cent in machine tools, and 1.6 per cent in furniture. In most cases, however, the rates of growth were very small and, therefore, similar orders of magnitude between the by-product rate of growth and the independent estimates are consistent with large percentage errors in the capital coefficients.

The checks made above on the capital coefficients by industry of origin should lead to cautious use of flow-based coefficients. In general where the bulk of the capital stock allocated in this manner went to a single industry of origin, as it did in most industries, this coefficient may be regarded as reasonable. The residual coefficients, fortunately of minor quantitative importance, must be regarded as unreliable, although estimated with the consideration of relevant economic factors. The fact that the computed rates of growth seem to be, at least in the small sample it was possible to check, of the same order of magnitude as independent estimates indicates that the indirect method of deriving coefficients very well may have produced reasonable numbers.

D. INTERNATIONAL COMPARISONS OF CAPITAL-OUTPUT RELATIONS

Previous studies suggest a considerable similarity of capital structure in a number of countries. Many countries have collected data on fixed capital on an individual industry basis. Studies of the relations between capital and production made by Lord Stamp, Paul Douglas, and Colin Clark indicate that the orders of magnitude are quite similar in each country investigated.[17] Clark, using Lord Stamp's study of fixed capital

[17] Clark, Colin, *Conditions of Economic Progress*, 1940, p 389. Computations from Clark's estimates on this page yield the following relationships between capital and real income for 1913:

COUNTRY	CAPITAL-INCOME	COUNTRY	CAPITAL-INCOME
Argentina	5.85	Italy	4.36
Sweden	5.65	United States	4.33
Australia	5.53	Canada	4.32
Hungary	5.05	Britain	3.72
France	4.82	Japan	3.57
Belgium	4.66	Spain	3.52
Germany	4.45	Austria	3.50

Handsaker, M. L., and Douglas, P H, 'The Theory of Marginal Productivity Tested by Data for Manufacturing in Victoria,' *Quarterly Journal of Economics*, November 1937 and February 1938.

TABLE 6

Equipment-Output Relationships, United States, Australia, New Zealand, Canada, Mexico, South Africa, and Peru

Industry[1]	Depreciated Value of Equipment / Value of Output						
	United States 1939	Australia 1939-40	New Zealand 1939-40	Canada 1939	Mexico 1940	South Africa 1938-39	Peru 1939
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1) Food processing and tobacco							
(a) Fishing	.57	.43	–	.65	–	–	–
(b) Flour and grist mill products	.07	.13	.12	–	.15	–	–
(c) Canning and preserving	.12	.11	.08	–	.14	–	–
(d) Bread and bakery products	.14	.18	.19	–	.06	–	–
(e) Sugar refining	.24	.21	–	.30	.47	–	–
(f) Alcoholic beverages	.13	.32	.16	.30	.20	–	.45
(g) Nonalcoholic beverages	.13	.20	.34	–	.20	–	–
(h) Tobacco manufactures	.03	.09	.09	–	.10	–	–
(i) Manufactured dairy products	.10	.10	.07	.20	.06	–	–
(j) Slaughtering and meat packing	.06	.07	.07	.08	.03	–	–
(2) Machinery							
(a) Agricultural machinery	.20	.27	–	.39	–	–	–
(b) Automobiles	.16	.13	.14	–	.04	–	–
(c) Transportation equipment, n.e.c.	.58	.41	–	–	–	–	–
(d) Machinery, n.e.c.[2]	.22	.22	–	.27	.20	–	–
(e) Electrical equipment, n.e.c.	.09	.12	–	–	–	–	–
(3) Iron and steel smelting and refining[3]	.35	.29	–	.80	.33	–	–
(4) Rubber products	.16	.16	–	.34	.11	–	–
(5) Nonmetallic mineral manufactures	.43	.38	.45	.64	.26	.37	–
(6) Chemicals	.21	.23	.22	.22	.08	.22	–
(7) Lumber and timber products	.32	.18	.25	.28	–	–	–
(8) Wood pulp, paper, and paper products	.26	.38	–	–	.23	–	–
(9) Printing and publishing	.26	.22	.33	–	–	.32	–
(10) Textiles and leather							
(a) Cotton yarn and cloth	.31	.25	–	.36	.24	–	.37
(b) Woolen and worsted manufactures	.32	.22	.36	.32	.17	–	.26
(c) Silk and rayon products	.22	.35	–	–	.11	–	.26
(d) Clothing	.02	.08	.09	.03	.10	–	–
(e) Leather	.15	.09	.11	.08	–	–	–
(f) Leather shoes	.06	.10	.09	–	.06	–	.07
(11) Shipbuilding	.16	–	–	–	–	.04	–
(12) Construction	.02	–	.02	–	–	.03	–

[1] The industrial classifications used are those of the Leontief 96-industry classification, except in cases noted.

[2] Consolidates Industries 30 (engines and turbines), 34 (industrial and household equipment, n.e.c.), 35 (machine tools), and 36 (merchandising and service machines).

[3] Consolidates Industries 23 (blast furnaces) and 24 (steel works and rolling mills).

TABLE 6 (Continued)

Equipment-Output Relationships, United States, Australia, New Zealand, Canada, Mexico, South Africa, and Peru

Industry	Depreciated Value of Equipment / Value of Output						
	United States 1939	Australia 1939-40	New Zealand 1939-40	Canada 1939	Mexico 1940	South Africa 1938-39	Peru 1939
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(13) Automotive repair and services	.23	–	–	–	.19	–	–
(14) Fuel and power							
(a) Coke and manufactured solid fuel	1.17	1.22	–	1.69	.18	–	–
(b) Manufactured gas	2.24	1.48	2.20		–	–	–
(c) Electric public utilities	2.99	2.11	2.31	–	3.81	–	–
(d) Petroleum refining	.12	–	–	.14	–	–	–
(15) Steam railroad transportation[4]	5.79	6.87	6.28	7.38	–	–	–

[4]For this industry the ratio is original capital cost/value of output.

Sources: Australia: Commonwealth Bureau of Census and Statistics, *Production* 1939-40, Bulletin #34. Part I—Secondary Industry, ibid. *Manufacturing Industries* 1945-6; ibid. *Official Yearbook of the Commonwealth of Australia*, 1941.

New Zealand: New Zealand Census and Statistics Department, *Statistical Report on the Factory and Building Production of the Dominion for the Year*, 1939-40; ibid. *The New Zealand Official Yearbook*, 1943.

Canada: Reports of the Dominion Bureau of Statistics; *The Canada Yearbook*, 1942; Dominion Bureau of Statistics, *Prices and Price Indexes*, 1913-39.

Mexico: *Tercer Censo Industrial de los Estados Unidos Mexicanos*, 1940.

South Africa: Office of Census and Statistics, *Census of Industrial Establishments* 1936-7.

Peru: *Extracto Estadistico del Peru*, 1943.

Canadian and Peruvian data were found for land and buildings, plant and machinery, and no separate figures were given. To make them comparable, the Australian ratio of plant and machinery to total fixed capital was applied.

stocks in 1913, compares these estimates with real incomes in the same year for 14 countries. The capital-income ratios ranged from 3.50 to 5.85; that for the United States was 4.33.

Census information on fixed capital stocks of American industries is not available for 1939, but our estimates may be compared with the figures given for other countries where such information exists. Capital-output relationships have been computed for 6 countries, as shown in Table 6. While recognizing that possible variations in capacity utilization, technology, price structure, and product-mix may weaken the validity of the comparisons, it is still of some significance that in most industries the orders of magnitude of these capital output ratios are similar from country

to country.[18] The fact that our estimates compare favorably with those of the other countries leads to the conclusion that the over-all fixed capital coefficients estimated for the United States may very well be reasonable.

E. REPLACEMENT REQUIREMENTS

The replacement requirements were computed through a multiplication of the depreciation rates by the corresponding capital stock estimates. Since the latter in their turn are the product of our capital coefficients and capacity estimates, the replacement requirements are in a sense the over-all result of all our estimates and calculations. Consequently, barring the possibility of errors in estimates of depreciation and stock cancelling-out, the reasonableness of our replacement requirements would tend to validate our other estimates.

The sum total of all capital replacements thus obtained amounts to $8.2 billion. It may be compared with that included in the Department of Commerce national product estimates under the heading of capital-consumption allowances.[19] This figure is $8.1 billion and composed of $7.1 billion depreciation charges, $0.2 billion accidental damages to fixed capital, and $0.8 billion capital outlays charged to current expense. Since the same methods are used in both estimates—depreciation rates applied to capital stock it is to be expected that the results would agree, especially as the same or similar data were undoubtedly used in the calculations incident to each. However, even if the over-all stock data and the depreciation rates were the same, a different distribution of our capital coefficients and stock among industries of origin could easily have resulted in a quite different over-all estimate of capital requirements. Consequently the fact that our results are similar to those of the Department of Commerce suggests that our capital coefficient allocations by industry of origin are reasonable.

F. COMPARISON OF FIXED CAPITAL STOCK ESTIMATES

The fixed capital stock estimates may be compared with the fixed capital stock components of national wealth estimates. Table 7 compares

[18] In one industry, coke, where the Mexican technology is known to be different, using the beehive method exclusively, the ratio of that country's coke industry is markedly different from the ratios of the other nations studied. In two industries it was possible to compare the replacement cost-capacity ratios of India and the United States. In iron and steel this ratio is, for the United States, 1.10 and for India, 1.00. In cotton mills it is .51 for the United States and .52 for India. The India data are from M. V. Divetia and H. M. Trivedi, *Industrial Capital in India*, 1938-39.

[19] National Income *Supplement to the Survey of Current Business*, July 1947, Table 4, p. 20.

TABLE 7

Comparison of Various Nongovernmental Depreciated Fixed Capital Stock Estimates, 1939 (in billions of 1939 dollars)

	Kuznets	National Industrial Conference Board	Goldsmith	Harvard Economic Research Project
	(1)	(2)	(3)	(4)
(1) Total	156.7		168.2	159.3
(2) construction			128.0	111.5
(3) equipment			40.2	47.8
(4) Agriculture	11.0		18.3[1]	10.0
(5) construction			9.1[1]	3.5
(6) equipment and livestock		6.6	9.2	6.5
(7) Mining, manufacturing, public utilities, and other industrial	107.0		76.7	77.5
(8) Mining and manufacturing	24.7			24.6
(9) manufacturing		22.0[2]		22.1
(10) equipment		11.3		
(11) Public utilities	55.2	46.7[3]		40.9
(12) Residential	38.6		73.2[4]	70.6

[1]Includes housing.

[2]Machinery and stocks of goods.

[3]Includes canals and waterworks.

[4]Nonfarm only, including land.

Sources: Kuznets, Simon, *National Product Since* 1869, Table IV-12-13, converted to 1939 prices on basis of explicit and implicit price indices from pp. 36, 40, 41, and 50, National Industrial Conference Board. *The Economic Almanac for* 1945-6, p. 57; National Industrial Conference Board's total manufacturing estimate presented by M. Gainsbrugh and L. Krasno to Conference on Research in Income and Wealth, January 1948; unpublished estimates by R. W. Goldsmith.

our estimates with those made by Kuznets, Goldsmith, and the National Industrial Conference Board. Such a comparison is not necessarily a check of our results, for no one claims a high degree of accuracy for any of the wealth estimates that have been made.

In order to make the various estimates comparable with ours, certain adjustments had to be made. First, capital stocks of institutions, government, and consumers were deducted. Second, Kuznets' data, originally presented in 1929 prices, were deflated to 1939 prices. Third, as other estimates of wealth were estimates of depreciated capital stock, estimates of depreciated capital stock were made for those 16 industries whose capital stock was estimated in terms of replacement costs.[20] This last adjustment reduced our estimate by about $11 billion.

[20] Source, *Statistics of Income*, 1939.

The estimates of the total capital stock by Kuznets and Goldsmith agree fairly well with ours. Kuznets' estimate differs by less than 2 per cent; Goldsmith's by less than 6 per cent. There is less apparent agreement, however, in the component sectors of the economy. Our figures are very like those of Goldsmith, except for agriculture, where the differences arise from his inclusion of farm housing (which we have considered 'residential') and of certain types of livestock other than work animals and breeders. The figures for manufacturing or manufacturing and mining combined are approximately the same in all cases. Kuznets' high figure for public utilities is probably due to the high valuation (in 1929 prices) placed on railway and street investment in 1919, and the failure to revalue assets in these industries downward between 1919 and 1939. The discrepancy between our figures and the National Industrial Conference Board's estimate for public utilities results from omission of canals, shore installations of the shipping industry, and 'other utility plants' and 'unclassified and undistributed' from our electric power estimates. Kuznets' 'other industrial' includes items which we have omitted —irrigation, waterworks, canals, and roads. The difference between our estimates and Goldsmith's is due almost solely to a difference in residential-stock estimates, while the closeness to Kuznets' total estimates is due to the offsetting of his larger public-utilities stock against his smaller residential.

The comparisons presented above make it seem likely that the capital-coefficient estimates are a set of 'reasonable' numbers of the right general order of magnitude. Internal checks on the use of average coefficients to represent incremental coefficients indicate that while errors are produced the orders of magnitude are sufficiently close to accept the average coefficients as a first approximation of the incremental coefficients. The use, although a limited one, of capital coefficients by industry of origin computed from the capital flows of a single year undoubtedly results in somewhat unreliable figures. However, the growth rates employed seem to be reasonable and the resulting coefficients are probably of the right order of magnitude if the over-all coefficients are correct.

The estimates of replacement requirements, capacity utilization, and capital stock, compared with other estimates of these quantities which have been made, indicate that our estimates of over-all capital coefficients are consistent with available economic information. The close agreement of our estimates with the capital-output data for the other countries gives a further basis for believing that the figures presented in our tables are reasonable.

The estimates, while they seem plausible in the light of available evidence, should, of course, be regarded as first approximations. Subsequent

revisions are to be expected when detailed industry studies, such as the ones presented in Chapters 7, 10, and 11, have been made. The present estimates are, in our opinion, the best that could be made from available economic data. The next step must be the collection of engineering information to derive the appropriate incremental coefficients. (See footnote 4.)

IV. Capital and Inventory Relations in 1939

In the preceding sections the capital and inventory coefficients have been presented and examined in the light of methodological checks and comparisons with other economic data. A brief investigation of their general economic significance may also be of interest. In this section some simple statistical analyses will be undertaken in order to indicate the comparative importance of various industries in the capital structure of the American economy.

A. SIZE OF THE CAPITAL COEFFICIENTS

Table 8 presents the capital coefficients (some average undepreciated, the others, incremental), ranked by size. An analysis of this table shows the following general pattern: home renting, transportation, communications, and public utilities tend to have the highest amounts of fixed capital per unit of capacity; next come the extractive industries—agriculture, mining, and fishing—and the processing of mineral products, including iron and steel; finally come all the other industries with construction and transportation equipment well down the list. As value is added by successive production processes, the capital-capacity ratio declines. To go much beyond this conclusion, an analysis of capital-value added ratios is required.

B. RELATIVE FIXED CAPITAL REQUIREMENTS

Fixed capital requirements depend on both the size of the industry and its use of capital. A large industry using little fixed capital per unit of capacity may need to spend less on adding to and replacing capital than a small one using much capital. On the other hand, its very size may still make it necessary for it to spend relatively large amounts for plant and equipment, despite its relatively small per cent of capacity use of capital.

As expected, the industries with the largest stock of fixed capital also are required to make the highest expenditure on replacement in order to maintain capacity. This is clear from a comparison of Columns 1 and 2 of Table 9.

TABLE 8

Ranking of Industries According to Size of Capital Coefficient Gross (in dollars)

Ind. No.	Industry	Capital Coefficient Gross
85	Home renting	7.119*
45	Petroleum and natural gas	4.308
51	Communications	3.564
73	Steam railroad transportation	3.226
70	Transportation, n.e.c.	3.004
52	Electric public utilities	2.671
72	Transoceanic transportation	2.577**
50	Manufactured gas	2.519
28	Munitions	2.000**
24	Steel works and rolling mills	1.798**
22	Iron mining	1.580
71	Coastwise and inland water transportation	1.562
23	Blast furnaces	1.531**
67	Industries, n.e.c.	1.502
47	Anthracite coal	1.418
48	Bituminous coal	1.370
1-8	Agriculture	1.354
49	Coke and manufactured solid fuel	1.250**
43	Nonmetallic mineral mining	1.093
39	Nonferrous metal mining	1.068**
27	Firearms	.991**
44	Nonmetallic mineral manufactures	.967
25	Iron and steel foundry products	.867**
58	Cotton yarn and cloth	.825**
14	Starch and glucose products	.820
9	Fishing	.754
59	Silk and rayon products	.703**
46	Petroleum refining	.659
56	Wood pulp, paper, and paper products	.605
40	Smelting and refining of nonferrous metals	.599**
41	Aluminum products	.597**
54	Lumber and timber products	.538
74	Trade	.534
35	Machine tools	.501
62	Other textile products	.495
30	Engines and turbines	.493
29	Agricultural machinery	.466
15	Alcoholic beverages	.447
57	Printing and publishing	.427
60	Woolen and worsted manufactures	.412
38	Iron and steel, n.e.c.	.403
34	Industrial and household equipment, n.e.c.	.398
36	Merchandising and service machines	.398
76-82, 86-91	Other services	.382
66	Rubber products	.373
53	Chemicals	.371
31	Automobiles	.353
16	Nonalcholic beverages	.345

*Market value
**Replacement Cost

TABLE 8 (Continued)

Ranking of Industries According to Size of Capital Coefficient Gross (in dollars)

Ind. No.	Industry	Capital Coefficient Gross
55	Furniture and other manufactures of wood	.339
13	Sugar refining	.334
42	Nonferrous metal manufactures and alloys	.314
12	Bread and bakery products	.301
11	Canning and preserving	.299
20	Edible fats and oils, n.e.c.	.286**
33	Transportation equipment, n.e.c.	.281
19	Manufactured dairy products	.275
26	Shipbuilding	.232
21	Other food products	.222*
37	Electrical equipment, n.e.c.	.222
63	Leather	.203
18	Slaughtering and meat packing	.200
10	Flour and grist mill products	.194
64	Leather shoes	.179
32	Aircraft	.168
17	Tobacco manufactures	.119
65	Leather products, n.e.c.	.110
68, 69	Construction	.076**
61	Clothing	.070

*Market value
**Replacement Cost

Column 4 of Table 9 ranks industries according to their 1939 purchases of fixed capital for replacement and additions combined. By and large the order is much the same as in Column 2 showing replacement requirements indicating the impact of replacement of capital outlays. No logical connection exists between the rate of growth ranked in Column 3 and the other rankings. Industries whose capital stock was large had either relatively large increases or decreases in stock in 1939 and are either at the very top or very bottom of the ranking of additions to stock Transportation, electric power, and construction industries show substantial growth, while trade and agriculture show substantial declines.

Table 10 ranks industries according to the life expectancies of their capital stock.[21] Construction and fishing, where fixed plant (as against equipment) is relatively small, and agriculture, which includes livestock, use up capital most quickly. At the other end of the scale are water and rail transportation, housing, and textiles, in which the capital stock is

[21] The estimates of life expectancies were derived from Bureau of Internal Revenue, Bulletin 'F,' *Income Tax Depreciation and Obsolescence, Estimated Useful Lives and Depreciation Rates* (revised January 1942). The Commissioner of Internal Revenue's Office has stated that 'the estimated useful lives referred to are the result of observation and study by the Bureau engineers over a period of twenty years supplemented by mortality studies made in connection with depreciable property records of those taxpayers where retirement histories were sufficiently complete and accurate to justify such studies.'

TABLE 9

Ranking of Industries of Use by Fixed Capital Stock (Undepreciated or Replacement), Replacement Requirements, Fixed Capital Growth, and Purchases of Fixed Capital, 1939

Ind No.	Industry	Fixed Capital Stock	Replacement Requirements	Additions to Stock, 1939	Capital Purchases
		(1)	(2)	(3)	(4)
85	Home renting	1	1	1	1
73	Steam railroad transportation	2	3	3	3
1-8	Agriculture	3	2	68	2
74	Trade	4	4	57	6
52	Electric public utilities	5	6	2	4
24	Steel works and rolling mills	6	8	67	13
45	Petroleum and natural gas	7	5	6	5
76-82, 86-91	Other services	8	7	65	10
51	Communications	9	9	5	8
70	Transportation, n.e.c.	10	10	4	7
67	Industries, n.e.c.	11	12	66	13
44	Nonmetallic mineral manufactures	12	15	17	15
46	Petroleum refining	13	19	9	12
31	Automobiles	14	13	47	14
53	Chemicals	15	14	10	11
58	Cotton yarn and cloth	16	22	55	25
38	Iron and steel, n.e.c.	17	16	61	19
48	Bituminous coal	18	17	64	27
56	Wood pulp, paper, and paper products	19	20	20	16
50	Manufactured gas	20	25	36	24
57	Printing and publishing	21	18	50	20
68, 69	Construction	22	11	7	9
34	Industrial and household equipment, n.e.c.	23	21	21	18
23	Blast furnaces	24	24	63	60
54	Lumber and timber products	25	26	48	29
40	Smelting and refining of nonferrous metals	26	27	62	58
25	Iron and steel foundry products	27	29	56	46
60	Woolen and worsted manufactures	28	30	43	32
12	Bread and bakery products	29	28	40	30
19	Manufactured dairy products	30	23	60	31
43	Nonmetallic mineral mining	31	31	26	26
18	Slaughtering and meat packing	32	34	45	38
72	Transoceanic transportation	33	40	58	68
37	Electrical equipment, n.e.c.	34	33	12	21
59	Silk and rayon products	35	39	39	41
49	Coke and manufactured solid fuel	36	36	53	51
55	Furniture and other manufactures of wood	37	43	34	39
22	Iron mining	38	32	54	50
33	Transportation equipment, n.e.c.	39	42	51	62

TABLE 9 (Continued)

Ranking of Industries of Use by Fixed Capital Stock (Undepreciated or Replacement), Replacement Requirements, Fixed Capital Growth, and Purchases of Fixed Capital, 1939

Ind. No.	Industry	Fixed Capital Stock	Replacement Requirements	Additions to Stock, 1939	Capital Purchases
		(1)	(2)	(3)	(4)
42	Nonferrous metal manufactures and alloys	40	44	33	40
66	Rubber products	41	37	19	28
21	Other food products	42	35	46	45
39	Nonferrous metal mining	43	38	41	42
47	Anthracite coal	44	41	52	59
71	Coastwise and inland water transportation	45	55	8	15
62	Other textile products	46	50	44	54
10	Flour and grist mill products	47	49	29	44
16	Alcoholic beverages	48	48	17	35
11	Canning and preserving	49	45	59	48
28	Munitions	50	46	49	65
35	Machine tools	51	51	15	33
13	Sugar refining	52	55	38	56
29	Agricultural machinery	53	52	30	47
61	Clothing	54	47	11	22
36	Merchandising and service machines	55	56	31	52
64	Leather shoes	56	57	14	36
26	Shipbuilding	57	60	23	49
17	Tobacco manufactures	58	59	28	55
14	Starch and glucose products	59	58	37	57
16	Nonalcoholic beverages	60	62	17	43
41	Aluminum products	61	61	24	53
32	Aircraft	62	63	13	34
9	Fishing	63	54	18	37
63	Leather	64	65	35	66
20	Edible fats and oils, n.e.c.	65	64	27	63
30	Engines and turbines	66	66	25	61
65	Leather products, n.e.c.	67	67	32	67
27	Firearms	68	68	22	64

notoriously obsolete, and industries using very large items of equipment like gas and electric utilities.

If one examines more closely the industrial groups with the largest amounts of fixed capital stock, it becomes apparent that a high degree of concentration existed in this respect in the American economy in 1939. Ten industrial groups controlled 80 per cent of the capital stock of the economy[22] (see Table 11). These same industries were also responsible

[22] The term 'economy' is used here to describe that part of the economy whose capital structure has been examined in relation to capacity. There are, of course, significant capital-using elements of society which have been omitted because the capital capacity concept did not seem applicable. These would include public works such as water-supply and sewerage systems, hospitals, schools, recreational facilities, highways, ports, waterways, and soil-conservation and flood-control programs The importance of these omitted items

TABLE 10

Composite Life Expectancies of Fixed Capital Stock
by Industries of Use (in years)

Years	Industry
9-10	68,69 Construction
	9 Fishing
	1-8 Agriculture
11-20	19 Manufactured dairy products
	45 Petroleum and natural gas
	14 Starch and glucose products
21-25	22 Iron mining
	61 Clothing
	21 Other food products
	53 Chemicals
	38 Iron and steel, n.e.c.
	11 Canning and preserving
	31 Automobiles
	48 Bituminous coal
	66 Rubber products
	39 Nonferrous metal mining
	17 Tobacco manufactures.
	41 Aluminum products
	28 Munitions
	16 Nonalcoholic beverages
	20 Edible fats and oils, n.e.c.
	30 Engines and turbines
	27 Firearms
	47 Anthracite coal
	74 Trade
	67 Industries, n.e.c.
	44 Nonmetallic mineral manufactures
	57 Printing and publishing
	65 Leather products
	12 Bread and bakery products
	43 Nonmetallic mineral mining
	49 Coke and manufactured solid fuel
	64 Leather shoes
26-30	36 Merchandising and service machines
	76-82, 86-91 Other services
	29 Agricultural machinery
26-30 Cont'd	51 Communications
	70 Transportation, n.e.c.
	37 Electrical equipment, n.e.c.
	15 Alcoholic beverages
	35 Machine tools
	56 Wood pulp, paper, and paper products
	34 Industrial and household equipment, n.e.c.
	40 Smelting and refining of nonferrous metals
	33 Transportation equipment, n.e.c.
	32 Aircraft
	24 Steel works and rolling mills
	23 Blast furnaces
	60 Woolen and worsted manufactures
	25 Iron and steel foundry products
	18 Slaughtering and mear packing
	42 Nonferrous metal manufactures and alloys
	59 Silk and rayon products
	10 Flour and grist mill products
	55 Furniture and other manufactures of wood
	26 Shipbuilding
31-40	58 Cotton yarn and cloth
	54 Lumber and timber products
	62 Other textile products
	52 Electric public utilities
	73 Steam railroad transportation
	72 Transoceanic transportation
	13 Sugar refining
	63 Leather
	46 Petroleum refining
	50 Manufactured gas
	85 Home renting
	71 Coastwise and inland water Transportation

can be indicated by the fact that the Twentieth Century Fund predicted that 40 per cent of the average annual capital expenditures between 1946 and 1960 will be made by this sector of the economy. See Hartley, R. W., et al., *America's Capital Requirements: Estimates for 1946-60*, The Twentieth Century Fund 1950, p. 23. Reeve has estimated that the value of reproducible physical assets of the federal, state, and local governments in 1939 was almost $66 billion. See Reeve, J. E., et al., 'Government Component in the National Wealth,' Conference on Research in Income and Wealth, *Studies in Income and Wealth*, XII, p. 467.

TABLE 11

Fixed Capital Stock by Industry of Use, 1939

Industry	Fixed Capital Stock (millions of dollars)	Per Cent
	(1)	(2)
(1) Home renting	$70,577.8	33.8
(2) Steam railroad transportation	25,747.6	12.3
(3) Agriculture	16,586.5	7.9
(4) Trade	10,614.3	5.1
(5) Electric public utilities	10,154.1	4.9
(6) Steel works and rolling mills	7,778.6	3.7
(7) Petroleum and natural gas	7,233.3	3.5
(8) Other services	7,221.7	3.5
(9) Communications	5,513.1	2.6
(10) Transportation, n.e.c.	4,851.8	2.3
(11) All other	42,232.0	20.2
Total	$208,510.8	100.0

for nearly 80 per cent of replacement requirements and some 84 per cent of fixed capital purchases.[23] Home renting and steam railways together accounted for 46 per cent of the capital stock.

It is of some interest to note the type of industry which is a heavy capital user. With the exception of steel works and rolling mills, no manufacturing or mining industry is included among the first 10. Most of the capital stock of the economy is in the hands of service industries such as home renting and trade, agriculture, transportation, electric public utilities, and the crude-oil industry. In 1939 practically all manufacturing and mining activity was carried on with some 20 per cent of the fixed capital stock.

The series of tabulations which follow give the percentage distribution of types (industries of origin) of fixed capital by industry of use. Again a high degree of concentration is found; about 75 per cent of the capital stock of the economy was produced by only 5 industries as shown in Table 12.[24]

Ninety per cent of the transportation equipment, n.e.c., is held by railways, and over 50 per cent of construction is held by the home-renting industry.

In view of the high concentration of capital stock originating in such a small number of industries, it is not surprising to find that most indus-

[23] In the ranking of the first 10 industries according to size of fixed capital purchases, steel works and rolling mills drop out, and construction enters. Otherwise the list is the same.

[24] It is true that this concept of concentration depends on the classification of the industrial groups. However, it may be noted that the 10 industrial groups which were the heaviest capital users, and the 4 which were the chief suppliers of capital, show a high degree of homogeneity.

TABLE 12

Fixed Capital Stock by Industry of Origin, 1939

Industry	Gross Capital Stock (millions of dollars)	Per Cent
	(1)	(2)
(1) Construction	$130,226	62.5
(2) Transportation equipment, n.e.c.	11,605	5.6
(3) Electrical equipment, n.e.c.	10,910	5.2
(4) Machine tools	6,579	3.1
(5) All others	49,191	23.6
Total[1]	$208,511[1]	100.0[1]

tries have capital-stock structures which are composed of a limited number of combinations of the dominant types of capital goods. Thirty-one industries possess capital structures of which well over 90 per cent are derived from the construction and industrial equipment, n.e.c., industries alone (see Tabulation A). These industries vary, in so far as our industrial categories are concerned, mainly in the relative importance of construction as opposed to machinery. Five additional industries are quite similar, except that approximately 10 per cent of their capital stock is derived from iron and steel, n.e.c. (see Tabulation B).

These 36 industries include all the mining industries, the textile and apparel industries, the food-processing industries (except canning and preserving, which has a stock of ships), lumber, paper and wood products, petroleum, chemicals, rubber, manufactured gas, coke, blast furnaces, iron and steel foundries, leather and leather products, n.e.c., printing and publishing, and nonmetallic mineral manufactures.

The metal-fabricating industries (see Tabulation C), with the exception of shipbuilding, derive almost all their capital from the machine-tool and metalworking-equipment industry and from construction, with iron and steel, n.e.c., which produces small tools, supplying small amounts as well.

Two industries, electrical equipment, n.e.c., and industries, n.e.c., have structures which are a combination of the two preceding ones. They use large amounts of capital from both the industrial and household equipment and machine tool industries, as well as construction, with about 5 per cent coming from iron and steel, n.e.c. (see Tabulation D).

The three industries producing rolled metal, steel works and rolling mills, aluminum products, and nonferrous metal manufactures and alloys, have similar structures, as their capital stock consists of goods purchased from construction, machine tools, industrial equipment, n.e.c., steel works and rolling mills, and iron and steel foundry products (see Tabulation E).

TABULATION A

Ind. No.	Industry	Per Cent of Capital Stock From: Industrial Equipment, n.e.c.	Construction	All Other
		(1)	(2)	(3)
22	Iron Mining	92.4	6.3	1.3
48	Bituminous coal	84.7	15.0	.3
39	Nonferrous metal mining	82.2	16.0	1.8
50	Manufactured gas	79.7	18.8	1.5
13	Sugar refining	73.4	24.0	2.6
23	Blast furnaces	71.4	27.0	1.6
60	Woolen and worsted manufactures	71.3	28.0	.7
59	Silk and rayon products	69.7	30.0	.3
47	Anthracite coal	69.6	27.0	3.4
25	Iron and steel foundry products	66.8	28.0	5.2
43	Nonmetallic mineral mining	65.4	33.2	1.3
49	Coke and manufactured solid fuel	64.8	34.6	.7
21	Other food products	62.6	34.0	3.5
56	Wood pulp, paper, and paper products	62.3	36.5	1.2
14	Starch and glucose products	62.0	36.1	1.9
66	Rubber products	62.0	36.1	1.9
44	Nonmetallic mineral manufactures	61.8	33.1	5.1
54	Lumber and timber products	57.6	40.0	2.4
45	Petroleum and natural gas	56.5	43.3	.1
58	Cotton yarn and cloth	55.7	37.5	6.8
53	Chemicals	54.8	36.9	8.3
57	Printing and publishing	51.4	41.3	7.3
63	Leather	51.2	47.0	.7
17	Tobacco manufactures	49.7	49.6	.7
19	Manufactured dairy products	47.7	46.9	5.4
16	Nonalcoholic beverages	46.8	51.0	2.2
18	Slaughtering and meat packing	46.5	53.2	.3
10	Flour and grist mill products	42.0	57.0	1.0
12	Bread and bakery products	41.7	57.6	.7
65	Leather products, n.e.c.	34.2	63.5	2.3
46	Petroleum refining	32.6	61.7	4.7

TABULATION B

Ind. No.	Industry	Per Cent of Capital Stock From: Industrial Equipment, n.e.c.	Iron and Steel, n.e.c.	Construction	All Other
20	Edible fats and oils, n.e.c.	63.1	13.6	23.1	.2
61	Clothing	39.7	8.7	46.5	5.1
62	Other textile products	35.2	12.8	46.0	4.0
15	Alcoholic beverages	25.6	9.6	59.8	4.9
55	Furniture and other manufactures of wood	19.9	8.7	70.0	1.4

TABULATION C

Ind. No.	Industry	Per Cent of Capital Stock From: Machine Tools	Iron and Steel, n.e.c.	Construction	All Other
27	Firearms	78.2	3.5	12.9	.4
30	Engines and turbines	62.2	8.3	28.4	1.1
36	Merchandising and service machines	57.3	7.6	34.8	.3
31	Automobiles	56.6	8.5	32.9	2.0
38	Iron and steel, n.e.c.	54.2	3.9	38.1	3.8
35	Machine tools	51.1	2.9	42.9	3.1
28	Munitions	48.9		51.0	.1
33	Transportation equipment, n.e.c.	47.6	6.3	45.7	.4
34	Industrial and household equipment, n.e.c.	47.0	6.0	46.6	.4
32	Aircraft	39.4	5.5	54.8	.3
29	Agricultural machinery	25.0	33.7	41.0	.3

TABULATION D

Ind. No.	Industry	Per Cent of Capital Stock From: Industrial Equipment, n.e.c.	Machine Tools	Iron and Steel, n.e.c.	Construction	All Other
		(1)	(2)	(3)	(4)	(5)
37	Electrical equipment, n.e.c.	22.9	26.4	6.5	42.7	1.5
67	Industries, n.e.c.	15.8	40.8	4.2	38.6	.6

TABULATION E

Ind. No.	Industry	Per Cent of Capital Stock From: Steel Works and Rolling Mills	Iron and Steel Foundry Products	Industrial Equipment, n.e.c.	Machine Tools	Construction
		(1)	(2)	(3)	(4)	(5)
24	Steel works and rolling mills	14.6	17.3	12.6	18.7	35.0
41	Aluminum products	12.0	14.1	16.8	43.5	13.3
42	Nonferrous metal manufactures and alloys	4.5	5.1	20.3	27.2	42.1

TABULATION F

Ind. No.	Industry	Per Cent of Capital Stock From: Electrical Equipment, n.e.c.	Construction	Engines and Turbines	Nonferrous Metal Manufacture and Alloys	All Other
		(1)	(2)	(3)	(4)	(5)
51	Communications	66.9	15.0	—	15.0	3.1
52	Electric public utilities	61.3	16.9	13.5	3.7	4.6

The two water transportation industries derive all their capital stock, in so far as our data show, from the shipbuilding industry.

Two industries, electric public utilities and communications, have between 60 and 70 per cent of their capital stock originating in the electrical equipment industry, and about 15 per cent in the construction industry. The remaining capital in the electric-utilities industry is mainly from the engines and turbines industry, while the communication industry has about 15 per cent of its capital stock from the nonferrous metal manufactures and alloys industry (Tabulation F).

The remaining industries, agriculture, fishing, canning and preserving, transportation, n.e.c., steam railways, construction, smelting of nonferrous metals, leather shoes, and the service industries have structures which do not conform closely to the patterns of other industries.

Table 13 shows fixed capital-replacement requirements by industry of origin. For replacement purposes the most important items are buildings, heavy and specialized machinery and equipment, electrical equipment, automobiles, railroad and transit equipment, tires and tubes, and agricultural machinery.

Table 14 shows life expectancies of fixed capital by industry of origin. Requiring to be replaced most frequently are tires, livestock, aircraft, leather and textiles used in or as fixed capital, and automobiles. The most lasting types of fixed capital are buildings, ships, railroad and transit equipment, engines and turbines.

Table 15 ranks industries of origin of fixed capital according to which were the most important suppliers in 1939. Construction was by far the most important. Of the equipment produced, the products of agricultural machinery, automobiles, other transportation equipment, machine tools and miscellaneous electrical equipment comprised 55 per cent of the total, indicating a smaller concentration of types of equipment than in the case of stock.

Table 16, giving capital growth rates in 1939 by industry of origin, shows the types of fixed capital goods for which the demand for pur-

TABLE 13

Ranking of Capital Replacement Requirements
by Industry of Origin
(millions of dollars)

68, 69 Construction	$2,853.5
34 Industrial and household equipment, n.e.c.	1,171.5
1-8 Agriculture	952.0
37 Electrical equipment, n.e.c.	438.2
31 Automobiles	429.5
33 Transportation equipment, n.e.c.	388.6
66 Rubber products	373.3
35 Machine tools	353.8
29 Agricultural machinery	233.9
36 Merchandising and service machines	132.9
38 Iron and steel, n.e.c.	127.1
24 Steel works and rolling mills	106.0
25 Iron and steel foundry products	91.4
67 Industries, n.e.c.	84.4
42 Nonferrous metal manufactures and alloys	83.5
55 Furniture and other manufactures of wood	70.7
30 Engines and turbines	66.2
26 Shipbuilding	39.4
32 Aircraft	17.7
65 Leather products, n.e.c.	12.4
62 Other textile products	10.8
58 Cotton yarn and cloth	4.3
60 Woolen and worsted manufactures	.4

poses of expansion was greatest in that year. These were primarily construction and agricultural machinery, and to a lesser extent shipbuilding and miscellaneous iron and steel used mostly in construction and buildings.

C. MAGNITUDE OF INVENTORIES AND INVENTORY COEFFICIENTS

Table 17 ranks the industries of the economy by the size of their inventory coefficient (the relation between inventories of raw materials held and their outputs). Tables 18 and 19 present estimates of the total inventories. Table 18 shows fixed capital stock and inventories ranked by industry of use; Table 19, by industry of origin. It should be noted that these latter two tables include inventories held for the final-demand industries, exports, government, and households, as well as stocks which were undistributed because of lack of data.

There appears to be a tendency for the heavy metalworking industries and industries which utilize large amounts of storable agricultural products to require relatively large inventories per unit of output. Low inventory coefficients are found in the service group, transportation, communication, and mining.

In absolute figures, the highest volume of inventories tends to be held

TABLE 14

Ranking of Composite Life Expectancies of Fixed Capital Stock by Industry of Origin
(in years)

Ind. No.	Industry	Number of Years
68	69 Construction	46
26	Shipbuilding	36
33	Transportation equipment, n.e.c.	30
30	Engines and turbines	29
37	Electrical equipment, n.e.c.	25
42	Nonferrous metal manufactures and alloys	23
60	Woolen and worsted manufactures	21
34	Industrial and household equipment, n.e.c.	20
25	Iron and steel foundry products	19
38	Iron and steel, n.e.c.	19
35	Machine tools	19
24	Steel works and rolling mills	18
55	Furniture and other manufactures of wood	17
29	Agricultural machinery	14
36	Merchandising and service machines	14
31	Automobiles	8
62	Other textile products	7
65	Leather products, n.e.c.	7
67	Industries, n.e.c.	7
58	Cotton yarn and cloth	6
32	Aircraft	5
1-8	Agriculture	4
66	Rubber products	1

in connection with the operations of households, foreign trade, flour mills, agriculture, construction, chemical, automobile, tobacco, cotton, and steel manufacture. At the other end of the scale are mining and transportation, with low inventories.

The types of inventories held are chiefly farm produce, clothing, petroleum, chemicals, iron and steel, automobiles, electrical equipment, lumber and timber products.

Considerable light can be thrown on the magnitude of the inventory coefficient by breaking it down into its major components. The inventory-output ratio of each industry is the product of two other relationships: the ratio of storable inputs to output, and the ratio of inventories held to storable inputs.[25]

The storable input-output ratio of each industry is a part of the pro-

[25] Storable inputs include all materials, supplies, and fuels.

$$\frac{\text{inventories}}{\text{output}} = \frac{\text{storable inputs}}{\text{output}} \times \frac{\text{inventories}}{\text{storable inputs}}$$

TABLE 15

1939 Ranking of Industries of Origin of Fixed Capital Flows
(millions of dollars)

Ind. No.	Industry	Fixed Capital Flows
68, 69	Construction	$5,921.6
34	Industrial and household equipment, n.e.c.	944.9
29	Agricultural machinery	441.5
31	Automobiles	276.5
35	Machine tools	257.0
37	Electrical equipment, n.e.c.	242.7
33	Transportation equipment, n.e.c.	177.0
38	Iron and steel, n.e.c.	164.0
36	Merchandising and service machines	126.2
66	Rubber products	124.2
24	Steel works and rolling mills	102.8
26	Shipbuilding	77.6
42	Nonferrous metal manufactures and alloys	69.0
67	Industries, n.e.c.	66.0
25	Iron and steel foundry products	59.1
55	Furniture and other manufactures of wood	34.7
30	Engines and turbines	21.8
32	Aircraft	19.1
62	Other textile products	18.0
65	Leather products, n.e.c.	14.6
58	Cotton yarn and cloth	5.7
60	Woolen and worsted manufactures	1.0
1-8	Agriculture	.7*

* Excludes production of livestock.

duction function, and may be assumed to be determined by the technique of production of that industry. The determinants of the inventories-storable inputs ratios are somewhat more obscure, and among them are such factors as cost of storage, variability in particular seasonality of the rate of production and consumption, and extent of integration. Table 20 ranks the industries of the economy by the ratios of inventories to the annual flow of storable inputs in each. Inventories here mean stocks of raw materials and supplies held for or by the industry, and goods-in-process within the industry. It will be noted that the bulk of manufacturing industries have ratios ranging from about 20 to 40 per cent. On the assumption that a ratio within this range has been found by firms to be the most economical, most of the discussion to follow will be concerned with those industries whose ratios are above or below this range. Table 21 ranks the industries by the ratios of stocks of raw materials, supplies, and goods-in-process held by the consuming industry alone.

Six industries, firearms, aircraft, electric public utilities, steam railways, engines and turbines, industrial and household equipment, n.e.c.,

TABLE 16

Ranking of Fixed Capital Growth in 1939 by Industry of Origin (Flows Minus Replacement Requirements)
(millions of dollars)

Ind. No.	Industry	Fixed Capital Growth
68, 69	Construction	$3,068.0
29	Agricultural machinery	207.6
26	Shipbuilding	38.2
38	Iron and steel, n.e.c.	36.9
62	Other textile products	7.2
65	Leather products, n.e.c.	2.2
32	Aircraft	1.4
58	Cotton yarn and cloth	1.4
60	Woolen and worsted manufactures	.6
24	Steel works and rolling mills	− 3.2
36	Merchandising and service machines	− 6.7
42	Nonferrous metal manufactures and alloys	−14.5
67	Industries, n.e.c.	−18.4
25	Iron and steel foundry products	−32.3
55	Furniture and other manufactures of wood	−36.0
30	Engines and turbines	−44.4
35	Machine tools	−96.8
31	Automobiles	−153.0
37	Electrical equipment, n.e.c.	−195.5
33	Transportation equipment, n.e.c.	−211.6
34	Industrial and household equipment, n.e.c.	−226.6
66	Rubber products	−249.1
1-8	Agriculture	−951.3*

* Excludes production of livestock.

have high inventory-storable input ratios whether measured in terms of inventories held by and for the industry or those held by the consuming industry alone. In the case of electric public utilities and steam railways, the high ratio is due to the large stocks of coal kept on hand. Coal is a vital input to these industries, is relatively inexpensive to store, but sometimes irregular in supply due to strikes and occasional transportation difficulties. The costs of storage by no means dominate their total expenditures. The other four industries are metal-fabricating industries which tend, in general, to have higher than usual inventory ratios. This is apparently due to the relatively large amounts of goods-in-process kept by these industries, which commonly do not stock finished items, but manufacture the parts and then assemble them upon receipt of orders."[28] It may be noted that machine tools also holds large stocks of inventories in comparison with storable inputs. It is probable that the necessity to have on hand a full assortment of the various components and

[28] Cf. Lewis, H. T., *Industrial Purchasing*, 1942, pp. 203-4.

TABLE 17

Ranking of Inventory Coefficients, 1939

Ind. No.	Industry	Inventory Coefficient
10	Flour and grist mill products	.6972
14	Starch and glucose products	.6046
27	Firearms	.4696
58	Cotton yarn and cloth	.3879
32	Aircraft	.3555
17	Tobbaco manufactures	.3080
13	Sugar refining	.2942
23	Blast furnaces	.2932
33	Transportation equipment, n.e.c.	.2843
30	Engines and turbines	.2468
63	Leather	.2401
29	Agricultural machinery	.2261
15	Alcoholic beverages	.2197
60	Woolen and worsted manufactures	.2187
36	Merchandising and service machines	.2150
53	Chemicals	.1960
41	Aluminum products	.1957
62	Other textile products	.1953
24	Steel works and rolling mills	.1917
34	Industrial and household equipment, n.e.c.	.1850
20	Edible fats and oils, n.e.c.	.1840
37	Electrical equipment, n.e.c.	.1813
55	Furniture and other manufactures of wood	.1794
38	Iron and steel, n.e.c.	.1731
40	Smelting and refining of nonferrous metals	.1672
42	Nonferrous metal manufactures and alloys	.1668
35	Machine tools	.1667
28	Munitions	.1644
9	Fishing	.1495
11	Canning and preserving	.1413
31	Automobiles	.1390
56	Wood pulp, paper, and paper products	.1380
65	Leather products, n.e.c.	.1360
54	Lumber and timber products	.1346
26	Shipbuilding	.1342
64	Leather shoes	.1332
67	Industries, n.e.c.	.1313
59	Silk and rayon products	.1286
61	Clothing	.1227
25	Iron and steel foundry products	.1177
16	Nonalcoholic beverages	.1131
66	Rubber products	.1122
44	Nonmetallic mineral manufactures	.1087
21	Other food products	.1039
49	Coke and manufactured solid fuel	.0992

TABLE 17 (Continued)

Ranking of Inventory Coefficients, 1939

Ind. No.	Industry	Inventory Coefficient
68, 69	Construction	.0859
57	Printing and publishing	.0846
73	Steam railroad transportation	.0839
50	Manufactured gas	.0795
46	Petroleum refining	.0792
12	Bread and bakery products	.0719
52	Electric public utilities	.0704
1-8	Agriculture	.0686
70	Transportation, n.e.c.	.0470
18	Slaughtering and meat packing	.1391
71	Coastwise and inland water transportation	.0276
19	Manufactured dairy products	.0259
76-82, 86-91	Other services	.0246
45	Petroleum and natural gas	.0109
72	Transoceanic transportation	.0098
74	Trade	.0069
47	Anthracite coal	.0053
39	Nonferrous metal mining	.0038
48	Bituminous coal	.0035
51	Communications	.0034
43	Nonmetallic mineral mining	.0031
85	Home renting	.0001

materials of the final product is the cause of the very large stock of goods-in-process. The high ratio in the aircraft industry is in part caused by the stock-piling of materials during 1939 in preparation for expected war expansion. Stocks of raw materials held by the aircraft industry rose 62 per cent from the beginning of 1939 to the end of the year, as compared with a rise of but 15 per cent in the raw-material holdings of all manufacturing industries combined.

The following industries have consistently low inventory-storable input ratios (under 20 per cent) according to both types of inventory measurement:

Coastwise and inland water transportation
Bread and bakery products
Other food products
Petroleum refining
Coke and manufactured solid fuel
Smelting and refining of nonferrous metals
Transoceanic transportation
Slaughtering and meat packing
Manufactured dairy products

TABLE 18

Fixed Capital Stocks and Inventories, 1939,
Ranked by Industry of Use
(millions of dollars)

Ind. No.	Industry	Inventories (1)	Fixed Capital Stocks (2)	Total (3)
85	Home renting	.8	70,577.8	70,578.6
73	Steam railroad transportation	378.1	25,750.0	26,128.1
1-8	Agriculture	674.2	16,585.3	17,259,5
74	Trade	118.9	10,602.4	10,721.3
52	Electric public utilities	180.6	10,154.1	10,334.7
94	Households[1]	8,392.1	- -	8,392.1
24	Steel works and rolling mills	535.9	7,782.0	8,317.9
76-82, 86-91	Other services	333.0	7,221.6	7,554.6
45	Petroleum and natural gas	18.3	7,233.3	7,251.6
51	Communications	.5	5,513.6	5,514.1
70	Transportation, n.e.c.	51.9	4,851.3	4,903.2
67	Industries, n.e.c.	230.1	2,873.1	3,103.2
53	Chemicals	733.5	1,879.6	2,613.1
31	Automobiles	563.4	1,924.5	2,487.9
68, 69	Construction	869.9	1,434.7	2,304.6
46	Petroleum refining	286.0	1,978.8	2,274.8
44	Nonmetallic mineral manufacturing	172.1	2,031.7	2,203.8
58	Cotton yarn and cloth	541.6	1,601.0	2,142.6
38	Iron and steel, n.e.c.	385.7	1,574.8	1,960.5
56	Wood pulp, paper, and paper products	278.7	1,532.6	1,811.3
34	Industrial and household equipment, n.e.c.	415.1	1,388.7	1,803.8
57	Printing and publishing	223.8	1,451.7	1,675.5
48	Bituminous coal	2.5	1,570.9	1,573.4
23	Blast furnaces	161.5	1,376.0	1,537.5
50	Manufactured Gas	31.2	1,468.1	1,499.3
54	Lumber and timber products	193.5	1,226.0	1,419.5
10	Flour and grist mill products	871.8	386.0	1,257.8
40	Smelting and refining of nonferrous metals	188.5	989.0	1,177.5
37	Electrical equipment, n.e.c.	386.6	630.6	1,017.2
60	Woolen and worsted manufacturers	195.8	805.0	1,000.8
25	Iron and steel foundry products	54.5	863.8	918.3
12	Bread and bakery products	104.9	782.0	886.9
19	Manufactured dairy products	54.4	761.0	815.4
55	Furniture and other manufactures of wood	210.4	566.7	777.1
18	Slaughtering and meat packing	117.4	652.4	769.8
61	Clothing	469.2	291.1	760.3
75	Foreign trade[1]	689.5	-	689.5
43	Nonmetallic mineral mining	1.2	660.9	662.1
17	Tobacco manufactures	502.0	158.2	660.2
59	Silk and rayon products	80.1	579.8	659.9

[1]No fixed capital estimates were made for households, foreign trade, or government. Inventories are only those held by trade and manufacturers for these industries.

TABLE 18 (Continued)

Fixed Capital Stocks and Inventories, 1939,
Ranked by Industry of Use
(millions of dollars)

Ind. No.	Industry	Inventories	Fixed Capital Stocks	Total
		(1)	(2)	(3)
72	Transoceanic transportation	2.0	641.5	643.5
42	Nonferrous metal manufactures and alloys	135.1	478.6	613.7
49	Coke and manufactured solid fuel	46.2	552.0	598.2
21	Other food products	126.7	470.8	597.5
66	Rubber products	101.4	472.6	574.0
33	Transportation equipment, n.e.c.	75.4	486.8	562.2
62	Other textile products	138.3	402.9	541.2
15	Alcoholic beverages	158.6	355.8	514.4
22	Iron mining	.3	489.6	489.9
13	Sugar refining	165.1	316.4	481.5
11	Canning and preserving	118.7	352.6	471.3
39	Nonferrous metal mining	1.4	465.1	466.5
47	Anthracite coal	1.0	439.5	440.5
71	Coastwise and inland water transportation	7.0	428.0	435.0
35	Machine tools	75.3	323.6	398.9
29	Agricultural machinery	95.5	292.2	387.7
28	Munitions	16.5	348.9	365.4
64	Leather shoes	115.2	199.0	314.2
36	Merchandising and service machines	70.4	202.3	272.4
26	Shipbuilding	58.7	192.2	250.9
14	Starch and glucose products	72.2	140.3	212.5
32	Aircraft	99.3	103.4	202.7
63	Leather	83.2	96.6	179.8
16	Nonalcoholic beverages	41.6	126.9	168.5
41	Aluminum products	40.5	123.6	164.1
20	Edible fats and oils, n.e.c.	40.5	72.9	113.4
9	Fishing	17.0	98.7	115.7
30	Engines and turbines	33.3	66.7	100.0
65	Leather products, n.e.c.	24.1	24.9	49.0
27	Firearms	8.3	21.4	29.7
92	Government[1]	19.2	—	19.2

[1]No fixed capital estimates were made for households, foreign trade, or government. Inventories are only those held by trade and manufacturers for these industries.

The food-processing industries themselves, as a whole, tend to hold small stocks of inputs, particularly in the case of highly perishable inputs such as in the meat and dairy industries. In some instances where crops are seasonal and easily stored, for example the corn and potatoes used in the starch and glucose industry, fairly large inventories are held by

TABLE 19

Fixed Capital Stocks and Inventories, 1939,
Ranked by Industry of Origin
(millions of dollars)

Ind. No.	Industry	Inventories	Fixed Capital Stocks	Total
		(1)	(2)	(3)
68, 69	Construction	0.0	130,225.5	130,225.5
34	Industrial and household equipment, n.e.c.	528.6	23,765.5	24,294.1
33	Transportation equipment, n.e.c.	12.4	11,604.8	11,617.2
37	Electrical equipment, n.e.c.	632.4	10,914.0	11,546.4
1-8	Agriculture	4,586.5	3,332.0	7,918.5
35	Machine tools	56.2	6,554.5	6,610.7
31	Automobiles	513.3	3,354.1	3,867.4
38	Iron and steel, n.e.c.	1,028.3	2,422.8	3,541.1
29	Agricultural machinery	177.1	3,235.3	3,412.4
24	Steel works and rolling mills	1,016.9	1,910.0	2,926.9
42	Nonferrous metal manufactures and alloys	202.0	1,886.2	2,088.2
36	Merchandising and service machines	186.1	1,842.3	2,028.4
30	Engines and turbines	20.5	1,889.0	1,909.5
25	Iron and steel foundry products	158.4	1,748.6	1,907.0
55	Furniture and other manufactures of wood	489.9	1,210.2	1,700.1
67	Industries, n.e.c.	941.9	584.6	1,526.5
46	Petroleum refining	1,430.4	0.0	1,430.4
26	Shipbuilding	15.5	1,385.6	1,401.1
53	Chemicals	1,238.0	0.0	1,238.0
61	Clothing	1,233.5	0.0	1,233.5
54	Lumber and timber products	863.8	0.0	863.8
56	Wood pulp, paper, and paper products	608.8	0.0	608.8
66	Rubber products	226.8	346.3	573.1
21	Other food products	545.1	0.0	545.1
11	Canning and preserving	497.6	0.0	497.6
62	Other textile products	402.6	78.1	480.7
15	Alcoholic beverages	458.2	0.0	458.2
45	Petroleum and natural gas	401.3	0.0	401.3
58	Cotton yarn and cloth	358.9	23.9	382.8
44	Nonmetallic mineral manufactures	376.7	0.0	376.7
18	Slaughtering and meat packing	331.8	0.0	331.8
13	Sugar refining	296.9	0.0	296.9
40	Smelting and refining of non-ferrous metals	291.8	0.0	291.8
57	Printing and publishing	274.9	0.0	274.9
48	Bituminous coal	267.3	0.0	267.3

TABLE 19 (Continued)

Fixed Capital Stocks and Inventories, 1939,
Ranked by Industry of Origin
(millions of dollars)

	Inventories	Fixed Capital Stocks	Total
	(1)	(2)	(3)
23 Blast furnaces	257.4	0.0	257.4
60 Woolen and worsted manufactures	244.9	8.6	253.5
64 Leather shoes	248.0	0.0	248.0
43 Nonmetallic mineral mining	218.9	0.0	218.9
10 Flour and grist mill products	190.7	0.0	190.7
59 Silk and rayon products	170.8	0.0	170.8
17 Tobacco manufactures	166.0	0.0	166.0
39 Nonferrous metal mining	149.1	0.0	149.1
19 Manufactured dairy products	132.1	0.0	132.1
32 Aircraft	37.8	88.7	126.5
65 Leather products, n.e.c.	27.6	88.8	116.4
63 Leather	113.7	0.0	113.7
49 Coke and manufactured solid fuel	104.2	0.0	104.2
41 Aluminum products	102.3	0.0	102.3
22 Iron mining	96.2	0.0	96.2
16 Nonalcoholic beverages	74.1	0.0	74.1
12 Bread and bakery products	67.8	0.0	67.8
20 Edible fats and oils, n.e.c.	64.3	0.0	64.3
47 Anthracite coal	61.6	0.0	61.6
75 Foreign trade	49.8	0.0	49.8
14 Starch and glucose products	26.0	0.0	26.0
9 Fishing	24.2	0.0	24.2
28 Munitions	10.7	0.0	10.7
50 Manufactured gas	0.0	0.0	0.0
51 Communications	0.0	0.0	0.0
52 Electric public utilities	0.0	0.0	0.0
27 Firearms	0.0	0.0	0.0
70 Transportation, n.e.c.	0.0	0.0	0.0
71 Coastwise and inland water transportation	0.0	0.0	0.0
72 Transoceanic transportation	0.0	0.0	0.0
73 Steam railroad transportation	0.0	0.0	0.0
74 Trade	0.0	0.0	0.0
76-82, 86-91 Other services	0.0	0.0	0.0
85 Home renting	0.0	0.0	0.0

other industries for them. Therefore starch and glucose shows a high ratio when inventories are measured on the basis of stocks held by and for it; a low ratio for inventories held by it alone. The same is true for flour and grist mill products and alcoholic beverages.

When petroleum is the dominant input, such as in transportation, petroleum refining, iron mining, etc., inventories tend to be low. Petro-

TABLE 20

Ratio of Inventory Stocks to Annual Flow of Storable Inputs, 1939

Ind. No.	Industry		Ind. No.	Industry	
27	Firearms	2.86	42	Nonferrous metal manufactures and alloys	.30
32	Aircraft	1.14	44	Nonmetallic mineral manufactures	.30
14	Starch and glucose products	1.08	16	Nonalcoholic beverages	.27
10	Flour and grist mill products	.97	1-8	Agriculture	.27
52	Electric public utilities	.93	59	Silk and rayon products	.27
58	Cotton yarn and cloth	.82	61	Clothing	.26
73	Steam railroad transportation	.80	65	Leather products, n.e.c.	.26
9	Fishing	.71	56	Wood pulp, paper, and paper products	.25
36	Merchandising and service machine	.65	64	Leather shoes	.24
35	Machine tools	.61	20	Edible fats and oils, n.e.c.	.23
15	Alcoholic beverages	.57	11	Canning and preserving	.23
76-82, 86-91	Other services	.55	31	Automobiles	.21
30	Engines and turbines	.54	66	Rubber products	.21
13	Sugar refining	.53	40	Smelting and refining of nonferrous metals	.20
17	Tobacco manufactures	.52	70	Transportation, n.e.c.	.18
34	Industrial and household equipment, n.e.c.	.50	68, 69	Construction	.18
33	Transportation equipment, n.e.c.	.49	49	Coke and manufactured solid fuel	.18
29	Agricultural machinery	.45	39	Nonferrous metal mining	.17
28	Munitions	.42	71	Coastwise and inland water transportation	.16
37	Electrical equipment, n.e.c.	.42	12	Bread and bakery products	.16
53	Chemicals	.41	22	Iron mining	.15
67	Industries, n.e.c.	.40	21	Other food products	.15
60	Woolen and worsted manufactures	.38	46	Petroleum refining	.15
55	Furniture and other manufactures of wood	.38	72	Transoceanic transportation	.12
63	Leather	.38	74	Trade	.12
26	Shipbuilding	.38	43	Nonmetallic mineral mining	.11
57	Printing and publishing	.36	48	Bituminous coal	.10
38	Iron and steel, n.e.c.	.35	45	Petroleum and natural gas	.07
54	Lumber and timber products	.35	18	Slaughtering and meat packing	.05
23	Blast furnaces	.35	19	Manufactured dairy products	.03
50	Manufactured gas	.35	47	Anthracite coal	.03
25	Iron and steel foundry products	.34	51	Communications	.01
24	Steel works and rolling mills	.34			
41	Aluminum products	.33			
62	Other textile products	.33			

TABLE 21[1]

Ratio of Inventories of Raw Materials and Supplies Held by the Consuming Industry and Its Annual Flow of Storable Inputs, 1939

Ind. No.	Industry		Ind. No.	Industry	
27	Firearms	2.73	44	Nonmetallic mineral manufactures	.23
32	Aircraft	1.09	53	Chemicals	.22
52	Electric public utilities	.85	62	Other textile products	.21
73	Steam railroad transportation	.72	54	Lumber and timber products	.19
36	Merchandising and service machines	.61	65	Leather products, n.e.c.	.19
35	Machine tools	.58	14	Starch and glucose products	.19
9	Fishing	.58	56	Wood pulp, paper, and paper products	.19
30	Engines and turbines	.49	66	Rubber products	.18
68, 69	Construction	.47	64	Leather shoes	.17
34	Industrial and household equipment, n.e.c.	.45	17	Tobacco manufactures	.17
33	Transportation equipment, n.e.c.	.41	59	Silk and rayon products	.16
29	Agricultural machinery	.39	20	Edible fats and oils, n.e.c.	.16
28	Munitions	.36	10	Flour and grist mill products	.15
76-82, 86-91	Other services	.34	61	Clothing	.13
37	Electrical equipment, n.e.c.	.33	49	Coke and manufactured solid fuel	.12
63	Leather	.33	11	Canning and preserving	.12
60	Woolen and worsted manufactures	.32	31	Automobiles	.12
67	Industries, n.e.c.	.32	16	Nonalcoholic beverages	.12
25	Iron and steel foundry products	.30	21	Other food products	.11
58	Cotton yarn and cloth	.30	40	Smelting and refining of nonferrous metals	.11
24	Steel works and rolling mills	.28	13	Sugar refining	.09
38	Iron and steel, n.e.c.	.28	12	Bread and bakery products.	.09
57	Printing and publishing	.27	46	Petroleum refining	.08
50	Manufactured gas	.27	70	Transportation, n.e.c.	.05
41	Aluminum products	.27	71	Coastwise and inland water transportation	.04
26	Shipbuilding	.26	19	Manufactured dairy products	.02
23	Blast furnaces	.26	18	Slaughtering and meat packing	.02
55	Furniture and other manufactures of wood	.26			
15	Alcoholic beverages	.25			
42	Nonferrous metal manufactures and alloys	.23			

[1] Industries not noted on this table were those for which data on stocks held were not available.

leum is an expensive material to store, as it is a liquid and subject to fire and explosion. In the coke industry the inventory coefficient is low. The explanation appears to lie in the fact that there is a high degree of integration between coal mines and coke ovens and that the cost per ton of coal storage may be higher for the coke industry than for railways and utilities.[27]

It is to be expected that inventory ratios would be low in the mining industries. In all of them except iron mining, explosives, which are dangerous and costly to store, are a major input.

The textile industries vary. Cotton yarn and cloth does not itself hold high inventories, but large stocks are held for it by other industries, as cotton is seasonally produced and cheaply stored. Wool is similar, but since much of it is imported, most of the stocks are held outside the country and its inventory ratio is rather low. The rayon and silk industry, which consumes rayon primarily, has a fairly low inventory ratio because rayon is a manufactured commodity whose output can be easily adjusted to its consumption.

Most of the other industries which have not been specifically mentioned have inventory-storable input ratios that fall within what may be called the 'normal' range—20 to 40 per cent.

This brief discussion of the variations in inventory coefficients, while far from a complete explanation, indicates that much of the variation is determined by factors which are subject to quantitative analysis, such as seasonality of production, costs of storage, and the relative size of stocks of goods-in-process.

No attempt will be made here to draw any general conclusions from this simple analysis of the capital and inventory coefficients. It is hoped that the material presented may be of interest particularly to those whose interests lie in the broader fields of economics and industry. The rankings of the coefficients indicate that certain patterns appear to characterize the capital and inventory structure of the American economy, patterns which seem not inconsistent with other economic information.

Of considerable interest is the dominance of a few industrial groups in both the use and supply of fixed capital. Further, with few exceptions, the heavy users of capital are not industries engaged in manufacturing or mining. Further analysis would require considerably more investigation and detailed industrial and technical data.

[27] American Engineering Council, *Industrial Coal, Purchase, Delivery and Storage*, 1925, pp. 27, 320-21.

Chapter 7

THE TELEPHONE INDUSTRY: A STUDY IN PRIVATE INVESTMENT

Paul G. Clark

I. Introduction

The study presented here is an attempt to test a simple theory of private investment, called the capital-requirements theory, against the investment practice in the telephone industry. The theory has been derived in a form suitable for integrating investment and disinvestment in capital equipment into interindustry input-output analysis. Therefore, although the study also has more general implications, in the context of this volume it is significant primarily as an empirical investigation of the validity of a theory suitable for this purpose.

The course of discussion will be as follows. In Section II the capital-requirements theory is developed as a modification of the pure acceleration principle. In Section III the procedures currently followed in reaching investment decisions in the Bell System are outlined, and the theory is tested qualitatively against these procedures. In Section IV the coefficients of the capital requirements theory are calculated statistically from data for the telephone industry, and the theory is tested quantitatively by evaluating the reasonableness of the statistical results. In Section V certain conclusions are drawn as to the empirical validity of the capital-requirements theory, and as to its usefulness in input-output analysis.

II. The Capital-Requirements Theory of Private Investment

A. The Pure Acceleration Principle

A distinctive strand in the theoretical analysis of private investment is the acceleration principle. The essential assumption of the principle in its pure form is that the firm must maintain for technological reasons a fixed ratio between its output and its stock of capital equipment. It follows from this assumption that the firm must undertake changes in its stock of capital equipment, i.e. must undertake net investment, in ac-

cordance with changes in its output. The principle in this pure form can be stated algebraically as follows:

$$K_t = kO_t \tag{7, 1}$$

$$\Delta K_t = k \, \Delta O_t \tag{7, 2}$$

in which

O_t = the firm's output in time period t,
K_t = the firm's stock of capital equipment at the end of the period,
ΔO_t = the difference between the firm's output in t and its output in $t - 1$,
ΔK_t = the difference between the firm's stock of capital equipment at the end of t and its stock at the end of $t - 1$, and
k = the fixed technological ratio which the firm is assumed to maintain between its output and its stock of capital equipment.

Both theoretical considerations and empirical tests, however, suggest that the pure acceleration principle is subject to several important qualifications as a realistic theory of private investment.

First, the pure acceleration principle implies that the firm is able to adjust its stock of capital equipment instantaneously to increases in demand for its output, but in practice a construction period of some length must usually intervene between the decision to purchase additional capital equipment and its actual installation. The firm might react to the time required for construction in either of two ways: by postponing investment decisions until additional demand actually materializes, thus postponing its ability adequately to meet the demand until after the construction period; or by making investment decisions on the basis of its expectations of what the demand will be after the construction period, thus beginning the construction process in advance of expected changes in demand. The latter reaction is assumed in the capital-requirements theory.

Second, the pure acceleration principle implies that the firm adjusts the capacity of its stock of capital equipment precisely to its output, but in practice a margin of spare capacity is commonly provided. One reason for maintaining some spare capacity is that demand is expected to fluctuate, and that the firm wishes to be able to handle temporary increases in demand without undue delay. This motive is particularly strong in cases in which the essential assumption of the pure acceleration principle is most realistic—cases in which the technological relation between capacity and capital equipment is quite rigid. Another reason for maintaining some spare capacity is that demand is expected to follow an upward secular trend, and that there are economies in making a small number of large installations of capital equipment rather than a large number of

small installations. The significance of the margin of spare capacity depends, of course, on whether it is regularly varied, or is kept at some normal level. The latter practice is assumed in the capital-requirements theory, and means that the ratio between the firm's output and its stock of capital equipment depends on the firm's decision to maintain a normal margin of spare capacity, as well as on the technological relation between capacity and capital equipment.

Third, the pure acceleration principle is perfectly symmetrical with respect to increases and decreases in output, but actually the firm frequently is unable to react in the same way to decreases in demand for its output as to increases. If demand rises, the firm can usually expand its stock of capital equipment as rapidly as the fixed ratio to output requires, although at times it may be restricted by supply shortages. If demand falls, on the other hand, the firm frequently cannot contract its stock of capital equipment rapidly enough to maintain a fixed ratio to output, since it can contract only as fast as retirement because of wear and tear and obsolescence permits. The firm might, of course, simply junk the equipment, or sell it at distress prices, but this acceptance of capital loss is usually not worth while, since the cost of retaining idle capital equipment is usually small relative to possible future returns if and when it can be used again. If demand falls, therefore, the firm frequently cannot make its net investment as negative as the pure acceleration principle asserts, but instead accumulates excess capacity. Moreover, if demand then rises, the firm need not make any positive net investment until this excess capacity has again been brought into use. The pure acceleration principle should be modified to take account of this asymmetry in the firm's reaction to increases and decreases in demand.

Fourth and finally, the assumed ratio between the firm's output and its stock of capital equipment is changed from time to time by technological developments. This qualification does not destroy the usefulness of the principle, provided that the technological changes are discontinuous, and that the ratio remains stable from one technological change to the next. This qualification does suggest two modifications in the form of the principle, however. (1) Since a technological change typically leads the firm to modify its purchases of new capital equipment, but not immediately to scrap and replace all its obsolescent plant, the principle should be stated in terms of ratios between changes in the firm's output and changes in its stocks of capital equipment, rather than in terms of ratios between the firm's total output and its total stocks. (2) Since a technological change directly affects a particular type of capital equipment, the principle should be stated in terms of a set of ratios, one for each of the various types of capital equipment.

B. THE CAPITAL-REQUIREMENTS THEORY

The acceleration principle in its pure form is thus not sufficiently realistic to warrant confidence. If it is modified to make allowance for the four qualifications just discussed, however, it can provide the basis for a more realistic, yet still simple, theory of private investment the capital-requirements theory.

The first modification of the pure acceleration principle introduces the firm's expectations of future demand. The firm is conceived to adjust its stock of capital equipment to the demand which it expects to prevail after the construction period. Algebraically, equation (7, 1) becomes:

$$K_{t+1} = kO^e{}_{t+1} \qquad (7, 3)$$

and equation (7, 2) becomes:

$$\Delta K_{t+1} = kO^e{}_{t+1} - K_t \qquad (7, 4)$$

in which

$O^e{}_{t+1}$ = the expected output in the period $t + 1$, and the length of the period is the time required for construction.

Once the firm's demand expectations have been introduced into the theory, however, it is necessary to find a relation between these subjective expectations and some measurable objective variables. Here a variant of Metzler's expectations coefficient may serve as a simple hypothesis. The expectations coefficient is defined as the ratio between the expected proportional change in output from the current period to the next period, and the actual proportional change from the last period to the current period. Thus the expected output in the next period equals the output in the current period, plus a positive or negative increment which depends upon the expectations coefficient and the actual proportional change from the last period to the current period. Algebraically,

$$\epsilon = \frac{O^e{}_{t+1} - O_t}{O_t} \bigg/ \frac{O_t - O_{t-1}}{O_{t-1}} \qquad (7, 5)$$

$$O^e{}_{t+1} = O_t\left[1 + \epsilon\left(\frac{O_t - O_{t-1}}{O_{t-1}}\right)\right] \qquad (7, 6)$$

in which

ϵ = the expectations coefficient.

Note that if the coefficient is 0, future output is expected to be the same as current output; if the coefficient is +1, the current proportional trend is expected to continue; if the coefficient is −1, the current proportional trend is expected to reverse itself.

The question may arise how the effect of uncertainty of the future may be recognized in this treatment of expectations. On the assumption that a firm becomes less willing to undertake investment if the uncertainty surrounding future yields increases, the effect of greater uncertainty can be represented by a smaller expectations coefficient for future output, so that a given increase in demand from the last period to the current period leads the firm to undertake less net investment than it would if the future were more certain.

The second modification of the pure acceleration principle provides for a normal margin of spare capacity. One fixed ratio, or capital coefficient, depending on technological considerations, is assumed to exist between the stock of capital equipment and its capacity. Another fixed ratio, or spare-capacity coefficient, depending on the firm's decision to maintain a normal margin of spare capacity, is assumed to exist between the capacity of the capital equipment and the output which the firm expects to produce. Algebraically, equation (7, 3) is modified to read

$$K_{t+1} = \beta C_{t+1} \tag{7, 7}$$

and

$$C_{t+1} = \gamma O^e{}_{t+1}, \qquad \gamma > 1 \tag{7, 8}$$

in which

β = the capital coefficient,
γ = the spare-capacity coefficient, and
C_{t+1} = the capacity of the stock of capital equipment, defined in the same units as output.

Equation (7, 4) is modified to read

$$\Delta K_{t+1} = \beta\gamma O^e{}_{t+1} - K_t = \beta\gamma(O^e{}_{t+1} - \frac{1}{\gamma} C_t) \tag{7, 9}$$

The third modification of the pure acceleration principle recognizes the asymmetry between the firm's reaction to a decrease in expected output and its reaction to an increase. This is accomplished by assuming that the firm's net investment cannot be more negative than its retirement of capital equipment because of wear and tear and obsolescence, and that such retirement is technologically determined as a fixed fraction of its stock of capital equipment. In the algebraic statement, equation (7, 9) is subjected to the limitation that

$$\Delta K_{t+1} \geq -R_{t+1} \tag{7, 10}$$

in which

$-R_{t+1}$ = the firm's retirement, conceived as a negative flow.

It should be noted that if this limitation becomes operative in any period,

the firm will be left with a capacity relative to its expected output greater than that indicated by the spare-capacity coefficient. If expected output subsequently rises, positive net investment will be undertaken only to the extent that expected output exceeds the normal fraction of capacity indicated by the spare-capacity coefficient. The firm's retirement in turn is determined by

$$-R_{t+1} = -\rho K_t \tag{7, 11}$$

in which

$-\rho$ = the retirement coefficient.

This assumption regarding retirement is a simple one, subject to a number of qualifications in practice. Retirement due to wear and tear is determined most rigidly for capital equipment with a well-defined service life. Even with a well-defined service life, however, retirement depends on the age distribution of the capital equipment, and on the degree of past utilization of its capacity, as well as on the current stock. On the other hand, the more durable the capital equipment the more variable its service life, until some types, given proper maintenance and spare-parts replacement, are essentially immortal. Finally, retirement due to obsolescence depends in part on the firm's policy regarding the rate of introduction of technological improvements. Thus a complete theory would have to introduce other variables affecting retirement in addition to the stock of capital equipment. Nonetheless in the capital-requirements theory a fixed retirement coefficient has been assumed, partly because retirement enters the theory primarily as a limitation on negative net investment, and partly because it seems worth while to test the simple assumption before developing a more complex one.

The fourth modification of the pure acceleration principle, suggested by the necessity of admitting technological change into the theory, defines the capital and spare-capacity coefficients as incremental coefficients, relating changes in expected output, capacity, and capital stock, rather than as average coefficients, relating total expected output, capacity, and capital stock. In other words, the capital-requirements theory is stated in terms of equation (7, 9), rather than in terms of equations (7, 7) and (7, 8). When a technological change occurs, the firm introduces it by modifying its purchases of new capital equipment of the various types; its net investment as determined by equation (7, 9) is modified by abrupt changes in β and γ. The firm does not immediately scrap and replace all its obsolescent equipment, however; its total stock of equipment and total capacity are not determined immediately by equations (7, 7) and (7, 8) using the changed β and γ, but only gradually as all its obsolescent equipment is replaced.

The necessity of recognizing technological change suggests another

modification which defines the capital, spare-capacity, and retirement coefficients so that they apply to particular types of capital equipment, rather than to all types lumped together. Algebraically, all the coefficients and stocks (but not expected output) in equations (7, 9), (7, 10), and (7, 11) are rewritten to read:

$$\Delta K^a{}_{t+1} = \beta^a \gamma^a (O^e{}_{t+1} - \frac{1}{\gamma^a} C^a{}_t), \quad \Delta K^a{}_{t+1} \geq -R^a{}_{t+1} \qquad (7, 12)$$

$$-R^a{}_{t+1} = -\rho^a K^a{}_t \qquad (7, 13)$$

in which the superscript

a refers to a particular type of capital equipment, and similar superscripts are used for other types.

Here a definitional problem is raised. If the types are defined narrowly, as physically identical units, then a technological change must be conceived as the introduction of an entirely new set of coefficients. If the types are defined broadly, however, as physically different units performing a similar function (which requires that they be measured in terms of value), then a technological change can be conceived as the alteration of the old set of coefficients. This broader definition is adopted here.

The capital-requirements theory as now formulated may be summarized in the following basic hypotheses, which are to be tested against actual practice in the telephone industry: (1) The firm's demand estimates depend via a fixed expectations coefficient upon the recent trend of its output. (2) The firm's net investment depends via a fixed capital coefficient and a fixed spare-capacity coefficient upon these demand estimates. (3) The firm's retirement depends via a fixed retirement coefficient upon its present stock of capital equipment. The second and third hypotheses are conceived to apply to each particular type of capital equipment. In the algebraic statement, these basic hypotheses are

$$O^e{}_{t+1} = O_t \left[1 + \epsilon \left(\frac{O_t - O_{t-1}}{O_{t-1}} \right) \right] \qquad (7, 6)$$

$$\Delta K^a{}_{t+1} = \beta^a \gamma^a (O^e{}_{t+1} - \frac{1}{\gamma^a} C^a{}_t), \quad \Delta K^a{}_{t+1} \geq -R^a{}_{t+1} \qquad (7, 12)$$

$$-R^a{}_{t+1} = -\rho^a K^a{}_t \qquad (7, 13)$$

C. THE RELATION OF THE CAPITAL-REQUIREMENTS THEORY TO A MORE GENERAL THEORY

The capital-requirements theory, although more complex than the pure acceleration principle, is still quite simple. What then is the relation

between this simple theory and a more general theory, based on the profit-maximizing equilibrium of the firm? Without going into detailed analysis, the question can be answered generally by indicating the principal alternative assumptions which are made in the two theories.

A more general theory usually assumes that in all markets the firm can sell, buy, and borrow either unlimited quantities at going prices, or varying quantities at varying prices as indicated by sloping supply and demand curves. On the other hand, the capital-requirements theory assumes that in the market for output the firm can sell only a limited quantity at the going price, and that the price is accepted as a stable parameter of its decisions. In other words, a sloping demand curve exists, but because of conventional price-setting, it can be represented simply by the quantity demanded. The theory assumes as before that in the markets for capital equipment, for other inputs, and for capital funds the firm can buy and borrow unlimited quantities at going prices, or at rising prices as indicated by sloping supply curves; and in addition that these prices or supply curves are so related that it is profitable to purchase whatever amount of capital equipment is required to produce the output which the firm can sell.

The production function assumed in the more general theory usually contains continuous decreasing marginal rates of substitution among inputs for given outputs, and among outputs for given inputs; and continuous decreasing marginal rates of transformation between input combinations and output combinations, i.e. decreasing returns to scale. In the capital-requirements theory, however, the production function contains fixed coefficients relating the firm's output to the stocks of various types of capital equipment in use, and hence relating its capacity to its total existing stocks. In addition, the firm as a matter of policy is supposed to maintain normal margins of spare capacity as fixed fractions of total capacity in the various types of equipment.

Although both theories adopt the hypothesis that the firm seeks to maximize its profit, the consequence of profit maximization differs. In a more general theory, profit maximization consists in purchasing additional units of capital equipment up to the point at which the expected profit from the marginal unit falls to zero. In the capital-requirements theory, on the other hand, profit maximization consists in purchasing additional units of capital equipment up to the point at which the sum of the additional outputs which they can produce equals the expected total increase in output in the next period. The expected profit from the marginal unit remains positive, but if the firm purchased still more units, their output could not be sold in the next period.

The particular assumptions of the capital-requirements theory also mean that changes in the rate of interest and in the prices of output, of

capital equipment, and of other inputs, have no effect upon the firm's investment. There can be no substitution effect so long as the production function contains fixed coefficients relating the firm's output to the stocks of various types of capital equipment in use, and the firm maintains normal margins of spare capacity. There can be no expansion effect so long as the firm's output is limited by a sloping demand curve and a conventional price; a change in any of these prices causes a change in profit, but not in investment.

It may be noted at this point that in two respects the capital-requirements theory is actually more complete than the more general theory as often formulated. It attempts to explain both the formation of expectations regarding future demand and the determination of retirement. The simpler capital-requirements theory also has certain advantages for empirical investigations. To begin with, it suggests a smaller number of relevant economic factors. In questioning businessmen about their investment decisions, a smaller number of variables reduces the problem of multi-collinearity. The capital-requirements theory also specifies linear relationships among its variables, which eases the statistical analysis. Most important of all, a number of existing empirical investigations of private investment have indicated that the relevant economic factors suggested by the capital-requirements theory are of major importance, while the additional factors suggested by a more general theory can be readily neglected.

III. A Qualitative Analysis of Telephone Investment

A. THE METHOD

The capital-requirements theory is tested in the course of this study of telephone investment in two different ways. In this section it is tested qualitatively by examining the procedures followed in reaching investment decisions in the Bell System, and judging whether these procedures can be approximated by the fixed coefficients of the theory. In the next section a quantitative test is made by calculating coefficients from statistical data for the entire industry, and evaluating the reasonableness of the computed coefficients.

The importance of the qualitative analysis presented in this section should be emphasized. Not only is it a direct observation of investment procedures which can be compared in its own right with the procedures envisioned in the capital-requirements theory, but also it provides a basis for the subsequent quantitative analysis. The procedural information serves as a guide for various details of the statistical calculations, and the chronology of technological developments in the industry serves as a standard for evaluating the statistical results.

The information for this qualitative analysis was obtained primarily in interviews with members of the staff of the American Telephone and Telegraph Company.[1] An attempt was made to avoid some of the common shortcomings of the questioning technique by relying upon interviews rather than questionnaires, so that ambiguous questions and answers could be re-explored; and by placing the emphasis upon procedures rather than motives, which is feasible in a large institutionalized organization. It may be, however, that the resulting qualitative description lays insufficient stress upon the judgment used in weighing less tangible considerations than those discussed here. The final statement, of course, is my own responsibility.

B. DEMAND ESTIMATES IN THE BELL SYSTEM

The first stage of the investment process in the Bell System consists in estimating future demand. These demand estimates are made by the commercial departments of the associated operating companies, and by corresponding personnel in the American company's long-lines operating department, with procedural assistance from the staff of the American company.

The basic demand estimates are stated in terms of three interrelated variables—the number of lines, the number of main telephones, and the number of total telephones. These three variables are in ascending order of size, the number of main telephones being larger than the number of lines because it includes main telephones on multi-party lines, and the number of total telephones being larger than the number of main telephones because it includes extension telephones. Therefore once one of the variables is estimated independently, the others can be calculated from it by the use of appropriate ratios. The common practice is to take the number of main telephones as the independent estimate. The ratios used in calculating the other two variables are obtained largely from past records, but modified by expected future changes. Thus they vary somewhat from year to year, and also between exchanges. From these basic estimates of lines, main telephones, and total telephones, more detailed estimates, broken down among all the various classes of telephone service, are derived by the use of still other ratios. In small-town exchanges,

[1] Published materials which relate to investment procedures in the Bell System, and which were also consulted, include Federal Communications Commission, *Investigation of the Telephone Industry in the United States*, House Document #340, 76th Cong., 1st Sess., Government Printing Office, Washington, 1939. Ibid *Special Investigation Docket* #1, 1936, Exhibit #135, 'Long-Lines Department, Financial and Operating Summary'; Exhibit #580, 'Long Lines Department, Property Not Used and Useful'; Exhibit #2096G, 'Effect of Control upon Telephone Service and Rates' American Telephone and Telegraph Company, *Comments on the Telephone Investigation*, 1937, #31, 'Comments on Exhibit #135'; #32, 'Comments on Exhibit #580.' Ibid. *Annual Reports*, 1913-48.

for example, the number of business telephones is often calculated in this way from the estimated number of residence telephones. In metropolitan areas, on the other hand, the number of business telephones is estimated directly.

The complete demand estimates are of two types, depending upon the use to which they are put. The first type, the general planning estimates, are usually made for a period of three years in advance, although the period may be shortened or lengthened according to the confidence which the analysts place in the expected future trends. They are made annually, and reviewed and modified quarterly, or whenever a more detailed special-project estimate has been made. Their main purpose is to serve as a rough guide in determining when an expansion of capacity in a particular exchange will be required. The second type, the special-project estimates, are made whenever an expansion of capacity in a particular exchange is planned in detail. They are more elaborate than the general planning estimates, and are fitted to the particular projects under consideration. Their chief purpose is to help determine, in conjunction with the policy of building ahead of demand, how much capital equipment is required; and hence the number of years for which demand is forecast depends upon the period of building in advance which is applied to that particular type of capital equipment.

The basic technique applied in making all of these demand estimates is a field survey of each individual exchange. The commercial departments of the associated companies have staffs of 'development engineers,' each of whom is responsible for making the periodic demand estimates for certain exchanges. The development engineer's analysis is usually based on a sample of typical streets or blocks, and involves the consideration of two factors. He must forecast the number of new families. This part of his analysis is usually based on existing or prospective building programs. He must also forecast the increase in 'development,' or percentage of families with telephones. This part of his analysis is usually based on prospective personal incomes of individuals and prospective business conditions, plus such less tangible considerations as changes in telephone habits, e.g. changes due to servicemen's wartime experience with long-distance calls.

Demand estimates formed in this way obviously contain large subjective elements. Their reliability depends on the reliability of the development engineer's judgment regarding future building programs, personal incomes, and business conditions: and also on the reliability of his judgment upon the relations which exist between these factors and the demand for telephone service. The advisory staff of the American company recognize this fact, but feel that subjective methods of this sort are the most dependable methods available. Therefore, although they suggest that the

development engineer make his assumptions and his line of reasoning explicit, so that his estimate can be modified if either is later found to be in error, they encourage him to rely primarily upon his own judgment, using statistical trends and correlations as only one element to be considered. Rough consistency of the assumptions and reasoning applied in the various exchanges is obtained in the course of consultations among the development engineers. There is also some evidence that a general feeling of optimism regarding future expansion of demand for telephone service has prevailed, which probably has affected these subjective estimates.[2] Such optimism is of course natural in view of the steady increase in number of telephones actually experienced.

Although the basic technique used in making demand estimates is the field survey, statistical methods are used to supplement this technique. Statistical projections for the entire area of an associated company are frequently used as a check on the totals emerging from forecasts in the individual exchanges. Economic statisticians on the staff of the American company, besides providing various data, also periodically make general studies of business conditions, and general statistical projections for the entire Bell System. These are circulated and discussed with personnel of the associated companies.

Moreover, a technique based upon trend extrapolation was used by the long-lines department in its demand estimates made in the fall of 1930. The American company stated in reply to criticism of these estimates[3] that five factors had been considered in making them: (1) Long-lines messages had had an average annual rate of growth of 13.5 per cent in the period 1915-29. (2) They had had an annual rate of growth of 18 per cent in 1928 and 1929. (3) In the first half of 1930 they had been 6.5 per cent above the first half of 1929, and in the last half of 1930 they had been only slightly below the last half of 1929. (4) In the 1921 depression they had merely failed to increase over 1920, and in 1922 they had increased 17 per cent over 1921. (5) A majority of economists and business forecasters had predicted recovery in the first half of 1931, which would have made the 1929 depression similar to the 1921 depression. The company presented the data from which these considerations were derived in the form of a semi-logarithmic chart, upon which were plotted index numbers on a 1910 base of long-lines toll traffic, local stations, local traffic, national production, and national population, all compared to exponential reference trends of 1 per cent up to 10 per cent. The company also stated that

[2] American Telephone and Telegraph Company, *Annual Reports*, 1919, p. 43; 1925, p. 8; 1935, pp. 10-11.

[3] Federal Communications Commission, *Docket*, Exhibit #580, 'Long-Lines Department, Property Not Used and Useful,' pp. 11-14.

American Telephone and Telegraph Company, *Comments*, #32, 'Comments on Exhibit #580,' pp. 16-24.

the pattern of the past had apparently re-established itself after the depression, for long-lines messages had increased 20 per cent in 1936 over 1935, and 29 per cent in 1937 (based on seven months' experience) over 1936.

In summary, demand estimates in the Bell System are based primarily upon field studies in each individual exchange, i.e. upon subjective methods, supplemented and checked by statistical calculations. Representing the formation of these estimates by a simple device like the expectations coefficient, as in the capital-requirements theory, is only an approximation. Nonetheless, a natural tendency for subjective forecasts to be influenced by current trends, plus the use of supplementary statistical analysis relying explicitly upon trend extrapolation, suggest that the expectations coefficient is a useful approximation. To this extent the first basic hypothesis of the capital-requirements theory—that demand estimates depend via a fixed expectations coefficient upon the recent trend of output—is supported by the Bell System procedures.

C. DETERMINATION OF REQUIRED CAPITAL EQUIPMENT IN THE BELL SYSTEM

The second stage of the investment process in the Bell System consists in determining the additional amounts of capital equipment required to handle the estimated demand. These calculations are handled by the engineering departments of the associated companies and corresponding personnel in the long-lines department, again with procedural assistance from the staff of the American company.

One feature of the engineering is that a new installation of capital equipment must be decided upon a certain period of time before it is needed in service, because of the time required to engineer, manufacture, and install new capital equipment. The construction period varies according to the size of the job, but it is now roughly a year for automatic switchboards, six months for manual switchboards, nine months for repeaters (amplifiers which restore the strength of the signal over long lines), three months for new cable on an existing pole line, and two years for a major new pole line requiring a new right-of-way. The necessity of allowing for these construction periods is in keeping with the capital-requirements theory.

Another, and quite important, feature of the engineering stage is the practice of building ahead of demand—of providing in each new installation of capital equipment a certain margin of spare capacity. This is accomplished by taking the demand estimate for several years in the future as the measure of the capacity which is to be provided in the new installation. The appropriate period of building ahead of demand varies from one type of capital equipment to another, the criterion in each case

being the relative cost of a single installation with large capacity and of a succession of installations with small capacity. Detailed studies are made in applying this criterion, and for each type of equipment a period of building in advance is selected which gives the lowest present value of the expected costs, discounted at a rate of 7 per cent.

The periods currently in use are approximately as follows: In the case of open-wire pole lines, poles are usually erected of sufficient size to carry the expected number of crossarms and wires 20 to 25 years in the future, and then the crossarms and wires are added to keep up with current demand. Underground conduit is also normally placed to provide capacity adequate for 20 to 25 years. In long toll cables, a sufficient number of conductors is usually provided to handle the expected traffic 15 to 20 years in the future, and then the carrier systems which are needed to convert these conductors into usable circuits are provided only 1 to 2 years in advance. Central office equipment is normally installed with a capacity adequate for 1 to 3 years; central office buildings are normally erected with sufficient floor space to house the expected expansion of equipment during the next 4 to 8 years; and when a new building site is acquired, land may be purchased for the anticipated building needs 20 or more years ahead.

This practice of building ahead of demand suggests that the concept of the spare-capacity coefficient adopted in the capital-requirements theory is reasonably applicable to the telephone industry. It should be remembered that the spare-capacity coefficient is defined as a marginal coefficient, relating changes in capacity to changes in expected output, and not as an average coefficient, relating total capacity and total expected output. New investment in a particular type of equipment is undertaken whenever demand in the near future, making allowance for the construction period, is expected to exceed current total capacity. This capacity was installed at some time in the past on the basis of demand estimates made then, rather than on the basis of demand estimates in the immediately preceding period, as indicated in the capital-requirements theory. When new investment is undertaken, a margin of spare capacity, measured by the period of building ahead of demand, is provided. With a fixed number of years of building in advance, the amount of additional capacity provided is proportional to the expected annual increase in output, and hence its determination can be represented by a fixed marginal spare-capacity coefficient. The total amount of spare capacity in this type of equipment then gradually diminishes as the expected future demand materializes, and the next new investment is required only when demand is again expected to exceed total capacity in the near future.

Once the desired increase in capacity has been determined, the amount

of additional capital equipment required to provide this increase in capacity must be determined. The procedure varies according to the type of equipment under consideration.

The required number of additional telephone instruments, to begin with, can be determined almost directly from the estimated increase in total telephones, the third item in the basic demand estimates. The ratio is somewhat higher than 1:1, however, because the required stock of instruments includes a certain number to absorb variations in the flow of connects and disconnects.

The required number of additional pairs of conductors in subscriber cables and open wires (i.e. those which connect subscribers' telephones to the central office of an exchange) can be determined similarly from the estimated increase in lines, the first item in the basic demand estimates. The ratio here is considerably higher than 1:1, however, because of the impossibility of filling each cable exactly to the 100 per cent point. The fill is of course better in heavily populated than in lightly populated areas, but a 75 per cent fill is considered a satisfactory objective.

The procedure by which the engineer determines the required number of additional circuits in trunk cables and open wires (i.e. those which connect the central office of one exchange to the central office of another exchange) is more complex. The calculation is further complicated by the fact that cables between two exchanges frequently include circuits which connect still other pairs of exchanges. The following discussion refers, for simplicity, only to circuits required to handle traffic between the two terminal exchanges. In this simple case the engineer uses, in essence, two ratios: (1) The average number of busy-hour calls per main telephone (counting only those from one exchange to the other) is determined from the records of the traffic department, modified by expected changes. Note that capacity is provided to handle the traffic during the busy hours of the average day of the busy month, and therefore that at other times there is necessarily some spare capacity. (2) The average holding time per call, in seconds, is likewise determined from the records of the traffic department, modified by any expected changes. Using these two ratios and the future demand estimates for the two exchanges under consideration, the engineer can compute the traffic which the trunk cables and open wires will have to carry, in terms of hundreds of call-seconds, or CCS, by means of the following formula:

$$\begin{bmatrix}\text{Estimated}\\ \text{Number}\\ \text{of Main}\\ \text{Telephones}\end{bmatrix} \times \begin{bmatrix}\text{Average Number}\\ \text{of Busy-Hour}\\ \text{Calls per Main}\\ \text{Telephone}\end{bmatrix} \times \begin{bmatrix}\text{Average}\\ \text{Holding}\\ \text{Time}\\ \text{per Call}\end{bmatrix} \div (100) = \begin{bmatrix}\text{Traffic}\\ \text{in CCS}\end{bmatrix}$$

TABLE 1

TRUNK CAPACITY TABLES

Hundred Call-Seconds and Per Cent. Use at Various Service Levels for Trunk Groups of 1 to 50

The Table Number Indicates the Number of Calls per 1000 Encountering All Trunks Busy

Trunks	Table 1		Table 5		Table 10		Table 20		Table 30		Table 40		Trunks
	CCS	% Usage	CCS	% Usage	CCS	% Usage	CCS	% Usage	CCS	% Usage	CCS	% Usage	
1	.1	.1	.2	.5	.4	1.0	.7	2.0	1.1	3.1	1.5	4.2	1
2	1.6	2.3	3.7	4.8	5.4	7.5	7.9	10.9	9.7	13.5	11.3	15.7	2
3	6.9	6.4	12.2	11.3	15.7	14.5	20.4	18.9	24.0	22.2	26.9	24.9	3
4	15.4	10.7	24.2	16.8	29.6	20.6	36.7	25.5	41.6	28.9	45.7	31.8	4
5	26.6	14.8	38.9	21.6	46.1	25.6	55.8	31.0	61.6	34.2	66.6	37.0	5
6	40.0	18.5	55.4	25.7	64.4	29.8	76.0	35.2	82.8	38.3	89.3	41.3	6
7	54.7	21.7	73.4	29.1	83.9	33.3	96.8	38.4	105	41.7	112	44.4	7
8	70.9	24.6	92.5	32.1	105	36.5	119	41.3	129	44.8	137	47.6	8
9	88.2	27.2	113	34.9	126	38.9	142	43.8	153	47.2	162	50.0	9
10	107	29.7	134	37.2	149	41.4	166	46.1	178	49.4	188	52.2	10
11	126	31.8	156	39.4	172	43.4	191	48.2	204	51.5	214	54.0	11
12	145	33.6	178	41.2	195	45.1	216	50.0	230	53.3	240	55.6	12
13	166	35.5	201	42.9	220	47.0	241	51.5	256	54.7	267	57.1	13
14	187	37.1	224	44.4	244	48.4	267	53.0	283	56.1	295	58.5	14
15	208	38.5	248	45.9	269	49.8	293	54.3	310	57.4	322	59.6	15
16	231	40.0	273	47.4	294	51.1	320	55.6	337	58.5	350	60.8	16
17	253	41.3	297	48.5	320	52.3	347	56.7	365	59.6	378	61.8	17
18	276	42.6	322	49.7	346	53.4	374	57.7	392	60.5	407	62.8	18
19	299	43.7	347	50.7	373	54.5	401	58.6	420	61.4	436	63.8	19
20	323	44.9	373	51.8	399	55.4	429	59.6	449	62.4	465	64.6	20
21	346	45.8	399	52.8	426	56.4	458	60.6	478	63.2	494	65.3	21
22	370	46.7	424	53.5	453	57.2	486	61.4	507	64.0	523	66.0	22
23	395	47.7	451	54.5	480	58.0	514	62.1	536	64.7	552	66.7	23
24	419	48.5	477	55.2	507	58.7	542	62.8	564	65.3	582	67.4	24
25	444	49.3	504	56.0	535	59.4	571	63.4	593	65.9	611	67.9	25

TABLE 1

TRUNK CAPACITY TABLES

The Table Number Indicates the Number of Calls per 1000 Encountering All Trunks Busy

Trunks	Table 1		Table 5		Table 10		Table 20		Table 30		Table 40		Trunks
	CCS	% Usage	CCS	% Usage	CCS	% Usage	CCS	% Usage	CCS	% Usage	CCS	% Usage	
26	469	50.1	531	56.7	562	60.1	599	64.0	623	66.6	641	68.5	26
27	495	50.9	558	57.4	590	60.7	627	64.5	652	67.1	671	68.9	27
28	520	51.6	585	58.0	618	61.3	656	65.1	682	67.7	701	69.6	28
29	545	52.2	612	58.6	647	62.0	685	65.6	711	68.1	731	70.0	29
30	571	52.9	640	59.3	675	62.5	715	66.2	741	68.6	762	70.6	30
31	597	53.5	667	59.8	703	63.0	744	66.7	771	69.1	792	71.0	31
32	624	54.2	695	60.3	732	63.6	773	67.1	801	69.5	822	71.4	32
33	650	54.7	723	60.8	760	64.0	803	67.6	831	69.9	852	71.7	33
34	676	55.2	751	61.4	789	64.5	832	68.0	861	70.3	883	72.1	34
35	703	55.8	779	61.8	818	64.9	862	68.4	891	70.7	913	72.5	35
36	729	56.3	807	62.3	847	65.4	892	68.8	922	71.1	944	72.8	36
37	756	56.8	836	62.8	876	65.8	922	69.2	952	71.5	975	73.2	37
38	783	57.3	864	63.2	905	66.2	952	69.6	982	71.8	1006	73.5	38
39	810	57.7	892	63.5	935	66.6	982	69.9	1013	72.1	1037	73.9	39
40	837	58.1	921	64.0	964	66.9	1012	70.3	1043	72.4	1069	74.2	40
41	865	58.6	950	64.4	993	67.3	1042	70.6	1074	72.8	1099	74.4	41
42	892	59.0	979	64.7	1023	67.7	1072	70.9	1104	73.0	1130	74.7	42
43	919	59.4	1008	65.1	1052	68.0	1103	71.3	1135	73.3	1161	75.0	43
44	947	59.8	1036	65.4	1082	68.3	1133	71.5	1166	73.6	1192	75.3	44
45	975	60.2	1066	65.8	1112	68.6	1164	71.8	1197	73.9	1223	75.5	45
46	1003	60.6	1095	66.1	1142	69.0	1194	72.1	1228	74.2	1255	75.8	46
47	1030	60.9	1124	66.4	1171	69.2	1225	72.4	1259	74.4	1286	76.0	47
48	1058	61.2	1153	66.7	1201	69.5	1255	72.6	1291	74.7	1317	76.2	48
49	1086	61.6	1183	67.1	1231	69.8	1286	72.9	1322	74.9	1349	76.5	49
50	1115	61.9	1212	67.3	1261	70.1	1317	73.2	1353	75.2	1381	76.7	50

The number of circuits required to carry this traffic can then be read off a set of trunk capacity tables, some of which are reproduced in Table 1. Each of these tables represents a particular level of availability of circuits, as measured by the number of calls per thousand which can be expected to encounter all circuits busy, on the assumption of random distribution of calls within the busy hours. To illustrate the use of these different tables, the availability of circuits is usually set lower for trunk cables in toll service than for trunk cables in local exchange service, so that the former will be in use a larger percentage of the time. Once the desired availability of circuits is specified, the engineer can read down the appropriate table to the required CCS, and there find the required number of circuits, as well as the percentage of time these circuits can be expected to be in use.

The required increase in central office equipment is determined differently according to whether the central office is equipped with manual switchboards or with automatic switchboards. In determining the required number of operator positions for a central office equipped with manual switchboards, the engineer uses in essence three ratios: (1) The average number of busy-hour calls per main telephone (counting all calls this time) is again determined from the records of the traffic department, modified by expected changes. (2) The number of work-units per call is determined from time studies of the work required to handle calls of different kinds, e.g. toll calls, local exchange calls, uncompleted calls. (3) The number of work-units per operator, i.e. the operator load, is determined for each central office on the basis of past experience and expected changes. The Bell System has a theoretical standard operator load, established in 1911, of 230 work-units in the busy hour, the standard being defined as the number of work-units which can be handled by a fully experienced operator of average ability while maintaining a satisfactory grade of service. In particular central offices, however, the actual load differs from the theoretical standard, primarily because of differing experience of the operators. Using these three ratios and the future demand estimate for the particular exchange under consideration, the required number of operator positions is computed by means of the following formula:

$$\begin{bmatrix}\text{Estimated}\\ \text{Number}\\ \text{of Main}\\ \text{Telephones}\end{bmatrix} \times \begin{bmatrix}\text{Average}\\ \text{Number of}\\ \text{Busy-Hour}\\ \text{Calls per}\\ \text{Main}\\ \text{Telephone}\end{bmatrix} \times \begin{bmatrix}\text{Number}\\ \text{of Work-}\\ \text{Units}\\ \text{per Call}\end{bmatrix} \div \begin{bmatrix}\text{Number}\\ \text{of Work-}\\ \text{Units}\\ \text{per}\\ \text{Operator}\end{bmatrix} = \begin{bmatrix}\text{Required}\\ \text{Number of}\\ \text{Operator}\\ \text{Positions}\end{bmatrix}$$

The determination of the required stock of automatic switchboards in a central office using the dial system is complicated by the fact that there are three main types of automatic switchboards, according to the type of switching mechanism employed step-by-step, panel, and crossbar. Each of these types consists of a group of more or less separate components—for example, in the step-by-step switchboard, line-finders, selectors, and connectors. For each of the components in each of these types, the engineer must determine the number required to meet the estimated demand. In each calculation, two ratios are used: (1) The average number of busy-hour calls per main telephone is determined as in the case of manual switchboards; (2) the average holding-time per call, either from the records of the traffic department, as before, or from engineering data, if the component is in use only part of the total time consumed in a call. Using these two ratios and the future demand estimate for the particular exchange under consideration, the traffic which the component will have to carry is estimated, in CCS, by the same formula used for trunk cables. Then the number of units of the component required to carry this traffic with a specified availability of circuits can be read off a trunk capacity table.

Through all these complexities, there exist certain fixed ratios between the increase in capacity desired in each particular type of capital equipment, and the number of additional units required to provide that increase. These ratios depend variously on spares for temporary replacements, on practicable cable fills, on the timing and duration of calls, on operators' experience, on random distribution of calls within the busy hour, and on the planned availability of circuits. They could in principle be calculated directly from the rather complicated engineering procedures, although because of lack of sufficiently complete data I have not been able to do this in the quantitative analysis below. In other words, the concept of the capital coefficient adopted in the capital-requirements theory is a close approximation to reality in the telephone industry.

The possibility of varying the number of CCS handled by a given stock of capital equipment by permitting the availability of circuits to vary should be noted. This possibility arises because the product of the telephone industry has more than one dimension, and these dimensions are to some extent substitutable for each other. The consequence of attempting to produce beyond capacity is not a rise in the marginal cost of an unchanged product, as in most manufacturing industries, but rather a change in one of the dimensions of the product. Specifically, if an automatic switchboard is made to handle a larger number of CCS in the busy hour than it was designed to handle, this is accomplished by keeping customers waiting a few moments until circuits are open, thus keeping the circuits in use more continuously. In the trunk capacity tables, this ap-

pears as a movement to the right, along the row for the existing number of trunks, to a table with a higher percentage of use, but also a higher number of calls per thousand encountering all trunks busy.

How consistent has been the practice in the Bell System of maintaining the planned level of availability of circuits? In time of war, clearly, it has been permitted to fall where necessary to the maximum practicable extent. Yet even in these circumstances the fall is limited technologically; in World War II maximum wartime capacity in central office equipment was only 5 per cent greater than peacetime capacity. The excess wartime demand has appeared to a large extent in a backlog of unfilled orders for service; the backlog has then been eliminated, and the availability of circuits raised again, in the course of postwar construction. This sequence of events can be traced during and after both World Wars I and II.[4]

To sum up, then, the engineering stage in the investment process in the Bell System approximates closely the second basic hypothesis of the capital-requirements theory that the firm's net investment depends via fixed capital and spare-capacity coefficients upon its demand estimates. Given the demand estimates, and making allowance for the construction period, the engineer determines the desired increase in capacity by referring to the appropriate period of building ahead of demand; this calculation can be represented by the spare-capacity coefficients. Given the desired increase in capacity, and a specified level of availability of circuits, the engineer determines the necessary net investment by referring to a complex of technological considerations; this calculation can be represented by the capital coefficients.

D. FINANCING INVESTMENT IN THE BELL SYSTEM

In the third stage of the investment process in the Bell System funds must be raised to finance the investment expenditures. The American company has the main responsibility for carrying out this function, because of an arrangement by which the associated companies and the long-lines department obtained a large part of their funds in the first instance as advances from the American company, while the American company provides these advances by raising funds from the public. The associated companies later liquidate the advances from the proceeds of new stock or bond issues.

The critical question here is: how important is the availability of funds in determining the final investment decision? The answer is not clear. The question must be considered at least implicitly before the boards of

[4] American Telephone and Telegraph Company, *Annual Reports*, 1916, pp. 7-9; 1917, pp. 7-8; 1919, pp. 16-17; 1920, pp. 27-8; 1923, pp. 22-3; 1924, pp. 11-15; 1925, p. 11; 1941, p. 3; 1942, pp. 3-8; 1943, pp. 2-8; 1944, p. 13; 1945, pp. 1-6; 1946, pp. 1-7; 1947, pp. 1-6; 1948, pp. 2-6.

directors can approve a suggested investment program, derived from the demand estimates and the engineering calculations. Yet there have apparently been no instances in which an investment program has been drawn up and then cut back because of lack of funds, or expanded because of the ease of obtaining funds. The fundamental reason why the availability of funds is not more explicitly considered, however, is probably that the Bell System has not experienced serious difficulty in financing whatever investment expenditures have been suggested by the demand estimates and the engineering calculations. On the basis of these considerations, no serious error seems to be involved in neglecting the influence of the availability of funds upon the final investment decisions, as suggested in the capital-requirements theory.

A more important consideration in the investment process of the Bell System seems to be that of deciding how the necessary funds for the investment program can best be raised. The historical pattern of Bell System financing can be seen in the following calculation. During the period from 1913 to 1935 the total financial requirements of the system[5] were supplied from various sources in the following percentages: depreciation—21 per cent; undistributed profits—7 per cent; other internal sources—4 per cent; associated company stock issues—4 per cent; associated company bond issues—12 per cent; other associated company sources—3 per cent; American company stock issues (including premiums obtained in the conversion of convertible bonds into stocks)—28 per cent; American company bond issues (including the principal of convertible bonds later converted into stocks)—19 per cent; other American company sources—6 per cent. The figures add up to more than 100 per cent because the definition of funds raised exceeded the definition of financial requirements in this calculation by about 4 per cent.[6]

E. THE INFLUENCE OF TECHNOLOGICAL PROGRESS AND OF PRICE CHANGES

The capital-requirements theory recognizes only one important source of change in the relationship between expected increases in demand and the firm's net investment—technological progress. What then have been the principal technological developments in the history of the telephone industry, and how important have they been in affecting investment in the Bell System?

A major technological change has been the development of the three existing types of automatic switchboards. The panel type, which uses selectors operating from a continuous power drive, was introduced in

[5] Defined as the funds used for capital expansion (75 per cent of the total), omitting the funds used for replacement of retired capital equipment; plus the funds used to retire maturing obligations, omitting retirements of convertible bonds by the issue of new stocks

[6] Federal Communications Commission, *Investigation*, pp. 423-8.

1917, primarily for large metropolitan exchanges. During the 'twenties, after a delay due to the war, manual switchboards in these exchanges were generally replaced. About the same time the step by-step type, which uses electromagnetic switches, was installed in various small cities where cost studies showed that replacement of the manual switchboards would be economical. This trend in the smaller exchanges has continued up to the present time. Then in 1938 the crossbar type, which uses all relays, was developed for use in the large metropolitan exchanges. While the existing panel type has not been generally replaced, the crossbar type has since been used for expansion of capacity in these exchanges. The latest development in this field permits toll operators to make long distance connections by dialing, and requires supplementary automatic switching apparatus which makes connection to the proper exchange.

The cost calculations made in deciding whether to convert from manual to dial operation have been quite detailed. A comparison was made between the present value of the expected annual costs of operation, maintenance, and taxes for the existing manual switchboards; and the present value of the expected total annual costs, including amortization (after allowance for salvage of the manuals), for the automatic switchboards — all costs being calculated over the engineering life of the equipment, and discounted at a rate of 7 per cent. Many of the cost calculations for the smaller exchanges revealed that it would be economical to continue to add operator positions to the existing manual switchboard until the cost of further additions rose significantly, e.g. because of the necessity of increasing the size of the building to house the additional positions. Hence replacement of the existing manual switchboards in these exchanges has been a gradual process, depending in each location upon the time when a major expansion of capacity became necessary.

A second technological change obvious to the customers has occurred in the design of telephone instruments. In 1927, a hand set with separate bell box was distributed among the associated companies as an alternative to the existing desk sets. Technical improvements followed, and in 1934 another hand set, with the bell in the base and with a new anti-sidetone circuit, was authorized. In both cases replacement of existing telephone sets was gradual. This technological change has repercussions on other types of equipment as well. The strength of the signal from the new instruments was greatly increased, and this made it practicable to reduce the diameter of the conductors in the connecting wires and cables. The increased strength of signal from the instruments more than compensated for the increased attenuation in the smaller conductors.

Third, a number of gradual improvements in the design and utilization of cables have occurred. The design of cables has been improved by decreasing the size of the wires, altering their insulation, increasing the

number of wires in the sheath, and altering the sheath. The utilization of cables has been intensified by the development of suitable loading techniques, which consist essentially in placing coils of the proper inductance at regular intervals along each conductor. Originally the attenuation of the signal in one mile of cable was as great as the attenuation in 50 to 100 miles of open wires, but successive improvements in loading and cable design gradually extended the distance over which it was economical to transmit by cables. In the late 'twenties in particular, open wires in toll service were widely replaced by cables.

A fourth major technological improvement was the development of the carrier system of transmission. By the use of this system a large number of messages can be transmitted simultaneously over a single circuit, and hence its introduction means that a smaller amount of outside plant is subsequently required to handle a given traffic load. The carrier system was developed in 1918, but at first could only be applied to circuits in open wires. In 1938, however, the K-type carrier system was applied on a commercial basis to toll cables. This system requires twin voice cables for transmission in both directions, but each pair of wires in the two cables provides 12 channels of voice communication ranging in 4-kilocycle steps from 16 to 60 kilocycles. A few years later, coaxial cables were put into commercial use. These cables contain eight coaxial conductors, each pair of which, because of their wide frequency response, is capable of providing 600 channels of voice communication. Both the K-type carrier system and the coaxial cables, however, are most economical over long distances, and cost studies have therefore been made to determine their proving-in points. The latest development in this field has been the use of microwave radio channels, beamed along a line of towers spaced at approximately 25-mile intervals, thus dispensing with cables entirely. New types of carrier systems are still being developed, and Bell System engineers expect that within another year or two a system suitable for operation over the existing single voice cables will be available. In general these new systems will be installed to handle expansion of demand rather than to replace older types.

Thus it is clear that technological progress has exerted an important influence upon investment in the Bell System. It should be remembered that a technological change is defined in the capital-requirements theory as an abrupt alteration of the marginal capital and spare-capacity coefficients for the affected types of capital equipment. Therefore these four major technological developments should be reflected in the statistical coefficients calculated in the next section from the historical course of telephone investment. The chronology of these developments is used there as a standard for evaluating the statistical results. Moreover, it should be noted that technological changes have been introduced in the Bell

System in the course of expanding capacity, and that replacement of the obsolescent equipment has been gradual. This is in accord with the emphasis upon marginal rather than average coefficients in the capital-requirements theory.

The capital-requirements theory, as pointed out before, neglects the influence of changes in prices upon the firm's investment. Does the investment practice in the Bell System throw any light upon the validity of this simplification?

The detailed calculations of the present values of expected costs for alternative periods of building ahead of demand indicate that changes in the prices of elements in installation costs may alter the spare-capacity coefficients. Indeed, since World War II the period of building ahead of demand in cable installations has been extended somewhat because of increases in labor costs. The periods of building ahead of demand in use today, however, are essentially the same as those in use in the early 'thirties, and hence in general price changes must have been either infrequent and small, or of minor importance in the investment process. It may be noted in passing that the market rate of interest is not relevant here, since a conventional 7 per cent rate of discount is used.

The detailed calculations of the present values of expected costs for alternative types of switchboards also suggest that changes in the relative prices of capital equipment and of other inputs may alter the capital coefficients. In the history of the Bell System, however, it is difficult to isolate instances in which relative price changes have had this effect. Possible instances are the shift from open wire to cable and the shift from manual switchboards to automatic switchboards, but both of these instances are so closely associated with technological developments that the influence of the relative price change by itself appears small.

In general the influence of price changes seems to have been of minor importance in the investment process in the Bell System, compared to the influence of technological changes. It is probably a reasonably close approximation to reality to assume, as in the capital-requirements theory, that all major changes in spare-capacity coefficients and capital coefficients are due to technological developments.

F. RETIREMENT IN THE BELL SYSTEM

According to the capital-requirements theory, retirement is a fixed fraction of the firm's stock of capital equipment. The retirement practice in the Bell System casts some light on the usefulness of this third basic hypothesis.

To begin with, although there seems to be no type of capital equipment which is retired when it reaches the end of a specified service life, there

is some tendency for retirement to be fairly regular for other reasons. Outside plant, such as telephone poles, is tested periodically for physical soundness, and a fairly stable number of units tends to be retired every year because of wear and tear discovered in these tests. Inside plant, such as switchboards, is retired primarily when new technological developments are introduced, but the policy of introducing technological developments gradually tends to make retirement of inside plant fairly regular.

On the other hand, almost every type of capital equipment is quite durable, which weakens the tendency toward regular retirement. In 1935 the estimated service lives used for depreciation accounting by the long-lines department were as follows: central office equipment—13 years; buildings—36 years; pole lines—20 years; aerial cable—25 years; underground cable—30 years; underground conduit—75 years.[7] The durability of capital equipment also makes it difficult to contract the existing stock when demand falls, which is in keeping with the asymmetry of the capital-requirements theory.

There is also some indication that once a unit of capital equipment has been retired, its replacement is in most cases automatic. Inside plant is usually replaced automatically because the reason for its retirement is to substitute new equipment of improved design. Many types of outside plant are replaced automatically because they are links in a chain of capital equipment, and failure to replace the individual links would involve abandoning the capacity of the entire chain. Individual poles in pole lines and individual sections of cable have this characteristic, while individual open wires do not. Finally, the automatic replacement of both inside and outside plant may be based on the fact that demand has grown steadily for many years, and even when its growth has been interrupted, the expectation of future growth has justified maintaining the existing stock of capital equipment. The consequence of this tendency toward automatic replacement of retired capital equipment is that net investment has never become significantly negative in the Bell System. In terms of the capital-requirements theory, this means that the asymmetry between the effects of a rise and of a fall in expected demand has occurred at the point where net investment is approximately zero, rather than at the point where it is negative and equal to retirement.

IV. A Quantitative Analysis of Telephone Investment

A. THE METHOD

The capital-requirements theory is tested quantitatively in this section by calculating its coefficients from statistical data for the entire industry,

[7] Federal Communications Commission, *Docket*, Exhibit #135, 'Long-Lines Department, Financial and Operating Summary,' p. 85.

and then examining the statistical coefficients for stability and for consistency with the known technological history of the industry. This quantitative analysis uses the investment procedures in the Bell System as a guide for certain details of the calculations; more important, it relies upon the chronology of major technological developments in the Bell System as a standard for evaluating the statistical coefficients.

The method adopted for calculating the coefficients depends upon available statistical series for output of telephone service, stocks of various types of capital equipment, and retirement of various types of capital equipment. Because of lack of more complete data, I have not been able to calculate the coefficients directly from the investment procedures of the Bell System. In principle this could be done, and should give more precise results, but the quantitative analysis presented here is limited to a cruder method.

The method uses aggregate statistical series except in one instance. This has been done both because most of the available data are for the industry as a whole, and because the desired results, if obtainable, are aggregate coefficients. One of the distinctive characteristics of the telephone industry, however, is that both demand and capital equipment are geographically localized. The theoretical relation between net investment and expected increases in demand applies strictly only to each individual telephone exchange, and can be applied to the industry as a whole only as a statistical approximation. In particular, the stability of the statistical coefficients depends partly on a regular geographical distribution of increases in demand among the individual exchanges—a regularity which has not always prevailed in fact.

This is an instance of a general problem in statistical analysis—the problem of aggregation.[8] Only under special conditions has it been found possible to deduce valid relationships among macro-economic variables from theoretical relationships among micro-economic variables. Consequently, most aggregate relationships suitable for statistical analysis are derived by analogy rather than deduction, and should be conceived simply as approximations to the more fundamental micro-economic relationships. Specifically, the industry-wide coefficients calculated in this section are so conceived.

On the other hand, the use of data for a single industry largely avoids another general problem in statistical analysis—the problem of identification. The essence of this problem is how to isolate a single causal relationship from a system of mutually interdependent causal relationships, when the available statistical time series reflect the entire system rather than the single desired relationship. If analysis is limited to a single industry, however, the reflex effect of that industry upon itself, by way of

[8] The regional aspects of this problem are discussed in Chapters 4 and 5.

the system of mutually interdependent relationships outside that industry, is small and can be neglected without serious error. In other words, the analysis reverts from general equilibrium to partial equilibrium analysis. The use of data for a single industry also makes the analogy between aggregate and micro-economic relationships more plausible.

The principal justification in the past for statistical studies at the level of the entire economy has been that they describe the structure of the economy in terms of a comparatively small number of relationships, and hence reduce the analysis to manageable proportions. The development of the interindustry input-output scheme, however, which describes the structure of the economy in terms of a much larger number of relationships among its various industries, offers some hope that statistical studies can be shifted to the level of single industries, thus simplifying the problem of aggregation and largely avoiding the problem of identification. The results of these industry studies can then be tied together manageably in the input-output analysis.

Another feature of the statistical method is that it uses a one-dimensional measure of output and capacity. As indicated in the last section, however, one of the distinctive characteristics of the telephone industry is that its product is multi-dimensional, and in particular that the number of CCS handled by a given stock of capital equipment can be altered by permitting the availability of circuits to vary. Unfortunately the data needed to compensate for the other dimensions are not available. Using a one-dimensional measure of output and capacity, of course, implies that the other dimensions are held fairly stable—an implication which has not always been valid in the short run.

B. CALCULATION AND EVALUATION OF DEMAND ESTIMATES

The first part of the quantitative analysis should preferably consist in calculating a historical series of expectations coefficients from statistical data, and then examining their stability. Unfortunately, however, information about actual demand estimates in the telephone industry is not available except for one small sample. Therefore it has been necessary simply to assume a value for the expectations coefficient, and then to check this assumption against the one small sample.

The expectations coefficient is assumed to be +1, i.e. the recent proportional trend in output is expected to continue without diminution. This is compatible with the importance of trend extrapolation in the investment procedures of the Bell System, and with the fact that the growth in number of telephones in the United States actually has approximated an exponential trend. In addition, when the recent trend is either negative or unusually small, the trend which is extrapolated is assumed to be that of the last 'normal' year, set on the base of the current 'subnormal'

TABLE 2

Comparison of Actual and Calculated Demand Estimates

	Year	Number of Toll Messages (mil.)	Relative Increase in Toll Messages	Actual 1927 Estimates (mil.)	Calculated 1927 Estimates[1] (mil.)	Actual 1928 Estimates (mil.)	Calculated 1928 Estimates[2] (mil.)	Actual 1929 Estimates (mil.)	Calculated 1929 Estimates[3] (mil.)	Actual 1930 Estimates (mil.)	Calculated 1930 Estimates[4] (mil.)
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1)	1923	22.3									
(2)	1924	24.0									
(3)	1925	27.7									
(4)	1926	31.9									
(5)	1927	35.2	.103								
(6)	1928	41.7	.185	38.8	38.8						
(7)	1929	49.3	.192	42.7	42.8	47.8	49.4				
(8)	1930	50.5	.025	47.4	47.2	55.4	58.5	59.0	58.3		
(9)	1931	48.4		52.2	52.1	63.4	69.3	69.0	68.9	58.8	59.7
(10)	1932	38.0		57.4	57.5	71.0	82.1	80.0	81.4	68.2	71.6
(11)	1933	35.5				80.0	97.2	92.0	96.2	76.8	84.6
(12)	1934	37.5						105.0	113.7		

[1] Calculated from formula, $35.2 \times (1 + .103)^n$, in which n is number of years in future.

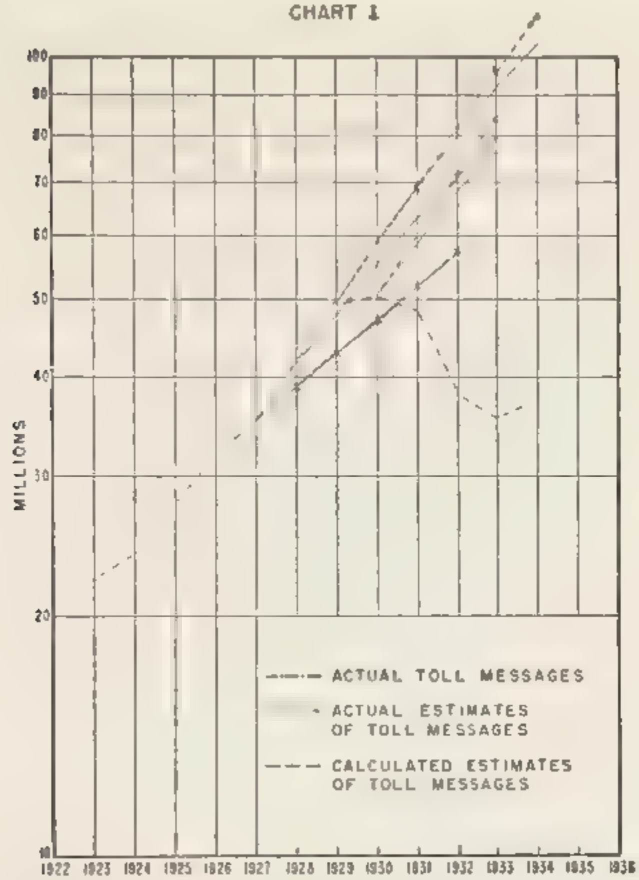
[2] Calculated from formula, $41.7 \times (1 + .185)^n$.

[3] Calculated from formula, $49.3 \times (1 + .182)^n$.

[4] Calculated from formula, $50.5 \times (1 + .182)^n$.

Source: (Col 1) Federal Communications Commission, *Docket*, Exhibit #135, p. 36; data are for long-lines department of Bell System. (Cols. 3, 5, 7, 9) Federal Communications Commission, *Docket*, Exhibit #580, p. 12; data are for long lines department of Bell System.

CHART 1



year. This is compatible with the optimism regarding future growth which seems to prevail in the Bell System.

In algebraic terms, the first basic hypothesis of the capital-requirements theory is

$$O^e_{t+1} = O_t\left[1 + \epsilon\left(\frac{O_t - O_{t-1}}{O_{t-1}}\right)\right] \tag{7, 6}$$

With the expectations coefficient assumed to be $+1$, and with the modification for optimism, this hypothesis becomes

$$O^e_{t+1} = O_t\left(\frac{O_t}{O_{t-1}}\right) \tag{7, 14}$$

in which

$\dfrac{O_t}{O_{t-1}}$ in a 'subnormal' year is replaced by the same ratio in the last 'normal' year.

The one small sample of actual demand estimates in the telephone industry which is available for checking the assumed expectations coefficient consists of the estimates of toll messages made in the Bell System in the four years 1927-30. The actual number of toll messages is also

TABLE 3

Calculation of Demand Estimates

	Year	Number of Telephones in Bell System (mil.)	Number of Telephones in United States (mil.)	Ratio of U. S. Telephones This Year to Last Year	Calculated Telephones Next Year[1] (mil.)	Calculated Present Capacity[2] (mil.)	Calculated Increase in Telephones over Capacity[3] (mil.)
		(1)	(2)	(3)	(4)	(5)	(6)
(1)	1912	4.76	8.73	–	–	–	–
(2)	1913	5.11	9.54	1.0928	10.43	9.54	.89
(3)	1914	5.44	10.05	1.0535	10.59	10.05	.54
(4)	1915	5.80	19.52	1.0468	11.01	10.52	.49
(5)	1916	6.37	11.24	1.0684	12.01	11.24	.77
(6)	1917	6.83	11.72	1.0427	–	11.72	–
(7)	1918	7.05	12.08	1.0307	–	12.08	–
(8)	1919	7.59	12.67	1.0488	13.21	12.67	.54
(9)	1920	8.13	13.41	1.0584	14.19	13.41	.78
(10)	1921	8.72	13.88	1.0350	14.69	13.88	.81
(11)	1922	9.32	14.35	1.0339	15.19	14.35	.84
(12)	1923	10.20	15.37	1.0711	16.46	15.37	1.09
(13)	1924	11.17	16.21	1.0547	17.10	16.21	.89
(14)	1925	11.91	16.94	1.0450	17.70	16.94	.76
(15)	1926	12.67	17.75	1.0478	18.60	17.75	.85
(16)	1927	13.41	18.52	1.0434	19.32	18.52	.80
(17)	1928	14.18	19.34	1.0443	20.20	19.34	.86
(18)	1929	15.04	20.23	1.0460	21.16	20.23	.93
(19)	1930	15.19	20.20	.9985	21.13	21.16	0
(20)	1931	14.91	19.71	.9757	20.62	21.16	0
(21)	1932	13.31	17.42	.8838	–	21.16	0
(22)	1933	12.82	16.71	.9592	–	21.16	0
(23)	1934	13.12	16.97	1.0156	–	21.16	0
(24)	1935	13.57	17.42	1.0265	–	21.16	0
(25)	1936	14.45	18.43	1.0580	19.50	21.16	0
(26)	1937	15.33	19.45	1.0553	20.53	21.16	0
(27)	1938	15.76	19.95	1.0257	21.05	21.16	0
(28)	1939	16.54	20.83	1.0441	21.75	21.16	.59
(29)	1940	17.48	21.93	1.0528	23.09	21.93	1.16
(30)	1941	18.84	23.52	1.0725	–	23.52	–
(31)	1942	20.01	24.92	1.0545	–	24.92	–
(32)	1943	21.25	26.38	1.0586	–	26.38	–
(33)	1944	21.58	26.86	1.0182	–	26.86	–
(34)	1945	22.45	27.87	1.0376	29.89	27.87	2.02
(35)	1946	25.71	31.61	1.1342	35.85	31.61	4.24
(36)	1947	28.51	34.87	1.1031	38.47	34.87	3.60
(37)	1948	31.36	38.35[4]	1.0998	42.18	38.35	3.83

[1] Calculated from formula, (Col. 2) x (Col. 3), 1919 calculation uses 1917 ratio; 1921 and 1922 calculations use 1920 ratio; 1930 and 1931 calculations use 1929 ratio; 1938 calculation uses 1937 ratio; 1945 calculation uses 1941 ratio.

[2] Entered from Col. 2, 1913-29 and 1940-48; from 1929 entry in Col. 4, 1930-39.

[3] Calculated from formula, (Col. 4) – (Col. 5).

[4] Calculated from number of telephones in Bell System and 1947 ratio between United States telephones and Bell System telephones, $31.36 \times \frac{34.87}{28.51}$

Source: (Col. 1) American Telephone and Telegraph Company. (Col. 2) Federal Communications Commission, *Statistics*, 1947, Table 9.

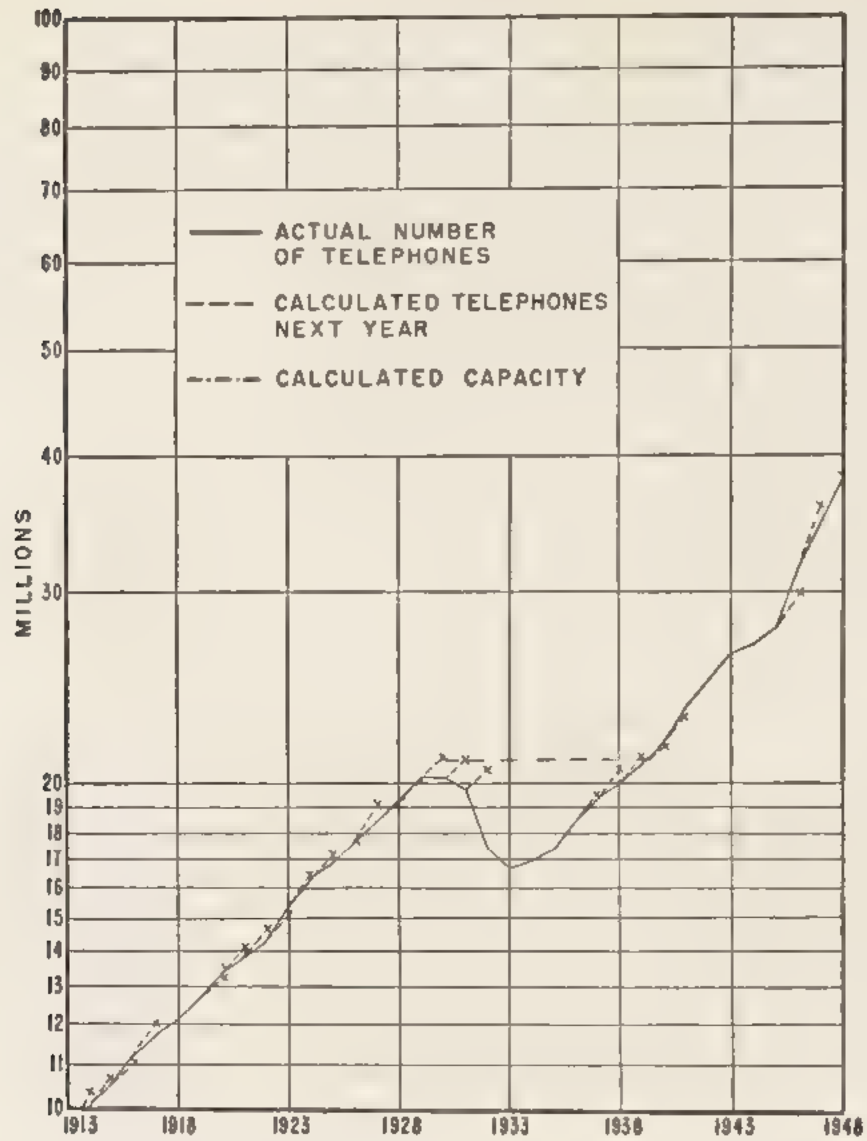
available, and hence it is possible to calculate demand estimates in these years, using equation (7, 14), and to compare them with the actual demand estimates. This comparison is shown in Table 2 and Chart 1. It is clear that the calculated demand estimates, based on the assumed expectations coefficient of +1, approximate the actual demand estimates closely. Note in particular that the calculated demand estimates for 1930, based on the 'normal' trend of 1929, forecast a rising trend comparable to that of the actual demand estimates, even though the number of toll messages showed a 'subnormal' increase in 1930. Thus in so far as such a short series of actual demand estimates is significant, it lends support to the assumed value of the expectations coefficient and to equation (7, 14). More generally, it lends support to the first basic hypothesis of the capital-requirements theory—that demand estimates depend, via a fixed expectations coefficient, upon the recent trend of output.

Three steps may now be taken in preparation for the calculation of capital and spare-capacity coefficients in the next part of the quantitative analysis. These steps are set forth in Table 3 and Chart 2.

First, equation (7, 14) is used to compute a series of demand estimates for the entire telephone industry over the period 1913-48. There is, of course, no possibility of checking these estimates directly, because of lack of information on actual demand estimates. The measure of industry-wide output used in deriving the recent trend for these calculations is the total number of telephones in the United States. This is compatible with the practice in the Bell System of estimating demand in terms of number of lines, main telephones, and total telephones. The 'subnormal' years for the purpose of the modification for optimism in equation (7, 14) are 1921-2, 1930-31, and 1938. It is not reasonable to expect such optimism in the depth of the depression, 1932-5, but since the demand estimates in those years, whether optimistic or pessimistic, were far below capacity, they can be neglected. No demand estimates are calculated for the years of war or government seizure, 1918-19 and 1942-5, and the immediate postwar or post-seizure estimates made in 1919 and 1945 are calculated from the trends of the last essentially peacetime years, 1917 and 1941, respectively.

Second, a series is obtained to represent effective industry-wide capacity over the same period 1913-48. According to the investment procedures of the Bell System, it should be remembered, new investment in each individual exchange is undertaken whenever expected demand exceeds current total capacity, but because of the policy of building ahead of demand, the next new investment is required only after several years in which the growth of demand gradually uses up the spare capacity. This means that at any one time in the industry as a whole there exists a distribution of spare capacity in the individual exchanges. Unfortunately, however, there

CHART 2



are no published figures for this distribution of spare capacity. Statistically, therefore, it seems desirable to represent effective industry-wide capacity by the current industry-wide output, as measured by the actual number of telephones in the United States. The theoretical justification for this is that an expected increase in industry-wide demand over current industry-wide output, provided it is distributed in some regular way among the individual exchanges, requires new investment in those particular exchanges, at one end of the distribution, which have used up their spare capacity. It would also be possible to represent effective industry-wide capacity by the calculated demand estimate made in the previous year, but since this is a derived figure, it seems less desirable in most years. The only exception to this rule is in 1930, when the actual

industry-wide output did not increase over 1929, yet a large net investment program was carried through; capacity in 1930 is therefore measured by the calculated demand estimate. The statistical series for effective industry-wide capacity is continued at its maximum previous level, even when expected demand subsequently falls. In other words, the telephone industry is assumed to make replacement investment equal to its retirement. This is compatible with the tendency to automatic replacement noted in the Bell System procedures.

Third, a series of differences is computed between the calculated demand estimates one year in the future and the calculated effective capacity in the current year. A one-year projection is used here because it is the closest approximation using annual data to the construction periods indicated in the last section. This series of differences—the calculated increases in expected demand over capacity—is one of the two main elements needed for computing the statistical capital and spare-capacity coefficients.

C. CALCULATION AND EVALUATION OF CAPITAL AND SPARE-CAPACITY COEFFICIENTS

The second part of the quantitative analysis consists in calculating a historical series of capital and spare-capacity coefficients for each type of telephone capital equipment. Then the statistical coefficients are examined to see if they vary in the proper direction and in the proper years as indicated by the known history of technological developments in the industry, but are stable from one technological change to the next. This is a key part of the quantitative analysis. It tests directly the second basic hypothesis of the capital-requirements theory—that net investment depends, via fixed capital and spare-capacity coefficients, upon changes in expected demand—and indirectly the first basic hypothesis, since the calculated demand estimates are used in computing the coefficients.

The evaluation of the stability of the statistical coefficients and their consistency with the technological history of the industry requires a subjective judgment. Here it is meaningful to think in terms of a scale of closeness of approximation of the hypothesis to the data, rather than in terms of the dichotomy of proof or disproof. This will be brought out in the evaluation below.

One of the two main elements needed for computing the statistical coefficients has already been obtained—the series of calculated increases in expected demand over capacity. The other is a series of deflated net investment for each of the various types of capital equipment used in the telephone industry. It is obtained as follows.

Data are available stating the value of six types of capital equipment in the Bell System at the end of each year 1912-48. These series have been corrected for changes in the number of companies included in the consolidated accounts; some important changes in accounting definitions occurred between 1932 and 1933, however. The six types are central office equipment (mainly switchboards), land and buildings (mainly central office buildings), exchange lines (outside plant used mainly for local service), toll lines (outside plant used mainly for toll service), station equipment (mainly telephone instruments and installations), and other plant (mainly vehicles and office furniture).

These six series are first blown up to the level of the entire industry by the use of a series of ratios between the total value of capital equipment in the industry and the total value in the Bell System. The ratios are computed directly for every fifth year from 1912 to 1937 (years of the *Census of Electrical Industries*), and by interpolation for the intervening years. Annual net investment in current prices in each type of equipment is then calculated by taking the annual increments of these six series. Finally, separate price indices on a 1926 base are derived for each type of equipment, and the six series of annual increments are deflated with these indices. It would have been preferable to use these indices to deflate gross investment, to use another index to deflate retirement, and to take the difference between the two deflated series, but this was not possible because of lack of data. The data and procedure for calculating deflated net investment in central office equipment are presented for illustrative purposes in Table 4.

Now that both of the main elements needed for computing the statistical capital and spare-capacity coefficients have been obtained, it is revealing to plot the two series for comparison of the timing of their variations. According to the capital-requirements theory, their timing should be synchronous. This comparison is shown in Charts 3 through 5. Note that the expected demand increases are plotted in the charts one year later than in Table 3 to refer to the year in which the increase is expected to occur rather than the year in which the estimate is made.

Turning to the individual comparisons, the timing of variations in net investment in central office equipment, as shown in Chart 3, is quite similar to the timing of variations in the expected demand increases, in particular at the peaks in 1914, 1917, 1924, 1930, 1941, and 1947. This correspondence indicates that net investment in central office equipment, which is the largest component of total telephone investment, is timed in accordance with the capital-requirements theory.

One discrepancy which recurs in the comparisons for the other types of equipment should be pointed out, however. According to the capital-

TABLE 4

Central Office Equipment: Calculation of Capital and Spare-Capacity Coefficients

	Year	C. O. Equip. in Bell System (mil. $)	Ratio of Industry to Bell System[1]	C. O. Equip. in Telephone Industry[2] (mil. $)	Increase in C.O. Equip. (mil. $)	Price Index for C.O. Equipment[3]	Deflated Increase in C.O. Equip.[4] (mil. 1926 $)	Calculated C & S-C Coefficient[5] (1926 $ per telephone)
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1)	1912	99	1.46	145	–	–	–	–
(2)	1913	108	1.440	156	11	69	16	–
(3)	1914	119	1.420	169	13	69	19	21.35
(4)	1915	125	1.400	175	6	69	9	16.67
(5)	1916	136	1.380	188	13	69	19	38.78
(6)	1917	157	1.36	214	26	69	38	49.35
(7)	1918	175	1.340	235	21	98	21	–
(8)	1919	191	1.320	252	17	98	17	–
(9)	1920	224	1.300	291	39	115	34	62.96
(10)	1921	273	1.280	349	58	127	46	58.97
(11)	1922	343	1.26	432	83	126	66	81.48
(12)	1923	436	1.242	542	110	112	98	116.67
(13)	1924	565	1.224	692	150	110	136	124.77
(14)	1925	652	1.206	786	94	103	91	102.25
(15)	1926	735	1.188	873	87	100	87	114.47
(16)	1927	795	1.17	930	57	97	59	69.41
(17)	1928	858	1.166	1000	70	90	78	97.50
(18)	1929	946	1.162	1099	99	84	118	137.21
(19)	1930	1041	1.158	1205	106	79	134	144.09
(20)	1931	1101	1.154	1271	66	89	74	–
(21)	1932	1099	1.15	1264	–7	89	–8	–
(22)	1933	1067	1.148	1225	––[6]	90	––[6]	–
(23)	1934	1067	1.146	1223	–2	103	–2	–
(24)	1935	1067	1.144	1221	–2	109	–2	–
(25)	1936	1082	1.142	1236	15	110	14	–
(26)	1937	1122	1.14	1279	43	109	39	–
(27)	1938	1162	1.140	1325	46	95	48	–
(28)	1939	1194	1.140	1361	36	96	38	–
(29)	1940	1245	1.140	1419	58	101	57	96.61
(30)	1941	1360	1.140	1550	131	100	131	112.93
(31)	1942	1463	1.140	1668	118	100	118	–
(32)	1943	1499	1.140	1709	41	100	41	–
(33)	1944	1542	1.140	1758	49	100	49	–
(34)	1945	1611	1.140	1837	79	100	79	–
(35)	1946	1816	1.140	2070	233	100	233	115.35
(36)	1947	2246	1.140	2560	490	122	402	94.81
(37)	1948	2718	1.140	3099	539	137	393	109.17

[1] Calculated from total capital equipment in Bell System and total capital equipment in telephone industry, at the five-year intervals of the *Census of Electrical Industries*, 1912-37, and by interpolation in the intervening years.

[2] Calculated from formula, (Col. 1) x (Col. 2).

[3] Calculated from data supplied by American Telephone and Telegraph Company.

[4] Calculated from formula, (Col. 4) – (Col. 5).

[5] Calculated from formula, (Col. 6) – (Col. 6, Table 2, previous year).

[6] Omitted because affected by accounting change between 1932 and 1933.

Source: (Col. 1) American Telephone and Telegraph Company.

CHART 3

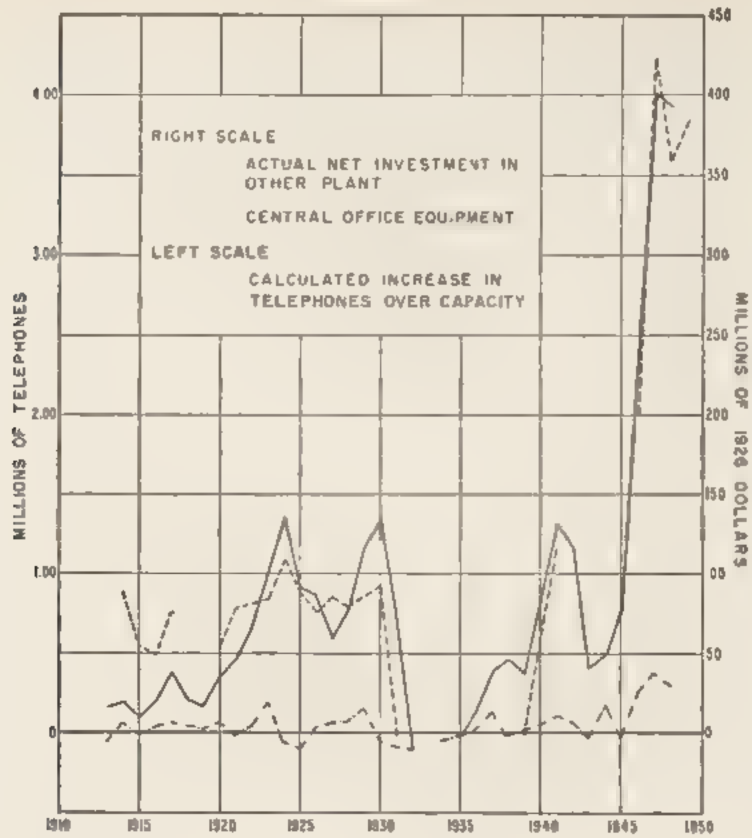
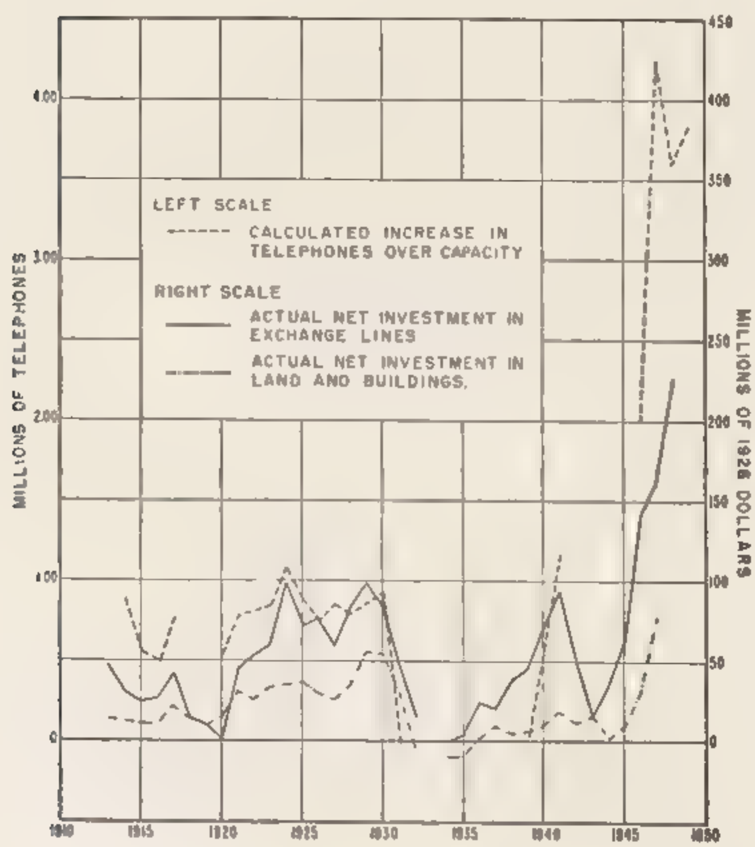


CHART 4



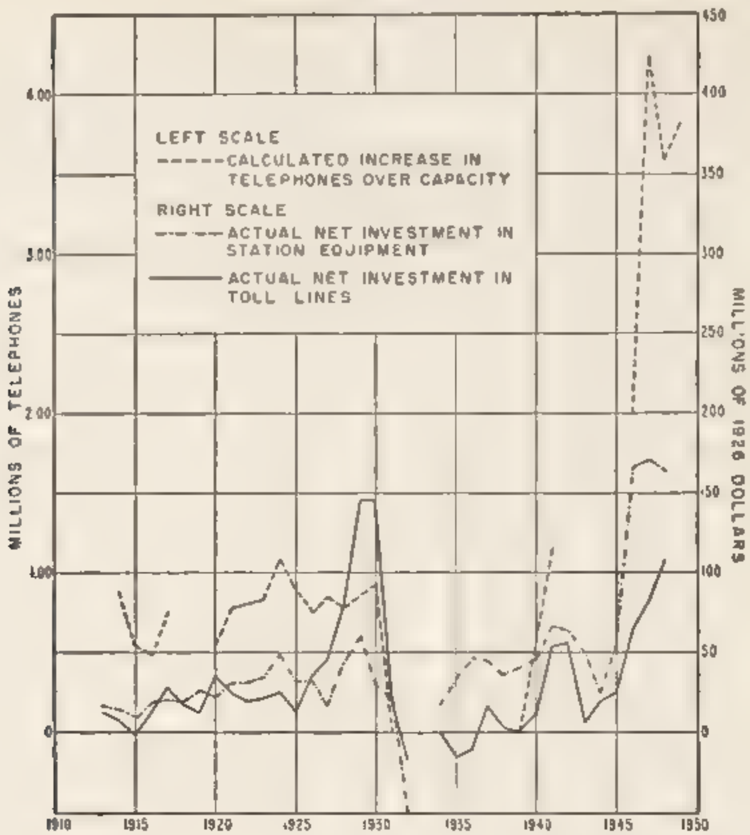
requirements theory, during the 'thirties no positive net investment should have been required, since actual demand remained below the 1929 peak until 1939 and expected demand remained below capacity until 1940. During the early 'thirties, in fact, net investment remained approximately zero, indicating both that no expansion was required and that retirement was almost entirely replaced. During the late 'thirties, on the other hand, some positive net investment was undertaken. This investment apparently was not due to lack of capacity in the industry as a whole; rather it was due in part to increases in demand in particular geographical areas, not typical of the entire country, and in part to the introduction of technological improvements even though existing capacity was adequate. Although this positive net investment began earlier than the capital-requirements theory indicates, it is significant that net investment rose sharply in 1940 and 1941, when expected demand definitely exceeded existing capacity.

The timing comparisons for the other kinds of capital equipment reveal a successively diminishing correspondence between net investment and expected demand increases, more or less in the following ranking: Exchange lines, as shown in Chart 4, rank second in correspondence, and are also the second largest component of total telephone investment. Land and buildings, as shown in Chart 4, rank third, partly because buildings have a longer construction period, and hence annual investment is more regular. Station equipment, as shown in Chart 5, ranks fourth in correspondence, with exceptionally large discrepancies over the depression years. Two distinctive characteristics of station equipment may account for these discrepancies: its construction period is short, so that investment can be rapidly adjusted to actual rather than expected changes in demand; and its retirement is large, so that disinvestment can be significant when demand falls, and excess capacity does not persist to bar investment when demand rises again. Toll lines, as shown in Chart 5, are fifth, primarily because they are 'lumpy' inputs. Each major new line is a comparatively large part of the total stock of toll lines, and includes a large margin of spare capacity. Finally other plant, as shown in Chart 3, ranks last in correspondence, because it simply does not have a definite technological relationship to demand. It is also the smallest component of total telephone investment.

To sum up, these comparisons indicate that the extent to which net investment is timed in accordance with the capital-requirements theory differs from one type of capital equipment to the next. The correspondence between net investment and expected demand increases is quite close, however, for the most important types—central office equipment and exchange lines.

The statistical capital and spare-capacity coefficients are now computed

CHART 5



by dividing the series of deflated net investment in each type of capital equipment by the series of calculated increases in expected demand over capacity, i.e. by finding the ratios between the two main elements plotted in Charts 3 through 5. This calculation is illustrated for central office equipment in Table 4. It should be noted that the statistical results are combined capital and spare-capacity coefficients, rather than the two kinds of coefficients taken separately. The reason for this has already been pointed out that data for the distribution of spare capacity in the industry are not available. The coefficients are marginal coefficients, stating the amount of capital equipment (in 1926 dollars) required for each additional telephone, rather than average coefficients, stating the amount of capital equipment required for each existing telephone. No coefficients have been calculated for other plant, since it seems clear that net investment in other plant is not explained by the capital-requirements theory.

In algebraic terms, the second basic hypothesis of the capital requirements theory is

$$\Delta K^a{}_{t+1} = \beta^a \gamma^a (O^e{}_{t+1} - \frac{1}{\gamma^a} C^a{}_t), \quad \Delta K^a{}_{t+1} \geq -R^a{}_{t+1} \qquad (7, 12)$$

For statistical analysis, using (a) the assumed value for the expectations coefficient and equation (7, 14) above, (b) the statistical measure for

CHART 6
CAPITAL AND SPARE CAPACITY COEFFICIENTS
CENTRAL OFFICE EQUIPMENT
LAND AND BUILDINGS

effective industry-wide capacity and the assumption of automatic replacement, and (c) the deflated value of increments in capital stock for a measure of net investment, the combined capital and spare-capacity coefficients are computed from

$$\frac{\Delta V^a{}_{t+1}}{P^a{}_{t+1}} = \beta^a\gamma^a(O^e{}_{t+1} - C'_t), \quad \frac{\Delta V^a{}_{t+1}}{P^a{}_{t+1}} \geq 0 \qquad (7, 15)$$

in which

ΔV = the increment in value of capital stock,
P = the price index, and,
C' = effective industry-wide capacity.

The historical course of the statistical capital and spare-capacity coefficients can be seen in Charts 6 to 8. How may these statistical results be evaluated? In general the calculated coefficients have two notable characteristics: they fluctuate significantly from year to year, and they trace out underlying patterns of variation over the entire period.

The annual fluctuations are, of course, at variance with the capital-requirements theory, which assumes that the coefficients are stable except

when altered by technological change. These fluctuations do not seem to be sufficiently extreme, however—at least for the more important types of capital equipment—to warrant rejection of the theory. The year-to-year fluctuations seem to arise from a combination of three sources. In part they are due to chance variations in the number of local exchanges requiring expansions of capacity as a result of a given industry-wide increase in expected demand, i.e. they stem from the aggregation problem discussed earlier. In part they result from variations in the availability of circuits, i.e. they stem from using a one-dimensional measure of output and capacity. And in part they arise from considerations affecting net investment which are abstracted from in the capital-requirements theory—in particular, from policy decisions to vary the rate of introduction of technological improvements. Such year-to-year fluctuations must be expected in view of the approximations of the statistical technique and the simplicity of the theory.

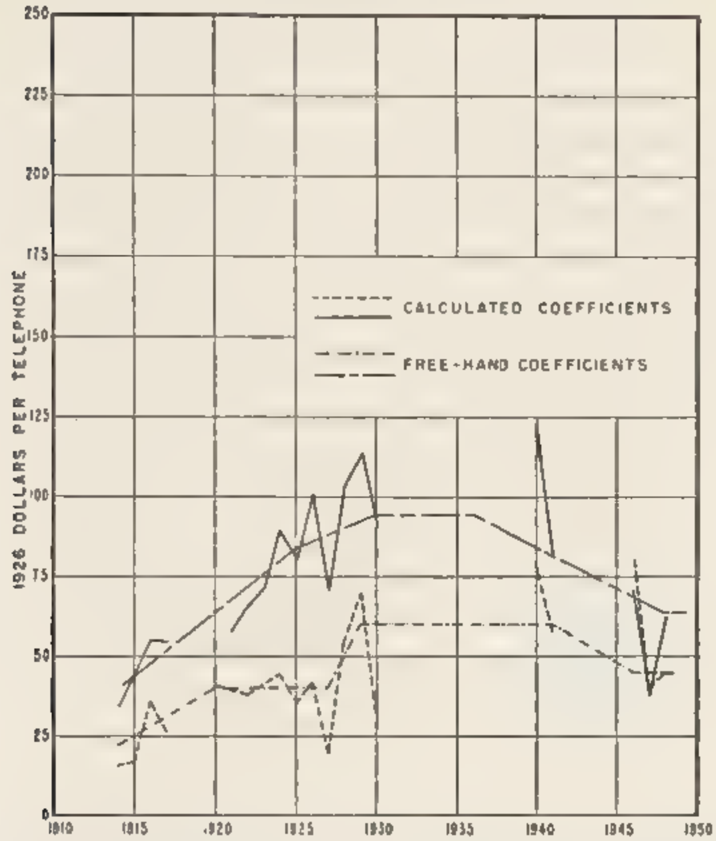
The underlying patterns of variation over the entire period, on the other hand, appear to reflect the fundamental influence of the capital coefficients and spare-capacity coefficients of the capital-requirements theory. These patterns can in general be explained in the light of the technological developments known to have taken place in the various types of capital equipment in the period 1913-48. In the charts, therefore, simple freehand lines have been drawn to represent these underlying patterns, consistent with the dates and directions of the known technological changes.

The statistical coefficients for the individual types of capital equipment reveal a successively diminishing stability and consistency wth the technological history, more or less in the same ranking noted earlier in the timing comparison.

As shown in Chart 6, the coefficients for central office equipment trace out an underlying pattern of variation which reflects clearly the technological change from manual to automatic switchboards. The coefficients rose sharply from 1920 to 1923, when the automatics, which have higher capital costs than the manuals (but lower operating costs), were first introduced; and then remained in the neighborhood of this higher level throughout the 'twenties, in 1940-41, and in 1946-8, when the automatics continued to be used for expansion and gradual replacement. Certain of the year-to-year fluctuations around this pattern are also understandable. The peak in 1923 and 1924 was due to the process of eliminating backlog demand and restoring the normal availability of circuits after wartime shortages, and the fall in 1925 followed the completion of this process. The peak in 1929 and 1930 was due to temporary acceleration of the technological shift from manual to automatic switchboards.

Chart 7 indicates that the coefficients for exchange lines trace out a

CHART 7
CAPITAL AND SPACE CAPACITY COEFFICIENTS
EXCHANGE LINES
STATION EQUIPMENT

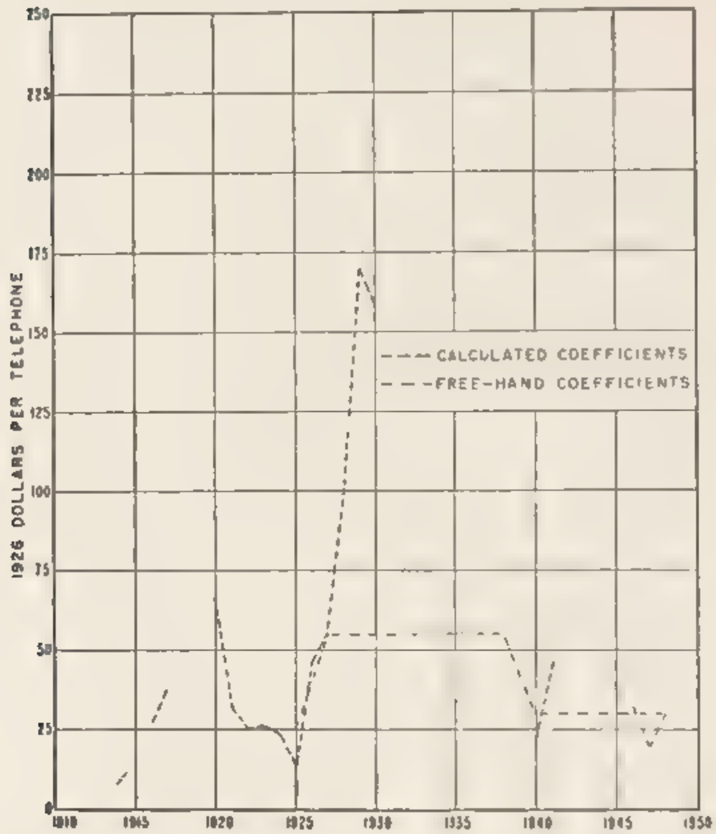


steadily rising pattern from 1914 through 1930, and then a falling pattern through 1941 to the years after World War II. The earlier upward trend reflects the technological shifts which took place in those years from open wire to cable, and from wire and cable above ground to cable underground, each of which require larger amounts of capital equipment per telephone. It also reflects the extension of telephone service to less heavily populated areas. The later falling trend stems from the technological changes in station equipment in those years, raising the strength of the signal from the telephone instruments and thus permitting a reduction in the size of the conductors in cables, i.e. a reduction in capital requirements in exchange lines.

In third rank, as shown in Chart 6, the coefficients for land and buildings trace out an underlying pattern which was high in the 'twenties, low in 1914-17 and 1940-41, and rising after World War II. This pattern arose from policy decisions as well as technological developments. The high level of the coefficients in the prosperous 'twenties (especially 1929 and 1930), and the low level in the post-depression years, were due partly to a program of constructing large administration buildings and making general improvements in architectural design, which was carried through

CHART 8

CAPITAL AND SPARE CAPACITY COEFFICIENT—TOLL LINES



in the 'twenties but later curtailed. The low level of the coefficients in later years was also partly a result of the technological trend to smaller and more compact central office equipment, reducing capital requirements in land and buildings.

The coefficients for station equipment (Chart 7) trace out an underlying pattern in variation which reflects the technological change from desk sets to hand sets, starting in 1927 and continuing through the 'thirties. The coefficients rose somewhat in the late 'twenties and continued at this level in 1940-41. The lower level prevailing after World War II is more difficult to explain, however. Perhaps it is due to the fact that the actual transition from desk sets to hand sets had been virtually completed.

The coefficients for toll lines shown in Chart 8 have been less stable from year to year than those for any other type of capital equipment. This reflects the 'lumpy' character of investment in toll lines. In so far as an underlying pattern of variation can be discerned, however, it is in accord with the known changes in technology. The rise of the coefficients in the late 'twenties was due fundamentally to the technological solution of the problem of long-distance transmission over cables; although, more immediately, to a policy decision to undertake general

replacement of open wire by aerial and underground cable at this time, and to provide a complete cable network in the northeastern part of the country. The technological change raised the underlying pattern of the coefficients, since cable requires more investment than open wire; while the concentrated replacement program raised the annual coefficients to the extraordinary peak of 1929 and 1930. The subsequent lower level of the coefficients before and after World War II reflects a second technological change, the development of carrier systems of transmission, which markedly reduced the amount of outside plant required for each toll circuit.

To sum up, the statistical capital and spare-capacity coefficients for the various types of capital equipment reveal both year-to-year fluctuations and underlying patterns of variation over the entire period. The annual fluctuations are at variance with the capital-requirements theory, but do not seem to be sufficient to reject the theory for the more important types of capital equipment. The underlying patterns of variation, on the other hand, are in general consistent with the known timing and direction of technological developments. All in all, this statistical analysis seems to lend considerable support to the second basic hypothesis of the capital-requirements theory—that net investment depends, via fixed capital and spare-capacity coefficients, upon changes in expected demand.

D. CALCULATION AND EVALUATION OF RETIREMENT COEFFICIENTS

In the third part of the quantitative analysis, annual retirement coefficients for the various types of capital equipment are derived, and the stability of the historical course of the statistical coefficients is examined. The capital-requirements theory assumes that retirement is a fixed fraction of the stock of capital equipment; therefore these coefficients should be essentially stable over the period studied, though perhaps modified by technological changes.

The essence of the statistical procedure is to take ratios between the retirement in each year and the stock of capital equipment in use at the end of the previous year; thus the coefficients are stated in pure numbers. Several aspects of the detailed procedure should be pointed out, however. Since statistical series for retirement by types of capital equipment are not available for the Bell System or for the industry as a whole, series have been compiled for the New York Telephone Company, the largest unit in the Bell System, from unpublished reports made to the Interstate Commerce Commission and the Federal Communications Commission. The classification of capital equipment is somewhat different from that used previously. Four types are essentially the same—central office equipment, land and buildings, station equipment, and other plant.

TABLE 5

Central Office Equipment: Calculation of Retirement Coefficients

Year	Retirement (thous.$)	Stock of Central Office Equipment (mil.$)	Retirement Coefficient[1]
	(1)	(2)	(3)
(1) 1915	758	25.65	---
(2) 1916	675	27.00	.02632
(3) 1917	272	29.76	.01007
(4) 1918	715	32.95	.02403
(5) 1919	663	36.91	.02012
(6) 1920	710	43.16	.01924
(7) 1921	1464	51.86	.03392
(8) 1922	757	72.87	.01460
(9) 1923	892	100.47	.01224
(10) 1924	1768	154.05	.01760
(11) 1925	2721	178.34	.01766
(12) 1926	3639	190.61	.02040
(13) 1927	30712	178.81	---[2]
(14) 1928	7461	186.39	.04173
(15) 1929	5811	204.91	.03118
(16) 1930	7822	220.39	.03817
(17) 1931	14298	230.68	.06488
(18) 1932	20345	230.50	.08820
(19) 1933	13306	231.29	---[3]
(20) 1934	4949	230.49	.02140
(21) 1935	6756	227.42	.02931
(22) 1936	2779	227.63	.01222
(23) 1937	5093	231.34	.02237
(24) 1938	7016	237.74	.03033
(25) 1939	10952	235.88	.04607
(26) 1940	8434	238.51	.03576
(27) 1941	4621	244.50	.01937
(28) 1942	3457	249.98	.01414
(29) 1943	1084	252.52	.00434
(30) 1944	858	254.30	.00340
(31) 1945	1179	258.02	.00464
(32) 1946	1433	275.06	.00555
(33) 1947	2590	340.50	.00942
(34) 1948	3365	423.84	.00988

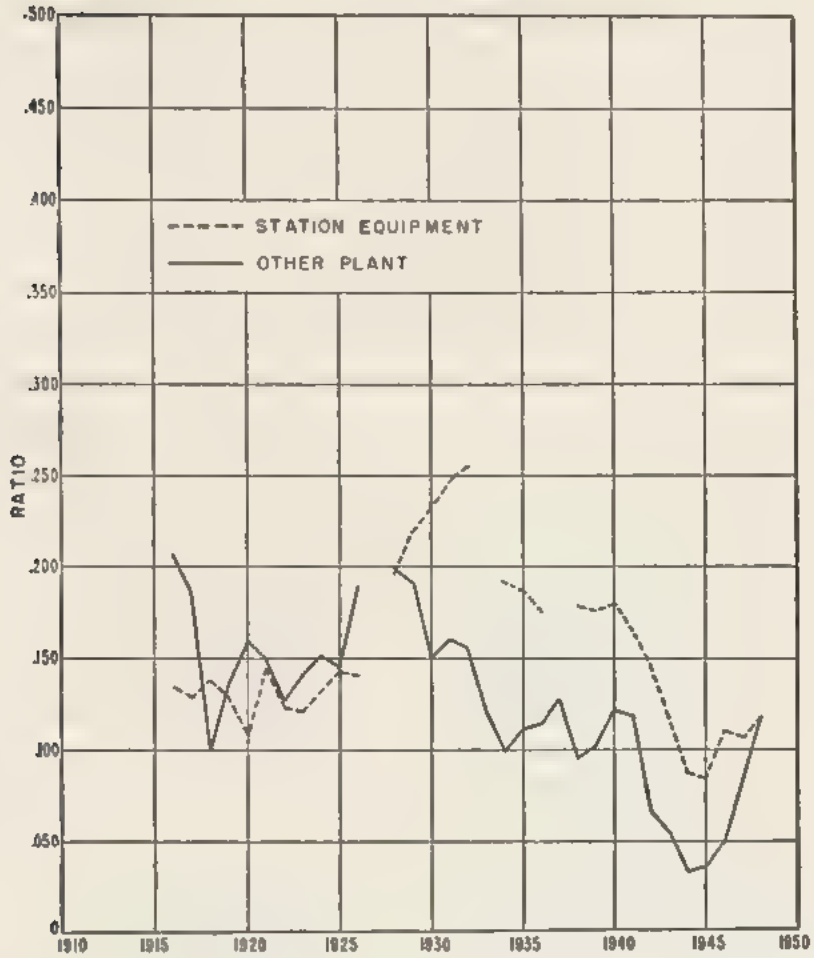
[1] Calculated from formula, (Col. 1) ÷ (Col. 2 previous year x 1000).

[2] Omitted because affected by change in area of operations in 1927.

[3] Omitted because affected by accounting change between 1932 and 1933.

Source: (Cols. 1, 2) Federal Communications Commission, unpublished reports of New York Telephone Company.

CHART 9
RETIREMENT COEFFICIENTS



Outside plant, however, which previously was divided between exchange lines and toll lines, is divided here among aerial wire, aerial cable, underground cable, pole lines, and underground conduit. The series are all stated in value terms. The value of the stock of capital equipment is essentially the sum of the original values, at the time they were first installed, of the various units still in service; no adjustment is made for depreciation. The value of retirement is essentially the original value of the units retired. Unfortunately these value series cannot be deflated, because information regarding the dates at which the retired units were originally purchased is lacking. The year 1927 has been omitted because in that year the New York Company transferred its properties in northern New Jersey, containing 10 to 30 per cent of the various types of capital equipment, to a wholly owned subsidiary, which later became the New Jersey Bell Telephone Company. The years 1933, in some cases 1937, and in other cases 1933-7, have been omitted because of changes in accounting definitions in those years. The entire procedure is illustrated in Table 5 for central office equipment.

In algebraic terms, the third basic hypothesis of the capital-requirements theory is

$$-R^a{}_{t+1} = -\rho^a K^a{}_t \qquad (7, 13)$$

Using undeflated value series for a single telephone company, the statistical retirement coefficients are calculated from

$$-U^a{}_{t+1} = -\rho^a V^a{}_t \qquad (7, 16)$$

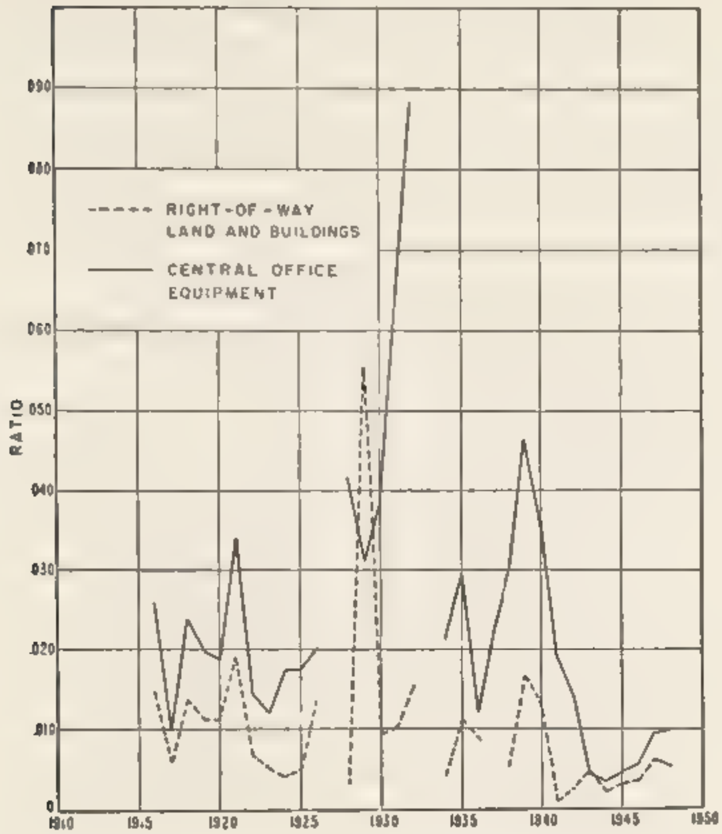
in which

U = the value of retirement, and
V = the value of the stock of capital equipment.

The historical course of the resulting statistical coefficients can be seen in Charts 9 through 13. In general the retirement coefficients fluctuate quite widely from year to year, indicating that annual retirement is subject to considerably more managerial discretion than the capital requirements theory suggests. All of the calculated coefficients, for example, fell very low during World War II, when new equipment was difficult to get for replacement; and most remained low in the postwar period, when new equipment was used mainly for expansion These year-to-year fluctuations are associated with the fact that the coefficients for most types of equipment are very small, indicating that the equipment is quite durable, and hence that there is scope for discretionary variation of retirement. Some of the fluctuations are also associated with the technological changes which are known to have occurred in this period. On the other hand, there appears to be some tendency for the year-to-year fluctuations to center around certain normal levels, even though the stocks of capital equipment increased manyfold over the entire period. This suggests that there may be a long-run tendency for retirement to be a certain fraction of the capital stock.

Turning to the individual types of capital equipment, the two types which have the most stable retirement coefficients from year to year and are most consistent with the capital-requirements theory are station equipment and other plant. It is significant that these two types also have the largest retirement coefficients. This supports the generalization that the less durable a type of equipment, the less scope there is for varying its retirement. Moreover, because large fractions of station equipment and other plant are retired in each year, replacement investment for these two types is more important than net investment. The statistical coefficients for station equipment are shown in Chart 9. The normal level of these coefficients seems to have varied, rising to a higher level beginning in 1928 and continuing through 1940. This reflects the technological transition from desk sets to hand sets in these years. The

CHART 10
RETIREMENT COEFFICIENTS

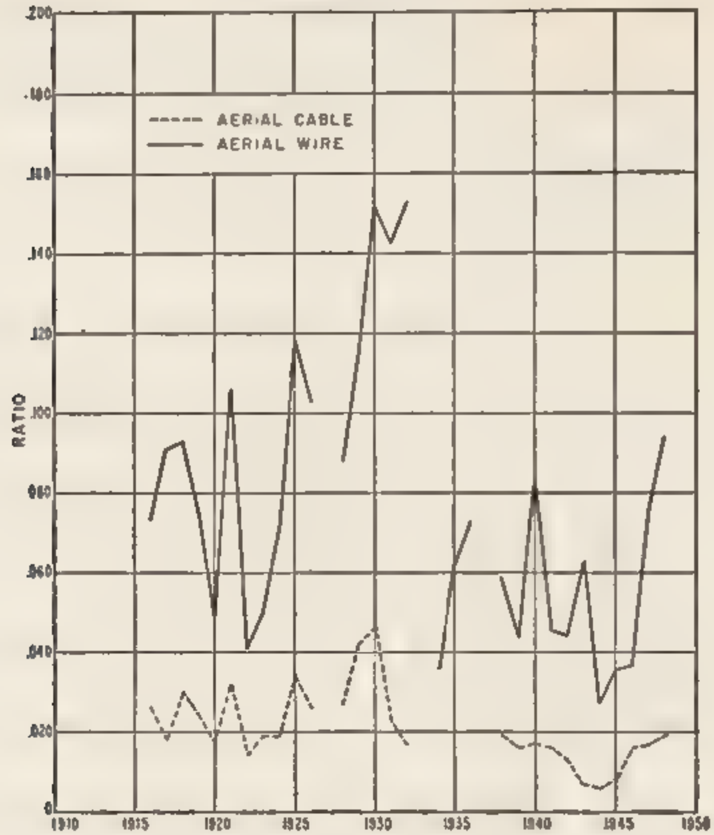


statistical coefficients for other plant are plotted in Chart 9. The normal level of these coefficients seems to have fallen from about 15 per cent in the 'twenties to about 11 per cent in the 'thirties.

The retirement coefficients for central office equipment (Chart 10) are among the most variable from year to year, and the least consistent with the capital-requirements theory. This may be explained by the fact that retirement of central office equipment is due largely to obsolescence, and is primarily affected by policy decisions regarding the rate of introduction of technological improvements. Thus retirement was increased from 1928 to 1932 as a result of an accelerated program of replacing manual with automatic switchboards; whereas in the eight years 1921-7 dial service had been introduced in 46 central offices, in the four years 1928-31 dial service was installed in 70 central offices.[9] An additional reason for increasing retirement in 1931 and 1932 was to reduce maintenance and operating expense by abandoning manual equipment no longer needed in view of the fall in demand. Over the entire period from 1915 to

[9] New York Telephone Company, *Annual Reports*, 1928, p. 5; 1929, p. 5; 1930, p. 6, 1931, p. 8.

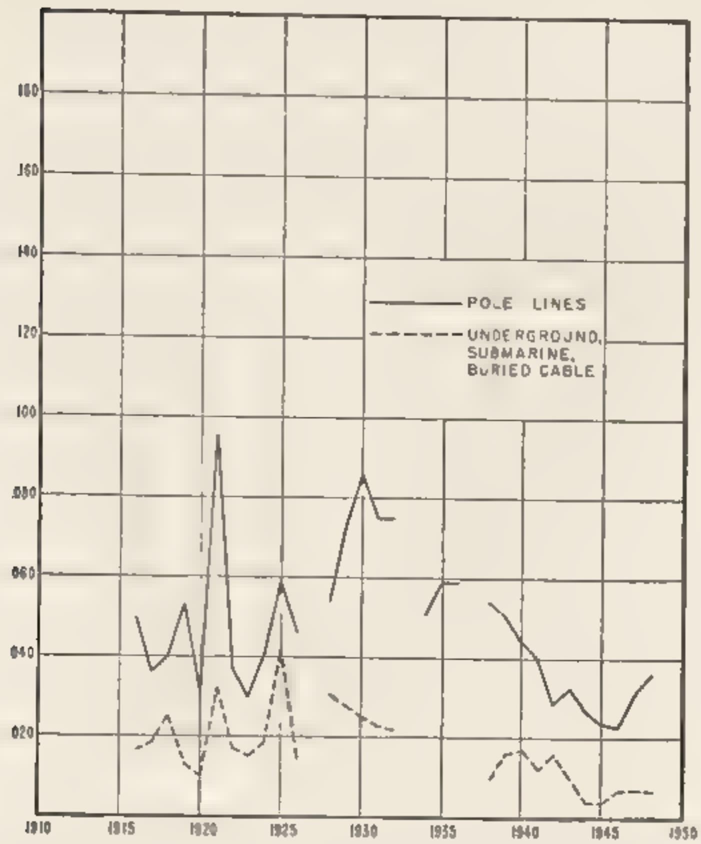
CHART 11
RETIREMENT COEFFICIENTS



1948, however, in which the value of central office equipment increased about 16 times, the statistical coefficients appear to have some tendency to center around a normal level of about 2½ per cent. The retirement coefficients for land and buildings, as shown in Chart 10, trace out a pattern of annual fluctuations similar to that of the coefficients for central office equipment, since this type of telephone plant consists mainly of buildings to house the central office equipment. The peak of retirement in 1929, however, was due to the transfer of many administrative and operating functions to a single new building in that year, and the normal level of the coefficients seems to have been about 1 per cent.

The retirement coefficients for aerial wire, aerial cable, and underground cable reflect in an interesting way the technological change which took place in this period from open wire to cable, both aerial and underground, and from aerial conductors, both wire and cable, to underground cable. These coefficients are plotted in Charts 11 and 12. In 1929 and 1930 the coefficients for both aerial wire and aerial cable rose well above normal, while the coefficients for underground cable did not. Then in 1931 and 1932 the coefficients for aerial wire remained high, while the coefficients for both aerial and underground cable fell. Over the entire

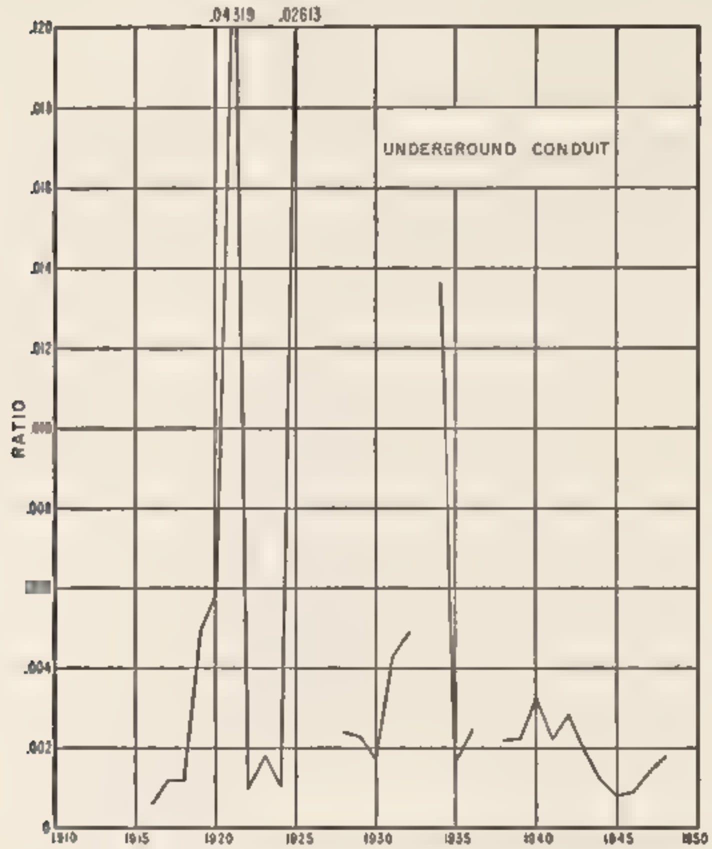
CHART 12
RETIREMENT COEFFICIENTS



period the coefficients for aerial wire were in the order of 8 per cent, and in contradiction to the capital-requirements theory, fluctuated markedly from year to year. The coefficients for both aerial cable and underground cable where in the order of 2 per cent, and were also more stable, particularly after 1938.

Pole lines, aerial wire, and aerial cable are complementary types of capital equipment, and hence the retirement coefficients for pole lines, as shown in Chart 12, trace out a pattern of annual fluctuations similar to that of the coefficients for aerial wire and aerial cable. Over the entire period from 1915 to 1948 the normal level of the coefficients appears to have been about 5 per cent, and except for 1921 the annual fluctuations are reasonably compatible with the capital-requirements theory. Underground conduit and underground cable have a similar complementary technological relationship, but annual retirement in these two types is not so closely related because the conduit is about 10 times as durable as the cable. As set forth in Chart 13 (note the change in scale), the retirement coefficients for conduit were in the order of .2 per cent in most years. Moreover, because of this durability the coefficients were more erratic from year to year than the coefficients for any other type of

CHART 13
RETIREMENT COEFFICIENTS



equipment, and hence less consistent with the capital-requirements theory.

In sum, the statistical retirement coefficients for the various types of capital equipment exhibit varying degrees of year-to-year fluctuations in the period studied. Station equipment and other plant are most stable; underground conduit, aerial wire, central office equipment, and land and buildings are least stable; and pole lines, aerial cable, and underground cable are intermediate. These fluctuations reflect the managerial discretion which enters retirement decisions, but which is abstracted from in the capital-requirements theory. Such discretion is inversely related to durabil.ty, and hence it is important that all types but station equipment and other plant are quite durable. On the other hand, over the period as a whole, while large increases in stocks of capital equipment were taking place, normal levels of the coefficients can in general be discerned. Thus there may be a long-run tendency toward normal retirement. All in all, however, this calculation of statistical retirement coefficients lends only limited support to the third basic hypothesis of the capital-requirements theory—that retirement depends, via a fixed coefficient, upon the existing stock of capital equipment.

V. Conclusion

This study points to one main conclusion—that the capital-requirements theory is a useful explanation of certain fundamental features of the investment process in the telephone industry. The hypothesis that demand estimates depend upon a fixed expectations coefficient is supported by the importance of trend extrapolation in the Bell System procedures, and also by the one short series of actual demand estimates. The hypothesis that net investment depends upon fixed capital and spare-capacity coefficients is supported by the policy of building ahead of demand and the essentially rigid engineering relationships in the Bell System procedures, and also by the consistency of the statistical capital and spare-capacity coefficients with the known technological developments in the period studied. The hypothesis that retirement depends upon fixed retirement coefficients, on the other hand, is supported to only a limited degree, largely because of the durability of most of the capital equipment in the industry.

An important question is the extent to which the capital-requirements theory can be extended to other industries. It should be recognized that the telephone industry has three important characteristics which are favorable to the theory. First, its productive process is heavily capital-intensive, with little scope for substitution of other inputs, and its product cannot be produced for inventory. This means that investment decisions are dominated by essentially rigid technological relationships. Second, the demand for telephone service has risen steadily and rapidly throughout the history of the industry, so that investment designed to expand capacity has bulked large in the investment program, technological improvements have been introduced primarily as alternative techniques for expansion, and it has been economical to build ahead of demand. Third, the telephone industry is essentially a monopoly, and is subject to public utility regulation. Its investment is little affected by the reactions of oligopolistic rivals, and must be adequate to meet demand at commission-determined prices.

These favorable characteristics, however, are clearly not limited to the telephone industry. The other public utilities, which together provide a significant fraction of total investment in our economy,[10] are similar in most of these respects. Many manufacturing industries as well are characterized by capital-intensive productive processes, historically rising demand, and nonaggressive market structures. Therefore, although each industry should be examined individually, it seems probable that the

[10] See ch. 6, sec. IV, Table 9.

capital-requirements theory can be extended to a considerable number of other industries.[11]

Moreover, in so far as this study of telephone investment can be generalized, it indicates that the capital-requirements theory can justifiably be used in integrating investment and disinvestment in capital equipment into the interindustry input-output scheme. The study has two implications for the resulting dynamic analysis, however. First, the theoretical coefficients were found to be more stable over periods of several years than in any single year, in which managerial discretion may affect investment. Therefore the dynamic input-output analysis itself will be more useful in determining what is required to accomplish a specified increase in total outputs over periods of several years than over a single year. Second, the coefficients were significantly altered by technological developments, which indicates, particularly when analysis is focused on periods of several years, that the most up-to-date engineering advice should be sought in calculating the coefficients, in order to make allowance for technological changes which are on the verge of being introduced. If these implications are recognized, however, this study of telephone investment offers a basis for hope that the capital-requirements theory can be successfully used in developing a dynamic input-output analysis.

[11] Throughout this chapter I have referred to *the* capital-requirements theory, meaning the particular theory tested here against the investment practice in the telephone industry. By modifications of particular assumptions, of course, it is possible to develop other variants within the general family of capital-requirements theories; and in extending the theory to other industries, such modifications may be desirable. The particular assumptions which I consider central to any capital-requirements theory are that investment depends upon expected output relative to capacity, that there are fixed technological coefficients relating changes in capacity to changes in stocks of capital equipment, and that negative net investment is limited by the durability of capital equipment to feasible retirement. On the other hand, modified assumptions can readily be introduced regarding the formation of demand expectations, the variability of margins of spare capacity, and the determination of retirement. For an example of another formulation within the family of capital-requirements theories, applied to a number of industries, see Hollis Chenery, 'Overcapacity and the Acceleration Principle,' *Econometrica*, January 1952. The principal difference between his capacity principle and the capital requirements theory developed here is that in his formulation net investment depends upon a certain fraction of the difference between output and normal utilization of capacity, so that the margin of spare capacity varies from year to year, approaching the normal margin as a goal, while in my formulation net investment depends upon the entire difference between output and normal utilization of capacity, so that the margin of spare capacity is constant from year to year. The choice between the two versions for particular industries probably depends primarily on whether demand has fluctuated seriously or has risen steadily in the past.

PART IV

EXPLORATIONS IN THE USE OF TECHNOLOGICAL DATA

Chapter 8

PROCESS AND PRODUCTION FUNCTIONS FROM ENGINEERING DATA

Hollis B. Chenery

THIS CHAPTER investigates the usefulness of engineering analysis for the economic study of production. It has two main objectives: to derive production functions from the type of engineering calculations made in designing industrial plants, and to apply this approach to input-output analysis.

The derivation of production functions from engineering calculations rather than from statistical examination of plant operations has obvious advantages if, in fact, it proves practical. The range of industrial processes to which the function applies is known in advance, and the effect of changing various technological parameters can be calculated. The results are less limited by the particular type of equipment installed in actual plants and approximate more closely the long-run production function of economic theory. Finally, some knowledge of production functions is of great value in classifying plants for input-output analysis so that a minimum of violence is done to the underlying assumptions, as was shown in the following chapter.

The present study began with an intensive examination of one industrial process in order to demonstrate that the project was feasible at all. When it had been shown that in a simple case a production function approximating that of economic theory could indeed be obtained from engineering calculations, a comparative study of all technology was begun to determine the range of application of the methods developed. This led to a division of engineering methods into two general types, analytical and experimental. While this dichotomy is of course not complete, the two elements are easily recognizable in all engineering design procedures. It is principally to the former that the methods outlined below apply.

The experimental or 'cut and try' method is that of extrapolation from similar systems. A new plant is designed from operating data on old plants without a satisfactory theory of the relation of the different elements to the process taking place. Since the analogy is never exact, the best combination of operating conditions determined for the old plant has to

be modified by trial and error to fit the new one. There is no assurance that a different design would not have produced better results.

The analytical method is based on laboratory experimentation. The process under consideration is first reduced to an ideal system. The method of idealization consists of selecting the characteristics of the physical system which are relevant to the performance of the equipment. Familiar elements of ideal systems are a 'perfect' gas, a 'black' body, a frictionless pendulum, the center of gravity, the resistance of a wire, and so forth. The laws of physics and chemistry describe the behavior of ideal systems as deduced from their conceptual elements. When all the important transformations which take place in a productive process can be represented by ideal systems, the alternatives available in the design of a set of equipment to perform this process can be determined with great generality.

Almost no industrial process is so completely understood that a new plant can be designed solely on the basis of an analytical or 'theoretical' solution. The analytical results lead to the construction of a pilot plant or experimental model which is modified on the basis of operating tests. The final result therefore combines the analytical and experimental methods in almost all cases. The analytical ingredient is of fundamental importance for economic analysis, however. It enables the engineer to say what would have been the best solution if some of the economic parameters had been different; if capital had been cheaper or fuel more expensive. Where processes have been developed by purely 'cut and try' methods, the way in which they could be adapted to changed economic conditions is usually known with considerably less accuracy.

Analytical methods of design have wide application in branches of technology utilizing electricity, chemistry, thermodynamics, and aerodynamics. In these fields there are a number of analytical 'design laws.' These are special applications of the more general laws of physics and chemistry and contain parameters representing properties of the apparatus which can be experimentally determined.[1] There are also synthetic design laws based on operating experience rather than on scientific theory. These are also used in designing equipment but are not valid beyond the range of experiments on which they are based. A formula of this type might describe the performance of a certain kind of engine, for example; it would be developed by a multiple-correlation study of operating results rather than from a thermodynamic and mechanical analysis of the components of the engine.

Design laws of either kind can be made the basis for economic production functions. The chief problems to be solved are: the selection of

[1] Ohm's law is a familiar example in the field of electricity, the magnitude of the resistance of any conductor is a parameter which is experimentally determined.

units of analysis; the relation of engineering variables to economic variables; and the combination of functions describing parts of plants into a production function for a whole plant. These methodological problems will be discussed in the following sections.

I. The Concept of an Industrial Process

To an economist 'production' means anything that happens to an object or set of objects which increases its value.[2] This action is most often a change in form, but it may be merely a change in space or in time. The basic physical condition necessary to effect any of these changes (except the last) is that energy must be applied to the material in some form. As a result a change in the energy configuration of the system takes place. The application of energy is one element common to both the economist's and the engineer's concept of production.

An industrial plant may be represented by a pattern of material flows and applications of energy—mechanical, thermal, electrical, or chemical—to the materials. For each energy application there exists a complementary set of inputs: a source of energy and a piece of equipment by means of which the energy is applied. Examples are laborer and tools, fuel and furnace, electricity and motor. From an analytical point of view, production might be broken down into single energy changes; for example, expansion of steam, exhaust, heat transfer, or a mechanical operation analyzed into its components of rotation, compression, etc.

For design purposes, the smallest feasible unit is a single piece of equipment: a motor, tank, pipe, wire, pump, etc. For economic analysis, the number of units may be greatly reduced by only considering variations in those major components in which there is the possibility of significant change in design as the price or quality of inputs and outputs changes. This results in combining in one unit all technologically complementary equipment.[3] The series of energy changes which takes place in such a major grouping of equipment will be called an *industrial process*.

Although it has not been possible to devise a precise definition of an industrial process, it is a design unit which is readily recognizable and widely used in most types of technology. The concept developed here corresponds approximately to the 'unit process' or 'unit operation' of chemical [4] and metallurgical [5] engineering and to the 'process' or 'opera-

[2] This definition of course excludes the 'production' of services, with which I am not concerned here.

[3] I.e. equipment whose design is determined once the design of the major element has been fixed: e.g. pipe, wiring, structures.

[4] See Shreve, R. N., *The Chemical Process Industries*, McGraw-Hill, New York, 1945, ch. 2.

[5] See Schuhmann, R., *The Unit Processes of Chemical Metallurgy*, AIMME, Technical Publication, No. 2363, June 1948.

tion' of the mechanical industries. No precise method of dividing up plants into processes is in use in any of these branches of technology; the division is made entirely on the basis of analytical convenience. The essential idea of a process is that it is associated with a given type of equipment and application of energy rather than with a specific material.

The concept of production as the application of energy to materials leads to a division of economic inputs according to their technological function. The basic distinction is between inputs which form part of the final product (materials) and those which do not (services). This distinction corresponds in general to the relation of the input to the energy transformation; the material inputs 'receive' energy while the service inputs supply it. The latter will be called 'processing factors.' (In the case of chemical change, the materials being processed may either gain or lose energy. The distinction can still be made between inputs which physically enter the final product and those which do not.)

There are two reasons for making this distinction. The processing services, since they do not physically enter the product, are not necessarily specific to any particular class of production. They may be divided into groups which are common to all productive processes (see below). The material inputs, however, are much more specialized according to the product since they enter it physically, and there is a more rigid minimum or limitational amount of each per unit of output. Secondly, the characteristic relations of substitution and complementarity are different as between factors (services) and materials.

Processing factors may have one of three functions: to supply energy; to transform the energy which has been supplied into another form; or to control the process. Under these three headings we may group the inputs which provide the principal processing services:

ENERGY SUPPLY	ENERGY TRANSFORMATION
a. Animals	a. Furnaces (fuel into heat)
b. Fuels	b. Boilers and reactors (heat transfer)
c. Water power	c. Prime movers (heat or electric energy into motion)
d. Chemicals	d. Electric generators (rotation into electric energy)
	e. Machinery (rotation to specialized motion)
CONTROL	f. Vehicles and conveyors, etc.
a. Men	
b. Instruments	

Most capital goods [6] may be placed in one of the last two categories. Each has a characteristic form of energy input and a different form of

[6] Goods-in-process and buildings are capital goods, but they are not processing factors as defined here.

energy output. These may be used to categorize the processes in which they are used.

In the remainder of this chapter, a process will usually be identified by the principal item of equipment with which it is associated, such as those in the second group above.

The method of analysis presented below was worked out by attempting to generalize from the way in which various types of apparatus are designed. Great differences were found among processes according to the energy input—heat, work, electricity, or chemical energy. The processes analyzed [7] include evaporation, electrolysis, gas compression, fluid flow, comminution, and air transport. In all these cases there exist analytical and synthetic design laws from which it is possible to derive general production functions.[8] The following method of analysis is therefore based on a variety of case studies and it appears to be applicable to a considerable range of industrial processes.

II. Analysis of a Single Process

A. THE ANALYTICAL SYSTEM

In one sense, a production function measures the effectiveness of various combinations of factors in producing a specified energy change. In some cases this energy change can be precisely calculated. The amount of heat necessary to evaporate a gallon of water (of a certain temperature and pressure) or the amount of work required to raise an object of a specified weight a given distance can be readily determined. The amount of energy which will actually be expended in performing these tasks, however, depends upon the combination of factors applied.

No ultimate definition of energy is available; we can only measure its transitory effects. We can say, however, that 'work and anything obtainable from or convertible into work are forms of energy.' [9] Potential energy, kinetic energy, heat, and work are different forms of energy in transition which are convertible into each other. The output of any productive process may be measured in terms of any of these forms of energy,[10] using units of either work (force $\times$ distance) or heat (calories).

The amount of work done per unit of product in most processes is fixed. In these cases, it is convenient to measure output in terms of what-

[7] I am greatly indebted to the following engineers from the Massachusetts Institute of Technology for their collaboration in this work Amos Shaler, Charles Satterfield, Barrie Potter, and John Enos.

[8] Some of the results are given in the appendix to this chapter.

[9] Everett, H. A., *Thermodynamics*, Van Nostrand, New York, 1941, p. 1.

[10] The work output is not to be confused with the input which supplies energy to the process.

ever quantity—weight, volume, etc.—requires a constant energy change per unit. In some processes, however, a variable energy change may be produced per unit of product. Examples of this type are heating, transportation, and some chemical reactions. In such cases the output may be measured in weight or in volume units by specifying an arbitrary amount of work or heat per unit: e.g. a ton mile. It is more convenient to keep the output of processes which supply energy to other processes rather than to materials (furnaces, prime movers, electric generators) in energy units: e.g. kilowatt hours, horsepower hours.

The 'design laws' discussed earlier contain the physical and chemical properties of materials, energy, and apparatus as variables. These dimensions can usually vary continuously over a certain range. To distinguish them from economic 'quantities' (i.e. dimensions to which prices can be attached) they will be called engineering variables or process variables. Process variables include both the quantity of an input or output and its other dimensions or 'qualities.' Examples are the following:

INPUT OR OUTPUT	PROPERTIES (DIMENSIONS)
Materials:	
a. Solids	Purity, hardness, density, weight
b. Fluids	Volume, specific gravity, viscosity
Equipment:	
a. Machinery	Size, weight, horsepower, speed
b. Structural elements	Size, density, tensile strength, heat resistance
Energy sources:	
a. Fuel	Heat content, flame temperature
b. Electricity	Voltage, frequency
Labor:	Skills

The central analytical problem of using engineering results is to transform a set of design laws describing a process into an economic production function involving only economic quantities. A method for doing this will first be stated formally and then illustrated in Section III. (The formal statement may be easier to follow if reference is made to the example.)

The conception and design of an industrial process may be separated into three distinct stages. First there is the idea of what has to be done to the materials to transform them into a given product. Analytically, this first conception of the process may be stated in terms of the properties of the materials and products alone; the processing factors enter only in the way in which energy must be applied to effect the transformation. The next step is to determine the possible combinations of factor services which will provide the energy requirement. Take, for example, the crushing of corn to separate its ingredients. The transformation of corn into meal can be described in terms of the properties of the corn and

the meal and the abrasion, impact, or other application of energy which is made to effect the change. The necessary amount of pressure and rotary motion applied by an abrasive surface can result from various factor combinations. These may be a man with two stones, a water wheel and two larger stones, an electric motor and steel rolls, and so forth. Finally we must specify the inputs needed to produce any given combination of factor services. To describe a flour mill, for example, we need to know how electricity, maintenance, operating labor, and depreciation vary with the rate of operation, the size of the apparatus, and various other process variables.

These three stages which relate the abstract conception of a change in form of materials to the inputs necessary to produce it can be described by three sets of equations. The first is the analytical model of the material transformation. It is a relation among the properties of the materials, the product, and the energy input. To take a simple case, if the material is water at a certain temperature and the energy is supplied as heat at a given temperature, an equation can be written relating the amount of steam produced in a given time to the amount of heat supplied (the source of heat is not specified). The amount of compression necessary to stamp an automobile fender out of a sheet of steel is conceptually similar although it is not so readily determined.

A second set of equations is required to describe the factor combination which supplies the energy. The output of this 'energy supply' function is energy and the independent variables are the properties of the factors used. If the energy requirement of the material transformation is known, the energy supply function can be determined experimentally by measuring the variation in the actual energy input as the design of the equipment is varied. In this case the result may be expressed as an 'efficiency' or ratio between the required energy and energy supplied. The process variables may be temperature, motor speed, dimensions of the equipment, and so forth.

The third set of equations, the input functions, relate economic quantities to the process variables. If the consumption of electricity is a function of the horsepower rating of the motor and its speed of operation, an input function for electricity can be determined statistically for each type of motor. Occasionally, input functions can be determined analytically, such as the equation for the number of tons of steel required to build a tank of specified dimensions. More often they are statistical performance curves which apply to a particular commercial type of apparatus.[11]

[11] The input functions usually must be determined statistically from actual apparatus because analytical expressions do not exist for such inputs as labor, maintenance, depreciation, and so forth. The only inputs which are likely to be analytically determined are the quantities of materials necessary to build the equipment.

The following symbols will be used to describe processes:

PROCESS ELEMENTS	'QUANTITY'	PROPERTIES	
1. Materials	m	μ	π
2. Factors:	y	ρ	
(a) Capital Goods	k		
(b) Labor	l		
(c) Energy	e		
3. Outputs	x	ξ	

If there are no side relations, the material transformation function (analytical model) and the energy supply function will each be a single equation. They may be stated as follows:

Material transformation function: [12]

$$\phi(X_i, \mu_i, \xi_i, E_r) = 0 \tag{8, 1}$$

Energy supply function:

$$E_r = E(\rho_i) \tag{8, 2}$$

where

E_r = required energy.

It is always possible to combine these two equations by eliminating the quantity of energy required between them:

$$\phi[X_i, \mu_i, \xi_i, E(\rho_i)] = 0 \tag{8, 3}$$

This combined function will be called the engineering production function.[13] It is useful to determine the energy required in the material transformation function even though this variable is always eliminated in the final production function. E_r (the required energy) indicates the minimum energy which must be utilized in the process. The energy supplied to the process will always be greater than this amount because of the inevitable energy losses in all transformations. Equation (8, 2) shows the limit to technological development aimed at the reduction of the gap between the energy supplied and the energy required by a given transformation—tons of coal per barrel of cement, for example,

The input functions have the form:

$$\begin{aligned} m &= m(\pi_i) \\ y &= y(\pi_i) \end{aligned} \tag{8, 4}$$

[12] The subscript 'i' is used to represent an indefinite number of variables ($\mu_1 \cdots, \mu_n$, $X_1 \cdots, X_m$, etc.) of each type in this and all following equations.

[13] Actual examples of such functions are given in the last section and in the appendix of this chapter.

(using π_i as a general symbol for all process variables). For example, the amount and quality of yarn input depends on the properties of the loom and the cloth to be produced, as do the inputs of energy or labor.

In the simplest case, the engineering production function of equation (8, 3) can be transformed directly into an economic production function. If we rewrite equation (8, 3) for a single output (using π_i for all process variables) it becomes:

$$\phi_1(X, \pi_i) = 0 \tag{8, 5}$$

If each input is a function of only one process variable, equation (8, 5) can be transformed by substituting the input functions. Assuming, for example, one input each of materials, capital, labor, and energy, it would become:

$$\phi_1(X, m_1, k_1, l_1, e_1) = 0 \tag{8, 6}$$

This is the form of the production function used in economic theory.

In general, two or more inputs will depend on some of the same variables, and such a consolidation will not be possible. The functional relations which correspond to the economist's 'production function' then consist of the engineering production function (equation 8, 3) and a set of input functions. They are much more general than the economist's function, however, because they describe a whole family of transformations of inputs into outputs which differ only in the value of certain parameters. In order to have the simplicity of a single function, the economist usually ignores the structure of the underlying physical relations, which may not permit a solution for the output in terms of given inputs alone. He also limits himself to a function between only one quality of each input and output.

The separation between the transformation function and the energy-supply function is useful when a greater degree of generality is desired. If several processes are available to perform the same transformation, a group of engineering production functions having different inputs but the same output can be determined. Several of these may be required to find the most economical way of achieving the transformation. A good example is an airplane (see Chapter 11). The analytical model does not depend upon the kind of motor which is attached, but only on the airframe. Energy supply functions for different kinds of motors can be combined with the same transformation function to determine the most economical way of propelling the aircraft.

B. DETERMINING THE ENGINEERING RELATIONS

The three-step analysis outlined above assumes that all necessary knowledge of the energy requirement and the process is available. When

this is so, the most general type of production function can be derived. In the more typical case, however, no complete material transformation function is known. It is usually possible to state certain restrictions of an analytical nature and to determine the most important process variables. The engineering production function is then determined experimentally for each combination of factors which has been tried. Many complicated chemical reactions are of this nature. For example, the complete analysis of the chemical change involved in making cement has not been worked out. The laws of heat transfer control the design of the cement kiln to a considerable extent, however. A similar condition exists for most mechanical operations, in which the laws of mechanics limit the design of machines but do not describe their functioning completely. In such cases, the engineering production function is determined statistically and only applies to the range of variation over which it has been tested.

The input functions are most accurate in the case of capital goods and energy and least accurate for certain labor inputs. In semi-automatic processes, however, labor is used almost entirely for maintenance and control. The control or operating labor is complementary to the machine and this relation gives a precise input function.

The selection of variables to appear in the engineering production function is partly a matter of analytical convenience. The variables should be those which are most readily related to the dimensions of the input which determine its cost. For example, in the process function for a tank it is better to use the dimensions of the tank than the pressure in the liquid because the dimensions determine the cost of the tank. Several extra relations are usually necessary to make the required transformations from the variables used in the analytical model to the dimensions of the inputs.

The final cost function specifies the effect on costs of each of the inputs and process variables. Formally, the cost function can be written entirely in terms of the process variables:

$$C = C(\pi_1, \cdots, \pi_n, P_1, \cdots, P_m) \tag{8, 7}$$

where

C = total cost
$P_1, \cdots, P_m$ = price parameters.

C. LEAST-COST SOLUTIONS

Although the use of process variables results in equations which are formally similar to the more usual economic functions, the differing characteristics of the two should be noted. The usual formulation in

terms of prices and quantities insures that the cost function will be easily determined because the units chosen are those of the market. In order to achieve a simple cost function, the production function is made discontinuous for the general case in which there are qualitative changes in inputs for different points on the same isoquant. The reformulation suggested here is made in the interest of achieving a continuous production function where the underlying technical law is continuous. Formally, the determination of the cost function requires testing all possible input combinations to fix the least cost of any combination of process variables. In actuality, the problem is usually much simpler because one or two inputs are likely to be more economical than all others for all relevant values of the process variables.[14] Therefore, the cost function may be continuous over the practical range involved.

Since the engineering production and cost functions are formally similar to the most general formulation in terms of economic variables, the least-cost solution is obtained in the same way for both. Following Professor Samuelson,[15] we can proceed to minimize the cost function, equation (8, 7), subject to X equals constant in equation (8, 5) (assuming a single output and continuous functions): Let

$$Z = C(\pi_1, \cdots, \pi_n) - \lambda\phi(\bar{X}_o, \pi_1, \cdots, \pi_n)$$

where

λ = a Lagrangean multiplier.

A necessary condition for a relative minimum is that:

$$\frac{\delta Z}{\delta \pi_i} = \frac{\delta C}{\delta \pi_i} - \lambda \frac{\delta \phi}{\delta \pi_i} = 0$$

or

$$C'_i = \lambda\phi'_i \qquad (i = 1, \cdots, n) \quad (8, 8)$$

These n equations[16] may be combined into $(n - 1)$, eliminating the Lagrangean multiplier:

$$\frac{C'_i}{C'_1} = \frac{\phi'_i}{\phi'_1} \qquad (i = 2, \cdots, n) \quad (8, 9)$$

[14] This dominance of a few inputs over all others would be apparent from the engineering analysis but not without the analytical model. For example, if the requisite tensile strength of a metal (process variable) is known from the engineering function, the cheapest metal input may readily be found.

[15] Samuelson, Paul A., *Foundations of Economic Analysis,* Harvard University Press, Cambridge, 1947, pp. 57-61.

[16] The only difference between the set of equations in (8, 8) and the usual solution for optimum input combinations is that the partial derivatives of the cost function with respect to the process variables need not be constant even if prices are constant.

Together with the production function (8, 5) these $(n-1)$ equations permit a solution for the n unknown process variables, π_i, if the secondary conditions for a minimum are fulfilled.[17]

The economic interpretation of this result is the same as for 'quantities' of inputs: the marginal cost of varying any process variable must be proportional to its marginal effect on output when total cost is at a minimum. The partial derivatives with respect to the process variables may be thought of by analogy as marginal productivities and marginal costs. They may, however, be either positive or negative.

Although it is often impossible to transform an engineering production function into a single function containing specific physical inputs,[18] it is possible to determine the quantity of inputs in aggregate terms implied by any point on the function. For example, it may be desirable to take aggregates such as materials, capital, labor, and energy as the independent variables. This transformation involves a minimization process similar to the one just outlined but more complicated. On the assumption of constant relative prices for inputs within each category, each aggregate can be measured in value terms. The amount of each aggregate can then be written in terms of the process variables. Assuming constant prices (P_m, P_k, etc.):

$$\begin{aligned} M &= \Sigma m_i P_{m_i} = M(\pi_1, \cdots, \pi_n) && \text{(Cost of materials)} \\ K &= \Sigma k_i P_{k_i} = K(\pi_1, \cdots, \pi_n) && \text{(Cost of capital)} \\ L &= \Sigma l_i P_{l_i} = L(\pi_1, \cdots, \pi_n) && \text{(Cost of labor)} \\ E &= \Sigma e_i P_{e_i} = E(\pi_1, \cdots, \pi_n) && \text{(Cost of energy)} \end{aligned} \qquad (8, 10)$$

While these functions are single-valued for given combinations of the process variables, there may be different combinations of process variables which can produce the same value of the aggregate. Therefore the isoquants can only be determined by minimizing one aggregate subject to the others remaining constant. For m aggregates, there will be m Lagrangean multipliers. If the functions are continuous, the solution will be of the following form (allowing K to vary and setting $M = M_o$, $L = L_o$, $E = E_o$, $X = X_o$):

Let

$$G = K - \lambda_1 M_o - \lambda_2 L_o - \lambda_3 E_o - \lambda_4 X_o$$

A necessary condition for a minimum amount for K is that:

$$\frac{\delta G}{\delta \pi_i} = \frac{\delta K}{\delta \pi_i} - \lambda_1 \frac{\delta M}{\delta \pi_i} - \lambda_2 \frac{\delta L}{\delta \pi_i} - \lambda_3 \frac{\delta E}{\delta \pi_i} - \lambda_4 \frac{\delta X}{\delta \pi_i} = 0 \quad (i = 1, \cdots, n) \qquad (8, 11)$$

[17] See Samuelson, op. cit. pp. 61-3. For the discontinuous case, see pp. 70-75.

[18] I.e. to secure a function of the form of equation (8, 6).

These n equations plus the m side conditions (each other aggregate remains constant) permit a solution for $\pi_1, \cdots, \pi_n, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ and hence for K in terms of X_o, M_o, L_o, and E_o. The result of such a solution is given in Chart 4 below.

This transformation into economic aggregates is likely to be very tedious even where the engineering production function is quite simple. The only kind of problem for which it is useful to make such a transformation is one in which the relative prices of the aggregates change while the relative prices within the aggregates remain constant. All other problems—the least-cost solution, the effect of changing single prices, and so forth—can be more conveniently treated in terms of the engineering production and cost functions.

Equation (8, 9) can be applied in some cases before the price parameters are specified to determine the combinations of the process variables which will produce a technologically efficient solution. Consider, for example, the following set of production and cost functions:

$$\begin{aligned} X &= \phi(a, b, c, d) \\ C &= P_1 f_1(a, b) + P_2 f_2(c, d) \end{aligned} \tag{8, 12}$$

In this case $\frac{C'_a}{C'_b}$ and $\frac{C'_c}{C'_d}$ do not depend on prices at all. The cost function involves two inputs, each of which is determined by two different process variables. An efficient solution requires using a minimum amount of each of these inputs for a given amount of the other and a given output. Therefore equation (8, 9) may be used to eliminate two of the four process variables from the production function without regard to prices. In effect, this means that whenever two process variables appear only in combination in the cost function one of them can be eliminated from the production function if it is desirable to do so. If this condition is imposed wherever possible, the production function will contain only technologically efficient possibilities.

D. Variation in Parameters

The preceding analysis has been conducted under the assumption that three sets of parameters remained constant: (1) output dimensions; (2) available input qualities; and (3) prices, P_i. Any one of these quantities can be considered as a variable and the effect of changing it on the least-cost position can be calculated. The first two cases are more interesting because they are not usually treated in economic literature [19] in a quantitative manner.

[19] See, however, Brems, H., 'The Interdependence of Quality Variations, Selling Efforts, and Price,' *Quarterly Journal of Economics*, May 1948, pp. 418–40.

1. *Output quality*

The effect of a particular characteristic of the product its purity, homogeneity, and so forth—on the input requirements can often be expressed as a side relation between the given dimension and one or two inputs. For example, the properties of steel may depend on the proportion of iron ore to scrap inputs. The number of defects in a particular textile product depend on the type of cotton used. If some minimum value of the product quality is specified, the available range of inputs is limited. The effect of varying the quality on the equilibrium value of the process variables can be calculated by adding an appropriate side relation which must be satisfied in the cost-minimizing procedure. The effect is to rule out certain areas of the production function.

The marginal cost of a variation in a dimension of a given output is particularly useful where the effect of the dimension on revenue can be calculated. Examples are the speed of travel, the load factor of utility service, the tensile strength of steel.

2. *Input quality*

The engineering production function includes the effect of changes in input dimensions on output. Some possible values of these dimensions have no physical counterpart, e.g. metals of a very high heat resistance, motors of great speed. The cost function is therefore discontinuous in certain areas, and a corner minimum may result. In other words, the lowest cost is achieved by using the highest value of the dimension—the metal of the greatest heat resistance or the motor of the greatest speed available. The effect of an increase in available input quality can be calculated by changing this limitation, since the production function itself is continuous. Some types of technological change, such as the development of a container which will withstand higher pressures, affect only the input function and not the production function.

III. Production Functions for Plants

A. The General Problem

Before applying this type of production analysis to an actual case, I shall consider briefly the problem of relating production functions for single processes (process functions) to functions for a whole plant.

Many groups of processes in a plant are substantially independent in the sense that the design of one does not affect the design of another if the plant output is given. The only variables common to the two pro-

duction functions in this case are the dimensions of the product. For example, the processes may produce different components of a single assembly. For a given rate of output and fixed product dimensions, the optimum design of each process can be determined separately. The production function for the plant will consist merely of all process functions.

In general there will be some processes in a plant which are interdependent; the design of one will affect the design of one or more processes even with a given product. In the simplest type of interdependence, the output of one becomes the input for a succeeding process. The production functions for two or more such processes can be combined into a single function. For example, if

$$X_A = \phi_A(m_1, y_1)$$

$$X_B = \phi_B(X_A, y_2)$$

then

$$X_B = \phi_B[\phi_A(m_1, y_1), y_2] \tag{8, 13}$$

(When the relation is of the form $X_B - kX_A$, the two can be considered a single process for most purposes.) A different type of interdependence exists when two processes have a common process variable or a necessary relation among two or more process variables. This may be a common property of the energy supplies of the two processes (steam pressure, voltage, etc.) or a joint use of a material input, for example. It is difficult to specify the general form of the mathematical relationships among such processes. In general, however, the interdependence will be expressed by one or more side relations.

The extent to which the existence of such processes complicates the analysis can only be determined from actual trials. In the design of an electric generating station, several processes are interdependent because they have steam pressure as a common variable. The optimum pressure can be fixed after considering all these processes together. It is not necessary to determine values of all the other variables to fix the steam pressure, however. Once this is done the processes can be treated as independent.

B. PRODUCTION AND COST FUNCTIONS FOR NATURAL-GAS TRANSMISSION

Production functions for the processes used in natural-gas transmission will be developed as an illustration of the engineering approach. From the two major process functions, a production function for the whole 'plant' will be determined; by the use of typical parameters in the input

functions, it will also be possible to state a cost function and to give the least-cost solution.[20]

In many respects this is an ideal case for the engineering approach because the problems of pipeline design have been well analyzed in the engineering literature. In addition, the effects of the major engineering variables on costs have been studied by the Federal Power Commission and by several engineering firms. The required technological information is therefore available in a suitable form. The technique of pipeline transmission is so standardized that the equations derived apply to the whole natural-gas industry.

1. *The analytical model*

We shall be concerned in this study only with the actual movement of gas over long distances through a pipeline. Auxiliary processes, such as measuring the gas and the various overhead costs, will be ignored because they do not affect the optimum combination of the major inputs. Since the cost of these auxiliary operations amounts to less than 20 per cent of the total cost, the relations among the inputs associated with the pipeline itself largely determine the production function of the plant as a whole.

The 'plants' of this industry consist of two major kinds of equipment: steel pipe of large diameter and compressors driven by gas engines. In the typical case, the investment in the pipeline is more than twice as great as that in compressor stations. We may take as a representative 'plant' one compressor station and the segment of pipe which extends to the next station. This is a valid simplification because the stations for a given line are usually identical in size and spacing so long as the volume transported remains the same. We are therefore concerned with the cost per mile of transporting a given quantity of gas.

Since a process has been defined as the series of energy transformations which occur in a single unit of equipment, the plants consist of two major processes. The complete series of energy transformation is as follows. Fuel (usually gas) is burned in the cylinders of the engine and its chemical energy transformed into heat, which is then converted into mechanical work by the expansion of the gas against the piston. Since the piston is rigidly connected to the compressor, the work of expansion is utilized directly in compressing the gas. The gas is discharged into the pipeline at a pressure which usually ranges between 400 and 1200 pounds per square inch. In flowing through the pipe it dissipates internal energy

[20] A full derivation of the equations involved has been published in my 'Engineering Production Functions,' *Quarterly Journal of Economics*, Vol. LXIII, No. 4, November 1949, pp. 507-31. The methodology will be developed in somewhat greater detail here, but the detailed calculations will be omitted.

as heat in overcoming frictional forces. (Assuming no change in elevation, this is the only energy used.) When the pressure has dropped to the original suction pressure of the first compressor, the process is repeated in the next station.

We can consider the engine and the compressor as a single process since there is no possibility of varying the output of one relative to the input of the next. There is, however, a large range of combinations of pipe and compression which will produce the same output. They are successive processes which can be combined in varying proportions. This is possible because in a large pipe the energy loss due to friction per unit of volume transported is less than in a smaller pipe carrying the same amount. Because of the greater energy loss (for the same volume), more power must be supplied in the latter case. The compressibility of gas makes the range of substitution of power for pipe much greater than in the case of liquids.

The models for the two processes can both be developed from thermodynamic theory on the assumption that natural gas acts as a perfect gas. (At high pressures, an empirical correction to this assumption is introduced in actual practice.) We need two functions to describe the transformation in the material: the work required to compress it and the work required to move it through an idealized pipe. As work is the product of pressure $\times$ volume, the output of the compression process will be measured in terms of the pressure increase (volume being given) since the formula for pipeline flow is most conveniently stated in terms of the pressure drop.

Process 1—*Compression*

The action of compression can be approximated by an adiabatic process (no heat loss) and the formula for the work done can be derived from the First Law of Thermodynamics.[21] The work done depends only on the properties of the gas and the ratio of the initial and final pressures. The formula therefore applies to any gas and any compressor. The general formula is

$$E_r = W_r = \left[\frac{n}{n-1}\right] P_s[R^{\left(\frac{n-1}{n}\right)} - 1\ X] \tag{8, 14}$$

where

n = ratio of specific heats
R = compression ratio (ratio of initial to final pressure)
W_r = work required
P_s = standard pressure of measurement
X = volume compressed

[21] See Pacific Gas Association, *Gas Engineer's Handbook*, p 760, and Lehn, H. C., 'An Analysis of Gas Pipeline Economics,' A.S.M E., *Transactions*, July 1943, p. 445.

The ideal model which is described by the transformation function includes no frictional losses in power and therefore does not specify any particular piece of apparatus.

A general form of the energy supply function, equation (8, 2), is

$$E_r = E_s - E_l \tag{8, 15}$$

where

E_s = energy supplied
E_r = energy required in transformation
E_l = energy loss

(This equation is often written by engineers in terms of the over-all efficiency: $\eta = \frac{E_r}{E_s}$.) To complete equation (8, 15), we need an expression relating the energy loss to one or more of the process variables. For the type of compressor in general use, this takes the form:[22]

$$E_l = aX \tag{8, 16}$$

where

X = capacity of the compressor.

The energy loss depends only on the capacity of the compressor and not on the amount of work done.

Substituting equation (8, 16) in equation (8, 15) gives the following energy supply function:

$$E_r = E_s - aX \tag{8, 17}$$

Process 2—Pipeline flow

The exact form of the function describing gas pipeline flow has been a matter of debate among engineers for at least 40 years. The general form of the equation is clear on theoretical grounds, however, and the issue concerns the corrections to be applied to it.[23] For a perfect gas, the following equation can be derived[24] for the volume rate of output[25] of the line:

[22] See Lehn, op. cit., and the discussion at the end of the article on the effect of the efficiency function on design.

[23] See American Gas Association, 'Proceedings of the Natural Gas Department,' 1947, pp. 4-12, for a comprehensive discussion of various formulas in use.

[24] Ibid. p. 5. The assumptions are steady isothermal flow, level line, ideal gas, and so forth.

[25] The separation is not made here between the transformation and energy supply functions because (as the equation shows) if there were no frictional forces there would be no energy requirement. In other words, there is no material transformation in the sense that there is in manufacturing.

$$X = a\frac{T_o}{P_o}\left[\frac{(P^2{}_1 - P^2{}_2)D^5}{GtLf}\right]^{1/2} \qquad (8, 18)$$

(see nomenclature below).

To apply this formula requires empirical determination of two relationships: how the friction factor, f, varies with the diameter and roughness of the pipe, and how the particular gas deviates from the perfect gas law under different conditions of pressure and velocity.

In order to simplify the mathematical treatment, the production function to be derived here will be limited to pressures below 500 pounds in which the second variable, the supercompressibility of the gas, is not important. The empirical formula for the coefficient of friction in terms of diameter derived by Weymouth [26] has proved satisfactory for most lines and will be used here. It is:

$$f = \frac{b}{\sqrt[3]{D}} \qquad (8, 19)$$

2. *Production and cost functions*

The engineering production function consists of the transformation and energy supply functions, which may be combined by eliminating the energy (or power) required. For most convenient use, it should be transformed by substituting the properties of the equipment for any process variables such as pressure, temperature, or speed which appear.

The following variables and parameters [27] will be used:

FACTOR DIMENSIONS (INDEPENDENT VARIABLES)

D = inside diameter of pipe (inches)
S = working stress in pipe (*p.s.i.*)
T = pipe thickness (inches)
R = compression ratio (P_1/P_2)
L = length of line between stations
k = efficiency constant of compressor (.98)

INPUT AND OUTPUT DIMENSIONS (PARAMETERS)

X = output (capacity) of pipeline in million cubic feet per day
g = specific gravity of gas (.607)
t = flowing temperature of gas (520°F. abs.)
P_o = pressure of measurement
t_o = temperature of measurement (520°)
n = ratio of specific heats
l = length of line (100 miles)

[26] A.S.M.E., op. cit. 1912, pp. 195-7.
[27] The parameters are taken from the *Gas Engineer's Handbook*, loc. cit.

OTHER (INTERMEDIATE) VARIABLES

P = initial pressure
f = friction coefficient
H = power required (horsepower)
W = work

a. *The engineering production functions*

(1) COMPRESSION: The production function for the compressor is obtained by eliminating E_r from equations (8, 14) and (8, 17) and inserting the values of the parameters which represent typical conditions for natural gas. The output is customarily measured in horsepower:[28]

$$H = \frac{14.2}{e} P_s(R^{.213} - k)X \qquad (8, 20)$$

where

k and e = efficiency factors depending on the type of compressor.

(2) PIPELINE: Equation (8, 18) can be transformed to include only the factor dimensions by substituting for the pressure from Barlow's formula for pipe stress: $P_1 = \frac{2ST}{D}$. Making this substitution in equation (8, 18) together with the expression for the friction factor, equation (8, 19), gives the following production function for the pipeline:

$$X = C_1 D^{5/3} ST \sqrt{\frac{R^2 - 1}{R^2 L}} \qquad (8, 21)$$

where

C_1 = a constant.

(3) PLANT FUNCTION: the output of the compressor is identical with the input of the pipeline, and in addition the two processes are linked by the common variable R (the compression ratio). It would be possible to combine the two functions by eliminating R between them but the solution for X is made more difficult. It is simpler mathematically to take equation (8, 21) alone as the production function for the combined process, substituting for the value of R from equation (8, 20) when necessary. This is possible because the only other variables in the compressor function are k, the compressor efficiency factor, and X (output). For any given stage of technology k may be considered as a constant. Therefore, any value of R implies a given amount of horsepower per unit of output.

b. *Input and cost functions*

Each process represents a complementary set of inputs, some fixed and

[28] Lehn, op. cit., p. 446.

some current. The cost of each of these inputs may be expressed in terms of one or more of the process variables. The input function relates these engineering variables and the economic variables usually employed in the cost function. Although the cost function may be written directly in terms of process variables, it is useful to identify the intermediate variables which represent economic 'quantities.'

In the present case, horsepower may be treated as if it were an economic commodity because, to a first approximation, it determines the amount of each of the joint factors used in the compressor station. In the case of the pipeline, the weight of pipe is such an intermediate variable. Not all the costs of the pipeline are associated with the weight of the pipe, however; some depend on the diameter and length. They must be introduced into the cost function separately.

Statistical studies employing these variables can be utilized to determine the cost function. To make current and durable inputs comparable in a cost function, the proper charge to be associated with the capital goods must be determined. One method would be to define the unit of time as the expected life of the most durable factor in the production function. For very durable equipment this procedure is not feasible. A more practical alternative is to take the average annual depreciation, obsolescence, and maintenance charges associated with each type of equipment in the production function. This is the usual form in which cost data are collected. In processes such as the present one where there is no possibility of varying the proportion of labor required for a given type of capital good, the labor charge may be treated in the same way.

On the basis of the cost analyses of the Federal Power Commission [29] and one of the manufacturers of pumping equipment, [30] costs in the present process will be assumed to fall into three categories only:

Proportionate to particular dimensions of capital goods,
Proportionate to total investment,
Proportionate to output.

Since we are only concerned with two types of very durable capital goods, pipe and pumps, we can simplify the formulation further. The actual physical life of each of these items is of indefinite length; the annual charge to be made represents largely obsolescence and the maintenance necessary to keep their efficiency unimpaired. The obsolescence charge may be included (if it is considered at all) with the other costs, such as interest and taxes, which depend on the total investment. All these costs may be expressed as a percentage, i, of the total investment.

The types of cost elements in the final cost equation may then be di-

[29] *Natural Gas Company Cost Units,* 1945.
[30] Clark Brothers Company, *High Pressure Pipe Line Research.*

vided into two groups: those which vary with the design variables, and those which are fixed by the quantity of output and the length of the line. The effect of the design variables on installation and operating costs is based on three engineering studies.[31] The simplified[32] cost equation adopted here is of the following form:

$$C = (a_1 i + b_1)H + (a_2 + a_4)iW + (b_2 + a_3 i)D + (d_1 l + d_2) \quad (8, 22)$$

where

a_1 = installation cost per horsepower
a_2 = cost per ton of pipe
a_3 = pipe installation cost depending only on diameter
a_4 = pipe installation cost depending only on weight
b_1 = annual operating cost per horsepower (labor, fuel, materials) for given load factor (f)
b_2 = annual pipeline maintenance cost per inch of diameter
d_1 = pipe installation cost depending only on length
d_2 = other operating costs (independent of design factors)
i = combined annual rate of interest, obsolescence, and taxes (as percentage of total investment)

Of these parameters, the a's represent installation costs and are always multiplied by i in the cost function; the b's are operating costs which vary with the design; the d's do not vary with the design chosen.

The 'quantities' of the capital goods have been introduced into the cost function as intermediate variables because some costs are proportional to them. These quantities may be expressed in terms of the factor dimensions as follows:[33]

$$\begin{aligned} W &= 28.2\, LDT \\ H &= (28.75R - 13.9)X \end{aligned} \quad (8, 23)$$

The latter formula is a linear approximation to equation (8, 20) which holds for the moderate variations in R which are economical.[34] The cost function corresponding to equation (8, 7) is determined by substituting these values for W and H into equation (8, 22).

In the following solution, the values taken for the cost parameters are:

[31] Clark Brothers Company, op. cit.; Kepler, W. R., 'Gas Pipe Line Factors Affecting Minimum Cost,' *American Gas Association Proceedings*, 1930, pp. 797, 819; Lehn, op. cit. pp. 445-60.

[32] Those variables which showed little effect on costs have been omitted.

[33] Lehn, op. cit., shows the cost of pipe to depend on: $(LT^{.8}D^{1.06})$, but this refinement has little effect on the final results.

[34] Using $e = .95$ and $k = .96$ in equation (8, 20).

a_1 = \$130 per horsepower
a_2 = \$100 per ton
a_3 = \$26,000 per inch of diameter per 100 miles
a_4 = \$12 per ton
b_1 = \$19 per horsepower
b_2 = \$1250 per inch per 100 miles
d_1 = \$3,000 per mile
$d_2 = .18C$
$i = .06$

These values are representative of the prewar conditions shown in the several cost studies referred to. The value of i, however, is abnormally low; the effect of increasing it is shown in Chart 4.

C. GRAPHICAL SOLUTIONS

In the interests of brevity, only graphical solutions and transformations of these equations are presented here.[35] Chart 1 shows a graph of equation (8, 21), including the least-cost solution obtained by applying equation (8, 9) to equations (8, 21) and (8, 22). The production function in terms of 'quantities' of the principal inputs—horsepower and tons of pipe—is given in Chart 2. This represents a transformation of equation (8, 21) to the form of equation (8, 6), using the intermediate variables in equation (8, 23). Such a transformation is possible in the present case if the value of i is taken as fixed.[36]

Chart 3 shows the various types of cost curve which can be derived by successive least-cost solutions to the production function. The curves shown are as follows:

- *LAC:* long-run average cost curve (all factors variable)
- *LMC:* long-run marginal cost
- *IAC:* intermediate average cost (fixed pipeline, variable amounts of horsepower)
- *IMC:* intermediate marginal cost
- *SAC:* short-run average cost (fixed plant)

The intermediate curve, which is not usually considered in economic theory, is useful in a dynamic analysis in which demand increases after the plant is already installed. In this case, as the curves illustrate, it is economical to vary the amount of horsepower over a considerable range before installing additional pipe.

Chart 4 gives the result of transforming equation (8, 21) into economic

[35] The detailed calculations are given in my 'Engineering Production Functions,' loc cit.
[36] It happens that the optimum value of T depends only on i.

CHART 1

ENGINEERING PRODUCTION FUNCTION: GAS TRANSMISSION

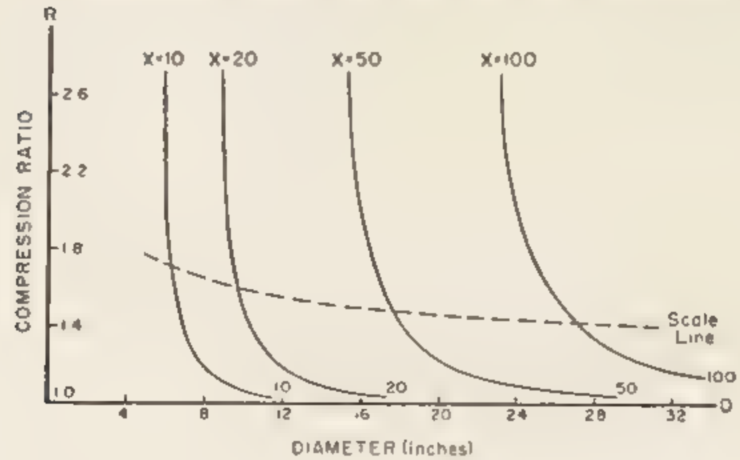
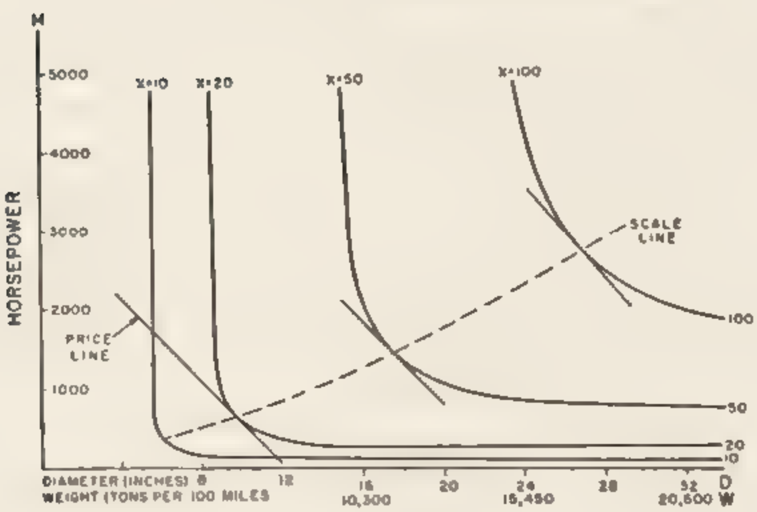


CHART 2

PRODUCTION FUNCTION (PHYSICAL INPUTS)



aggregates by the method of equation (8, 11). It also shows the effect of a variation in capital cost on the least-cost solution.

The combination of processes described here permits a fairly high degree of substitution between different types of equipment (see Chart 2). Since both pipe and pumping stations are very capital intensive, however, the possibility of substituting capital for current inputs (labor, maintenance materials, etc.) is more limited, as is shown in Chart 4. The effect of raising the capital charge, i, from 6 per cent to 10 per cent, for example, is very small.

The other notable feature of this production function is its great economy of scale, shown in the LAC of Chart 3. The combination of flexibility and economy of scale makes it economical to install a considerably larger

CHART 3

COST CURVES: GAS TRANSMISSION

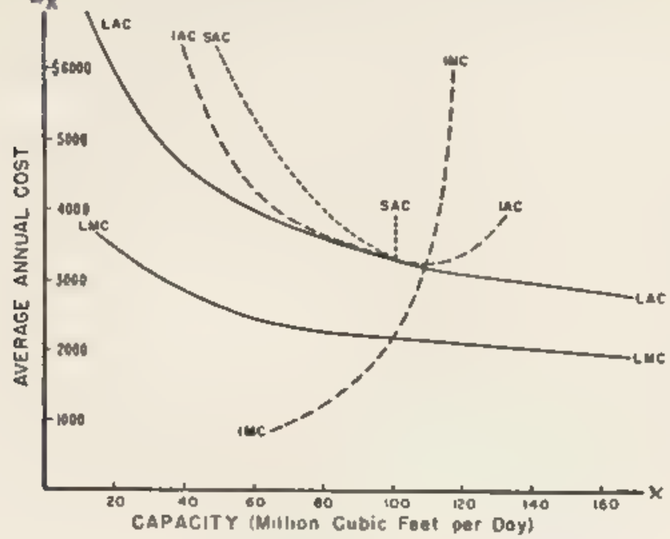
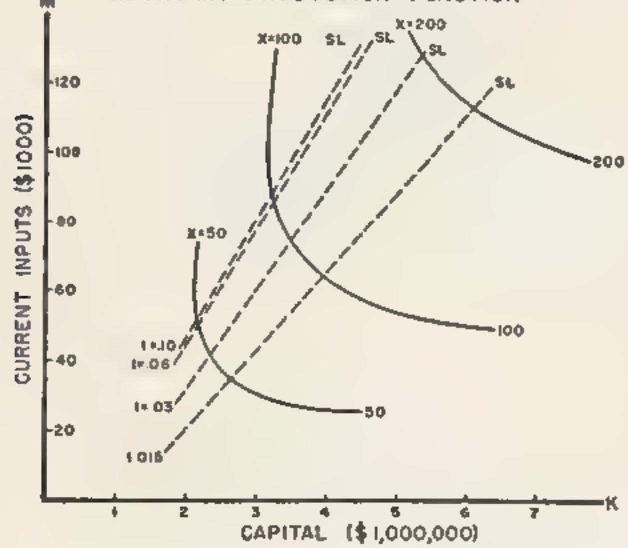


CHART 4

ECONOMIC PRODUCTION FUNCTION



pipe capacity than current demand requires if a rise in demand is anticipated.[37]

The advantages of an engineering analysis in deriving production and cost functions in the present case are apparent. A purely statistical analysis of the relation of output of all pipelines to the diameter, pressure, and horsepower, for example, would be a hopeless task because the observed range of variation in their proportions is small and it could not adequately take into account the different operating conditions for which

[37] The effect of these features of the production function on the timing of investment is analyzed in my 'Overcapacity and the Acceleration Principle,' *Econometrica,* January 1952, pp. 1-28.

the lines were designed. A correlation among capital, labor, and output would probably be even further from the true form of the production function. The advantage of the engineering approach is that it reveals the form of the design laws which permit variation in the inputs.

IV. The Use of Process Data

Various uses of engineering analysis and process results in future input-output work can be suggested.[38] These come under three headings:

(1) as a basis for classification and aggregation,
(2) as a supplement to other methods of calculating input coefficients,
(3) in testing and modifying the underlying assumptions of fixed proportions among inputs and outputs.

The first of these is discussed in the next chapter.

The principal advantage of an engineering approach in determining input coefficients lies in its greater generality. The coefficients depend less on the particular sample studied than do purely statistical results, and the effects of varying any of the parameters can be estimated. This advantage is clearly shown in the natural-gas case and in Chapters 10 and 11 below. These cases suggest that a combination of engineering and statistical techniques will give the best results, with the proportion of each depending on the type of technology. In the gas case just described, the engineering ingredient is probably larger than it can be in most other industries.

The possibility of determining the shape of certain production functions suggests various lines of development of the input-output technique. From five processes already studied, it appears that energy and capital goods have a fairly high elasticity of substitution in the long run, between .4 and 1.0 in various types of technology. It may be possible to determine mean elasticities of this sort for several categories of processes. These could be taken into account in applications in which a significant change in price ratios was expected. In other cases, knowledge of economies of scale of the magnitude shown in the present example might be utilized in estimating variations in the capital coefficient.

Aside from modifying the basic input assumptions for certain applications, the measurement of process functions should permit an estimate to be made of the error involved when constant coefficients are assumed. In the pipeline case, an assumption about expected price changes would permit the variation in input and capital coefficients to be readily deter-

[38] Other applications of this method are discussed in my 'Engineering Production Functions,' loc. cit.

mined. The composition of the capital stock is subject to considerable variations, for example, but the total amount per unit of output is not.

Finally, the process function is a promising tool for getting at the difficult problem of technological change. The three sets of equations on which it is based are affected quite differently by changes in techniques. In the pipeline example, the transformation functions of the pipe and compressor have not changed in forty years or more, and do not appear subject to change since they are forms of basic laws of thermodynamics. There have been modifications in the form of the energy supply functions as the processes have been studied, and a steady increase in efficiency of equipment. The input functions themselves have not changed greatly, but the possible combinations of process variables have expanded as the technique of pipe manufacture has improved. This type of technological change is represented by an increase in S, the allowable working stress in the pipe. Working pressures have more than doubled in recent years with a great change in the type of equipment used. This has all happened without much change in the engineering production function but by expanding the range of allowable variations in the parameters.

This type of technological change is discernible in many industries and to a large extent it is predictable. The engineering production function for electric generation stays constant, but the design of turbines and boilers is improved and the fuel consumption per unit of output declines as progress is made in metallurgy. Tracing out such changes in parameters in the process functions may give a clue to the economic effect of future engineering developments.

APPENDIX

Examples of Process Functions

In the following table, examples of functions of the type of equations (8, 1), (8, 2), and (8, 3) are given. The input functions, although not shown, can all be written in terms of the variables in equation (8, 3) in each case. The chief problem in deriving these functions was to make the transformations necessary to eliminate all variables except the properties of the inputs and output.

The third process, electrical transmission, is a case in which no mate-

rial transformation takes place. Other such examples are gears, pulleys, electrical transformers, steam pipes, and other processes whose function is to transmit energy. In all these cases the energy required is equal to the energy loss in the equipment, the amount needed in material transformation being zero. The energy supply function, in effect, becomes the engineering production function.

Together with the gas compressor, in which the input is work, these processes illustrate the principal forms of energy input: mechanical, thermal, and electrical. Although the type of equipment involved in utilizing each kind of energy is quite different, there are considerable similarities among the design laws themselves and in the techniques involved in transforming them into production functions.

Examples of Engineering Production Functions for Processes

Process	Energy input (1)	Change in material (2)	Transformation function equation (1) (3)	Energy supply function equation (2) (4)	Engineering production function equation (3) (5)	Symbols used
Evaporation	Heat	Liquid to vapor	$X = \frac{E_r}{l}$	$E_r = \frac{UA\Delta t}{N}$	$X = \left(\frac{U\Delta t}{l}\right)\frac{A}{N}$	E_r = energy required X = amount evaporated l = latent heat A = area of evaporator N = number of 'effects' (evaporators) Δt = temperature change U = heat transfer coefficients
Electrolysis	Electric energy	Chemical decomposition	$X = \frac{cP_r}{V_o}$	$P_s = P_r + NI(V_l + IR)$ $V = a + b \log I$	$P_s - \frac{X}{d}\left[V_o + a + b \log\left(\frac{X}{dN}\right)\right] - \frac{R}{N}\left(\frac{X}{d}\right)^2 = 0$	V_o = decomposition voltage P_r = power required P_s = power supplied N = number of cells I = current V_l = electrode voltage R = cell resistance X = output a,b,c,d = parameters
Electrical transmission	Electric energy	None (heat loss in line)	$E_r = o$	$E_s = E_r \frac{aX^2\rho Lt}{V^2 \cos^2 \theta A}$	$X = V \cos\theta \sqrt{\frac{aAE_o}{\rho Lt}}$	V = voltage A = conductor area E_l = energy loss E_r = energy required E_s = energy supplied X = output (kw) a = constant ρ = resistivity of wire L = length of line $\cos\theta$ = power factor t = time

Chapter 9

PROBLEMS OF CLASSIFICATION AND AGGREGATION

Mathilda Holzman

I. Classification and Input-Output Analysis

A. INDUSTRIAL CLASSIFICATION REQUIREMENTS FOR INPUT-OUTPUT ANALYSIS

The principles governing the definition of industries for input-output analysis are implicit in the theory and techniques of the analysis itself. An analytical scheme based on the interdependence of the various parts of a national economy implies some basic characteristics of the economy which give rise to consistent patterns of interdependence. In delimiting the industries of the economy, such characteristics may be used to define industries so that they will be functionally related. In the open system the particular functional relationship upon which input-output analysis is based is the technical production function relating products to inputs.[1] Since the industrial classification system employed must by definition determine what are called products and inputs, the greater the knowledge of the technical and natural conditions of production upon which the definition of industries is based, the more accurate will be the relationships to be measured, i.e. the technical coefficients of production.

In theory, disregarding the practical impossibility of such an undertaking, if the technical production function for each of the many million products produced by the individual enterprises were known, technical coefficients of production could be derived without recourse to statistical data such as those obtained from the *Census of Manufactures*.

For the analysis of most problems conceived to be important in economics such detailed coefficients would scarcely be useful. The detail with which the technical coefficients are computed and their aggregation obviously depend upon the purpose for which they are to be used.

[1] It is not meant to imply that the production function is not basic to the closed system but rather that in addition in a closed system the structure of consumer and other bill-of-goods demands must also be included in the analysis in the same way as the technical production function. See Chapter 1.

Input-output analysis is concerned in general with being able to take account of changes in a datum which will significantly affect the variables of the system. A change in the level of final demand changes the level of indirect output. A change in the structure of final demand changes the relative amounts and perhaps the level of indirect output. A technological change in an industry alters its input requirements with corresponding repercussions on the industries from which it buys, and conceivably alters other relationships as well. When any of the aforementioned changes takes place so as to increase the output demands being made on an industry or industries, there is always the possibility that such an increase may be beyond the productive capacity of the industry, either because of the limits of its own processing capacity or because of the lack of availability of some ultimate raw material input.

These considerations suggest that an adequate delimitation of industries must identify certain crucial products of the economy. In the case of the products which comprise final demand, if enough is known to form a group of products whose use is strictly complementary, the whole group of products is sufficiently identified by one of its members. If the structure of final demand changes and the use of the group of products is really complementary—such a change will affect the whole group in the same way. Therefore, no information is sacrificed by lumping them together. By the same token, then, products which are substitutable but have dissimilar production functions [2] must be separately identified since a change in their relative consumption will have different effects on the economy.

In addition to final product identification ultimate raw materials, for which there exists the possibility of shortages and which do not have close substitutes (substitutes whose use would not change the cost structure of consuming industries), must be separately identified.

Finally, there are the considerations having to do with cost structure suggested by, but much more numerous than, possible technological changes in an industry.

If the industrial classification set-up identified each ultimate raw material at each stage of fabrication or distribution and thereby every intermediate and final product of the economy, the problem of instability of technical coefficients over time due to changing product-mix would not exist in input-output analysis.

This problem arises only because industries are aggregates of products which have different production functions and which are not complementary in use. If the classification system precludes the possibility of identifying each single raw material and product of the economy and their

[2] Substitutable products having similar production functions can, for all analytical purposes, be considered as the same product.

alternative production functions, each industry as defined must be some kind of an aggregate of products.

B. MATHEMATICAL CRITERIA FOR AGGREGATION

In terms of purely mathematical considerations, the criteria for the grouping of products into industries are few and obvious. Starting analytically with an idealization of all the structural input-output relations between the flow of goods and services in the form of a matrix,[3] it can be determined mathematically what consolidations and rearrangements of the data would not affect the solution of the system or rather of alternative systems of equations based on that matrix. Such a matrix would not be square, but would have an excess of columns over rows corresponding to the alternative production functions yielding *identical* outputs. This matrix could be squared if the information for maximizing were available. This matrix would yield 'real'[4] technical coefficients. The matrix defined above could be consolidated without loss of information if some duplication of information were present (linear dependence).

C. A TECHNICAL SCHEMATIZATION OF THE ECONOMY

Such a matrix is not available and an industrial classification must be developed on the basis of theoretical considerations—economic and technological—which corresponds as well as possible to the industries of this hypothetical consolidated matrix. The problem is to decide, on the basis of technical and economic considerations, which products are likely to have linearly dependent production functions, i.e. identical or proportional input structures.

Examination of the technical processes of manufacture may assist in making this decision. For example, in the United States four general operations are performed on almost all materials: the cultivation or extraction of the raw material, its purification or conversion into an economically useful form, the combination of these purified materials into the basic materials of industry, and finally, the fabrication of these basic materials into final products.

[3] If capital requirements are being taken into account, two such matrices are required, one for flows, and the other for stocks.

[4] In general, industries, as defined in the 96-industry classification, are aggregates of products having different production functions so that the technical coefficients for the industry are (base-year quantity) weighted averages of the coefficients appropriate to the separate products and in this sense are not 'real' coefficients. Should the relative quantities of the products—the product-mix—change, the technical coefficients may no longer be appropriate. However, 'real' coefficients may change because of technological change. They may also change because an unjustifiable assumption of fixed coefficients (linear homogeneous production function) for the product has been made.

1. Extraction includes all operations which are necessarily performed at the raw material source, such as growing and harvesting of crops and mining of minerals. Other operations which may be (and in some cases usually are) performed at the source—grain winnowing, ore concentration—are not extractive functions.

2. Purification consists in those operations by which the extracted raw material undergoes whatever changes from its natural state are necessary for its further use. For fruits and vegetables this involves nothing more than cleaning them if they are destined for household consumption. For most materials there are two stages of purification, a series of mechanical operations followed by chemical processing involving some degree of chemical change. This stage ends when the material reaches the purest form which is desirable. In general, different degrees of purity for a given material are usable, depending on the particular product to be made; and hence a diminishing amount of the material goes through the latter purifying processes.

3. Material combination is distinguished from material fabrication in that the product is designed for a wide variety of final uses and is sold on a weight basis with only a few quality specifications. Examples of products of this stage are alloys, steel ingots, papers, cotton gray goods, window glass, cement, canned and frozen foods, basic plastics, manufactured gas, electricity.

4. Fabrication consists of three sets of processes: the forming of product parts, whether by cutting as for textiles, or stamping, pressing, forging, and machining for automobiles; the assembly of the parts into final products; and finally whatever finishing—painting, dyeing, etc.—must be reserved until after assembly. Assembly processes apply only to the fabrics, wood, and metals groups of fabricated products. They are distinguished from the forming operations of fabrication in that little change in the form or shape of the materials takes place. Assembly of complicated products involves one or more stages of subassembly.

Deviations from this pattern consist primarily in the omission of one or both of the last two operations in the case of materials which require no more than initial purifications: e.g. sugar, meat, gasoline. For many materials, 15 or 20 separate processing stages may be observed, however, and there are border-line decisions which are not clear cut whether a particular process is part of one of the above-mentioned operations or of the operation immediately following.

D. TECHNICALLY DETERMINED PRODUCT GROUPINGS

In terms of such operations as those described in C above, the flows of goods and services are seen to be functionally related by the technical

structure of production. If the unconsolidated matrix of B above were available, linear dependence could be ascertained simply by inspection and the appropriate rows or columns consolidated. Such linear dependence, however, can be regarded as a reflection, except where it is the result of chance, of characteristics of the underlying technical structure of production for certain products. At this point linear dependence and the possible consolidations arising from it can be considered from the point of view of the schematization of the process of production given in C above. There are, logically speaking, two different principles of aggregation of products which will yield industries appropriate to the theoretical scheme.

1. *Vertical and horizontal aggregation*

When the output of one industry is wholly absorbed by a second industry, and the production function of the second industry permits no substitution of any other product for the output of the first industry, the two industries may be combined in a single industry. A third industry could be added if it were related to the second in the same way as the second to the first.

The metals, for example, might be treated in this way, with industries for steel ingots, copper ingots, etc., which could include all stages of manufacture for the particular metal from extraction through the material combination stage. A similar aggregation for steel would include iron ore, pig iron, and steel ingots.

For horizontal aggregation, products in the same stage of manufacture are grouped together in one industry. This is appropriate only to the extent that technical coefficients are not unstable because of changing product-mix, and indicates either strictly joint products or products whose technical coefficients are proportional. Horizontal aggregation is suggested by this schematization of manufacture because the products for which the same type of material is at the same stage of manufacture are most likely to have similar production functions, therefore identical or proportional input structures.

2. *Partial horizontal aggregation, the process-service industry* [5]

Suppose that a certain array of products had production functions such that each product used labor and machinery in the same ratio and that 'labor' and 'machinery' have been defined as two separate inputs. If the remaining inputs used with labor and machinery had not entered these productive operations also in identical proportions, linear dependence of

[5] The initial impetus to the investigation of the use of process service industries for input-output analysis was given the Harvard Economic Research Project by Hollis Chenery, who suggested the technique and processes for which it might prove useful.

the production functions for the products would not exist. However, if it were desired for analytical purposes to make use of the fact that each product consumed labor and machinery in the same ratio, one could do so by defining a new fictitious industry.[6] The only inputs to this fictitious industry would be labor and machinery. The output would be the service of labor and machinery in performing the particular operation by which the final products are made. Labor and machinery would be removed from the original input structures, and instead, the output of the new fictitious industry would be distributed to the final products.

As an example of this, consider rayon, nylon, silk, and cotton hosiery. Suppose separate industries exist for each of these products, and that these industries are so defined that the only process included in each of them is knitting. A fictitious industry, the output of which is the service, knitting, could be conveniently defined. Its inputs would be knitting machines, operator labor, and energy. If the assumption is correct that these input requirements are the same per stocking, whether the stocking is cotton, rayon, silk, or nylon (i.e. the technique of knitting a stocking is identical regardless of the textile of which the stocking is made), the service, knitting, could be distributed to each of the hosiery industries proportionately to its output of stockings. At the same time, knitting machines, operator labor, and energy associated with the use of knitting machines would be eliminated from each hosiery industry's production function since these now constitute the input structure of the knitting industry whose service the hosiery industries buy.

This knitting industry is a partial horizontal aggregation of industries consisting of a group of input coefficients which occur in fixed proportions for several products. Such an aggregation defines a processing industry, whose output is a service distributed to the several products.[7] Technologically, this implies that the production functions of the products are similar except for the materials being processed.

The use of such process-service industries must depend on the degree to which the same process is applied to different products. Process-service categories should be sought among the auxiliary processes which are common to many industries because they are not specific to any material. In addition to transportation and distribution (trade), steam production, electric generation, materials handling, and storage are such processes.

[6] In his article 'Introduction to a Theory of the Internal Structure of Functional Relationships,' *Econometrica*, October 1947, Professor Leontief develops the general mathematical conditions which must be met to establish such an industry. The fictitious industry must, in his terminology, be composed of a 'separable subset' of the variables occurring in the production function of each of the products.

[7] In the classification for the 1939 input-output study, trade and transportation are examples, logically, of this type of industry. However, this kind of aggregation can be applied to a wider range of processes than these two.

Selection of specific processes for treatment as process-service categories requires a detailed technological analysis in each case. Tentative suggestions based on the schematization of products outlines above are: (a) deep level mining, (b) strip mining, (c) crushing and grinding, (d) flotation, (e) electrolytic separation, (f) foundry work, (g) machining, (h) line assembly, (i) canning, (j) spinning and weaving, and (k) dyeing.

3. *Relative merits of the three methods of aggregation*

The advantages of the three possible principles of building up industries from original raw information can be evaluated in terms of the relative invariance of coefficients for a given *number* of industries secured by the alternative methods of aggregating. The use of a process-service industry for a particular group of products as opposed to a horizontal aggregation of the products avoids instability in material input coefficients due to changes in product-mix over time. It also has the advantage, as opposed to defining a separate industry for each material at each stage of manufacture (or process), that it reduces the number of industries which have to be included in the matrix in order to convey the same amount of primary information.[8] If 10 raw materials and 5 processes (or stages of manufacture) through which each material goes are to be separately identified, the number of industries when the processes are treated as process service industries is 15. When separate industries are defined for each material in each process, that number increases to 50.

The advantage of identifying separate processes by which a product is made as opposed to a vertical integration of the product and its intermediate products the outputs of the separate processes—lies in identifying the cost structures of the separate processes. This is important because technological change, in general, will affect cost structures differentially.

E. EFFECT OF THE THREE TYPES OF AGGREGATION ON AN IDEAL MATRIX

To illustrate what is implied by the different methods of aggregating, consider the schematic input-output distribution matrix given below:

[8] It must be stressed that this sentence refers to an ideal classification. The introduction of process-service industries into the 96-industry classification would *increase* the number of industries. At the same time, however, they would serve either to (1) identify cost structures of separate processes in the case where they are used as a substitute for vertically integrated industries; (2) avoid product-mix where they are used as a substitute for horizontally integrated industries. In either case they would be used only in those instances in which their use gave rise to a smaller increase in the total number of industries, for a given precision of information (number of technical relations), than the alternative definitions.

	X_1	X_2	X_3	X_1B_1	X_1B_2	X_1B_3	X_2B_1	X_2B_2	X_2B_3	X_3B_1	X_3B_2	X_3B_3	M	E	Bill of Goods
X_1				•											
X_2	•		•				•								
X_3										•					
X_1B_1					•										
X_1B_2						•									
X_1B_3															•
X_2B_1								•							
X_2B_2									•						
X_2B_3															•
X_3B_1											•				
X_3B_2												•			
X_3B_3	•	•													
L				•	•	•	•	•	•	•	•	•	•	•	
M				•	•	•	•	•	•	•	•	•	•	•	
E				•	•	•	•	•	•	•	•	•	•	•	

X_1, X_2, X_3 are raw materials. L, M, and E are respectively labor, machinery, and energy. B_1, B_2, B_3 are successive processes of manufacture so that, for example, X_1B_2 represents an intermediate product made with the material, X_1. X_1B_3 and X_2B_3 are thus the only products which are items of final demand.

For simplicity's sake, dots have been used to indicate distribution of products to consuming industries.

In this distribution matrix X_1, X_1B_1, X_1B_2, and X_1B_3 are products which represent the ideal case for vertical aggregation since the whole of X_1 is consumed by X_1B_1, all of X_1B_1 by X_1B_2, and all of X_1B_2 by X_1B_3.

If X_2B_1 and X_3B_1 are products whose input structures are the same or proportional except for the material inputs, which are respectively X_2 and X_3, a process-service industry, B_1, can be set up. Its output is distributed to the materials X_2 and X_3, either with the same amount of processing per unit of product to both materials if consumption of labor, machinery, and energy (the processing inputs) is the *same* per unit output, or an amount

in the same proportion as the proportion which the processing inputs for X_2B_1 and X_3B_1 are to each other.

Under the same assumptions for X_2B_2 and X_3B_2 and for X_2B_3 and X_3B_3, two more process-service industries, B_2 and B_3, can be set up.

The consolidation of the matrix resulting from the vertical aggregation of products X_1, X_1B_1, X_1B_2, and X_1B_3 and from setting up the process-service industries B_1, B_2, and B_3 is given below.

A horizontal aggregation of products, of which no example is given in this consolidation, has formally the same effect on the matrix as the vertical consolidation if n industries are aggregated either vertically or horizontally the number of rows and columns in the matrix are each reduced by $n - 1$.[9]

F. SUMMARY

Thus far, this chapter has been concerned with the determination of a completely valid industrial classification for input-output purposes. Such a classification is designed to secure unequivocable information about the production of goods and services. To secure such information, intra-industry product-mix must be eliminated in all cases in which the product-mix may change. In addition, separate processes must be identified in order to isolate[10] the effects of technological change.[11]

The consolidated matrix of the ideal type discussed above would result from exploiting all opportunities for aggregation which did not obscure information. To the extent that an industrial classification represents more gross aggregation, it must be constructed on the basis of less exact principles.

G. FINAL DEMAND INDUSTRIES

Before passing to consideration of the actual work on classification which has been done by the Harvard Economic Research Project, the effect on classification principles of final demand remains to be considered. Thus far, problems have been dealt with almost entirely from the point of view of the structure of production. The general role of consumption in input-output analysis is considered in Chapter 12. A very brief statement of problems in industrial classification of industries whose output consists in goods consumed by households will be given here.

[9] W. W. Leontief, *The Structure of American Economy*, 1919-39, New York, 1951, pp 14-16

[10] A change in technique would cause an input-output prediction to be incorrect. Such errors in prediction are by and large unavoidable without prior knowledge of technological change However, if processes are separately identified, the particular process affected by the change can be identified and thus the source of the error in prediction.

[11] This latter notion is circular unless processes have a well-defined technical meaning The concept of an industrial process is discussed at length in Chapter 8.

	X_1B_3	X_2	X_3	B_1	B_2	B_3	M	E	Bill of Goods
X_1B_3									•
X_2	•		•						•
X_3	•	•							
B_1		•	•						
B_2		•	•						
B_3		•	•						
L	•			•	•	•	•	•	
M	•			•	•	•	•	•	
E	•			•	•	•	•	•	

Data are likely to be collected by those primarily interested in household consumption on the basis of a different classification of industries of origin from that used by those concerned with the technical aspects of production. For use in the open system, all that is required of data on consumer expenditures is that they be available in enough detail to permit assignment to the appropriate industry of origin on the basis of the Leontief classification.

However, in the closed system—with households treated as an industry with a technically determined input structure—a conflict of criteria for industry definition arises. In dealing with problems of estimating consumption patterns, an adequate classification is one which identifies *substitutable* groups of consumption goods. Complementary goods can be grouped together with no loss of information. But if a group of complementary goods have different production functions, so that a change in one production function, and, therefore, its cost structure, is not accompanied by similar changes in the production functions for the rest of the group, the classification is inadequate from the point of view of production.

Adjustments between classifications appropriate for the two purposes within the limits set by available data require finding 'least common denominator' industries. These are industries producing goods complemen-

tary in final demand and having similar production functions. It is obvious that in cases where goods are wholly consumed by non-bill-of-goods industries the concept of complementarity in use is logically the same and the procedure for defining industries is identical.

II. The 96-Industry Classification

This section is concerned with the characteristics of the 96-industry classification of the United States economy used for the 1939 input-output study and proposals for the refinement of the classification.[12]

The aggregative criteria implied for determining the 96-industry classification are identity of product and quantitative similarity of cost structure of products included in an industry.[13] The degree to which these requirements may be met depends ultimately on the statistical data available for determining products and cost structures. Given the limitations of the basic statistical data, the classification problem is to choose, for a determined number of industries, that aggregation of products into industries which will show the greatest conformity to the requirements of identity of product and quantitative similarity of cost structure formulated in Section I.

Analysis of the industrial groups · food, beverage, and tobacco; textiles; and chemicals will serve to point out the considerations involved in expanding the 96-industry classification. The changes suggested imply no changes from the usual sources of data and assumptions about input structure.

The basic sources used in determining outputs and cost structures are the *Census of Manufactures*, 1939, and the *Census of Mineral Industries*, 1939. From the point of view of our aggregative criteria, the fundamental limitation imposed by Census data arises because Census information is gathered on an establishment basis, and the establishments are aggregated to form Census industries, the unit from which Leontief industries are formed. To the extent that separate establishments produce a variable array of products, which does not conform to our aggregative criteria, Census industries in themselves cannot conform to these criteria.[14] No decision about the manner in which to aggregate Census industries to form Leontief industries can overcome this limitation in the

[12] The committee on classification of the Harvard Economic Research was composed of Mrs. Sara F. Clark, Hollis B. Chenery, Irwin Leff, and myself.

[13] Leontief, op. cit. p. 20.

[14] In some instances it is conceivable that to get better cost data for a particular product, the retabulation of Census data could be run, for example, for all establishments in a Census industry whose output consists entirely in that product.

Census data, the importance of which has never been assessed independently of the general predictive ability of the input-output scheme.

A. FOOD, BEVERAGES AND TOBACCO

Industries 10 through 21 of the 96-industry classification are made up of the food-, beverage-, and tobacco-processing industries. In a very general way, the products of these industries may be considered as substitutes for each other. The actual structuring of these industries for the 96-industry classification has been done for the most part on the basis of similarity of the principal raw material being processed. This has led to industries whose products [15] would be considered to be closer substitutes for each other than they would be for the products of other industries in this group.[16] That the Census industries combined in Leontief industries do not meet the criterion of quantitatively similar cost structures may be seen from the figures entered in Columns 2 through 10 of Table 1, inserted in pocket at back of book. In these columns the average values of three elements of the cost structure of each Census industry in the Leontief industry classification, together with the range of values for these elements and the per cent which the range is of the relevant average, are given. Corresponding to each Leontief industry, an industry has been formed for which the criterion has been to secure, using no more than the given number (12) of Leontief industries, the greatest possible quantitative similarity of input structure for each industry. These input structure industries do not precisely fulfill the requirement of identical input structure of Census industries and in addition are very different, productwise,[17] from the Leontief industries. For precise fulfillment of the criteria, the number of industries in this group would have to be greatly increased. Such precise fulfillment requires industries not only quantitatively similar in input structure but qualitatively similar as well, whose products were identical.[18] If each industry fulfilled the two aggregative criteria, quanti-

[15] Neglecting joint products.

[16] The industries and the principal raw material (where a principal raw material exists) being processed in each are: flour and grist mill products, wheat; canning and preserving, fruits and vegetables; bread and bakery products, flour; sugar refining, sugar; starch and glucose products, corn; alcoholic beverages; nonalcoholic beverages; tobacco manufactures, tobacco; slaughtering and meat packing, livestock; manufactured dairy products, milk; edible fats and oils, n.e.c.; other food products.

[17] See Table 1.

[18] Such comparisons have been made for all the mining and manufacturing industries with results similar to those obtained for the food, tobacco, and beverage industries. Because it is not feasible at this time to increase sufficiently the number of industries defined by the industrial classification to meet the criteria, the results summarized in Table 1 for the other than food, tobacco, and beverage industries will not be included in the discussion.

tatively similar inputs would be qualitatively similar.[19] In spite of the deficiencies of the 96-industry classification of the food-, beverage-, and tobacco-processing group of industries, changes proposed are few. The decision to leave the group unaltered has a purely empirical basis. The primary empirical consideration is whether changing the 96-industry classification will significantly affect empirical results obtained. In Table 2, for instance, which gives the breakdown into Census industries of the Leontief canning and preserving industry, it can be seen that the extreme values of the ranges of the three input ratios are from the salad-dressing and quick-frozen-food industries. These two industries together account for only 5.9 per cent of the total value of product in the Leontief industry and their inclusion in the industry is not likely to be a significant source of error.[20]

The changes which were made in the classification of this group of industries were: establishment of a separate industry for confectionery and related products—chewing gum, cocoa, and chocolate products—formerly included in the residual industry, other food products, and the addition to this industry of the products of the starch and glucose and edible fats and oils, n.e.c., industries. These two changes were proposed to bring the Leontief classification into line with the new Standard Industrial Classification.[21] These changes make all the industries in this group equivalent to Standard Industrial Classification three-digit industries.

The Standard Industrial Classification is constructed so that industries are classified on a two-digit, three digit, and four digit basis, according to the degree of detail of information. For our purposes we have in many instances cut across the three-digit classification, i.e. combined four-digit industries from more than one three-digit industry. Since statistical information collected by government agencies is guaranteed in general only for three-digit industries, such statistics will be certain to be applicable to our classification only in so far as it corresponds to the three-digit

[19] Measured on a finer basis than they have been measured for Table 1, i.e. a basis fine enough for the input to be distributed from the appropriate Leontief industry.

[20] To date, no formal analysis of the permissible variation of the values of the input ratios of Census industries to be aggregated into Leontief industries has been done. However, the theoretical requirement of identical or proportional cost structures will be fulfilled so rarely that it is necessary to explore the problem of permissible variation of the values of input ratios for a given precision of prediction. For the present all that can be said is that when the extreme values of the range are contributed by Census industries whose value of product is small compared to the other Census industries in the aggregate, and these latter industries show small dispersion from the averages of the input ratios, the possible error in prediction arising from including the industries having the extreme values in the aggregate is not likely to be important.

[21] Executive Office of the President, Bureau of the Budget, Division of Statistical Standards, Technical Committee on Industrial Classification, *Standard Industrial Classification Manual*, 'Manufacturing Industries,' Vol. 1, November 1945.

TABLE 2

Leontief Industry 11 — Canning and Preserving

Census Industry	Value of Product (in millions of dollars)	Materials/ Value of Product	Wages and Salaries/ Value of Product	Fuel, etc./ Value of Product
	(1)	(2)	(3)	(4)
(1) Canned and dried fruits and vegetables	587	79.8	18.9	1.3
(2) Pickled fruits and vegetables	73	77.2	21.5	1.3
(3) Canned fish, etc.	65	79.9	18.9	1.2
(4) Salad dressing.	49	89.5	10.0	0.5
(5) Preserves, jams, and jellies	38	82.2	16.6	1.2
(6) Cured fish	16	79.4	19.1	1.1
(7) Quick frozen food	10	73.2	24.4	2.2
Averages		80.2	18.5	1.3

Standard Industrial Classification. However, there are instances in which either (1) it is reasonably certain that the government statistics required for purposes of this classification will be available on a four-digit basis or (2) the three-digit classification fits so badly our aggregative criteria that it is worth while to sacrifice the certainty of comparability of government-collected statistics.[22]

B. CHEMICALS

The chemicals industry (53) in the 96-industry classification has been subdivided into 11 new industries. Industrial inorganic chemicals and fertilizers have been grouped in a single industry because they employ

[22] If statistics were available in all cases for the Four-digit Standard Industrial Classification industries, an industrial classification identifying several thousands rather than less than 200 separate industries would be feasible. Whether a matrix solution identifying this many separate industries could be obtained would depend on the existence of digital computers for solving such a system. Even if such a machine did not exist, and the solution to the system had to be obtained by a matrix representing some degree of aggregation of the original several thousand industry matrix, the original information would permit the reweighting of technical coefficients of the aggregated industries in cases where product-mix changed.

In the 1947 input-output study currently being carried out by the Bureau of Labor Statistics, industries are being defined simply on the basis of data available on material flows. The indications are that 1947 Census data and supplementary statistical data will yield an industrial classification identifying approximately 192 industries. In this study the classification of food, beverages, and tobacco industries is the same as our amended classification except that nonalcoholic beverages and confectionery and related products are included in the miscellaneous food products industry instead of being classified as separate industries.

similar processes and raw materials. Although the plastics-materials and elastomers (except synthetic rubber) and the synthetic-rubber industries use similar processes and materials they have been made into two separate industries because of the strategic importance of synthetic rubber. Cyclic (coal tar) crudes have been combined with intermediates, dyes, color lakes, and toners to get a vertical integration of coal tar products, and to this industry has been added the gum and wood chemicals because the uses of the two classes of products are similar. To explosives have been added fireworks from the miscellaneous manufacturing-industries group. The various vegetable and animal oil and fat industries have been integrated with the particular group of finished chemical products which principally uses them. Thus marine animal oils together with drugs and medicines form one industry; cottonseed oil mills, grease and tallow, fatty acids, and animal oils, n.e.c., are grouped with soap and glycerine, cleaning and polishing preparations, and sulfonated oils and assistants to form another; and linseed oil mills, soybean oil mills, and vegetable oil mills, n.e.c., are grouped with paints, varnishes, lacquers, japans, enamels, inorganic color pigments, and wood fillers to form a third industry. The remaining new chemical industries are synthetic fibers; industrial organic chemicals, n.e.c.,[23] and miscellaneous chemicals, n.e.c.[24]

The Bureau of Labor Statistics classification [25] of the chemical industries is composed of 14 separate industries. The 3 additional industries are fertilizers, which we have grouped with industrial inorganic chemicals; vegetable oils, which we have split between soaps, etc., and paints, etc.; and animal oils, which we have split between drugs and medicines and soaps, etc. While we have grouped cyclic crudes and intermediates, dyes, lakes, and toners with gum and wood chemicals, the Bureau of Labor Statistics has grouped cyclic crudes with coke and products (a non-chemical industry) and intermediates, dyes, lakes, and toners with industrial organic chemicals.

C. TEXTILES

In the 96-industry classification, five industries—cotton, yarn and cloth, silk and rayon products, woolen and worsted manufactures, clothing, and other textile products—comprise the textile industry. This classification is a vertical integration for each principal raw material

[23] Includes noncyclic inorganic chemicals, solvents, synthetic perfume and flavoring materials, rubber chemicals, plasticizers, synthetic tanning materials, and chemical warfare gases.

[24] Includes printing ink, essential oils, perfumes, cosmetics, glue, gelatin, boneblack, carbon black, lampblack, compressed and liquefied gases, insecticides, fungicides, salt, etc.

[25] See footnote 22, p. 339.

through the finished raw material stage and a horizontal integration of fabricated products which distinguishes two classes of these. The definition of the first three industries has been inadequate in that these industries do not conform to the existing structure of textile production. Many products are made of textiles which are a mixture of cotton and wool; cotton, rayon, and wool; etc. Under the old industry definitions there is no way to handle these fabrics. Furthermore, an establishment (the basic data-collecting unit) may be processing more than one textile. Finally, the advent of synthetic fibers and their radically increasing importance in the total consumption of textiles pose problems which are not easily managed in terms of the old definitions. The new Standard Industrial Classification takes account of these inadequacies by a basic reclassification of the textile industries which does not, even on a four-digit basis, identify separately the different textile raw materials, except in the case of the four-digit industry, broad woven fabric mills, woolen and worsted. Instead, four-digit industries identify the separate processes in textile manufacture; namely, scouring and combing plants, yarn mills, thread mills, broad woven fabric mills (cotton, silk, and synthetic fibers), broad woven fabric mills (woolen and worsted), narrow fabric, and other small wares mills.

The manner in which the industries for textile products should be defined presents a problem which as yet has not received the attention necessary to arrive at any solution. The appropriate product groupings depend on a thorough analysis of household textile consumption habits to determine the patterns of substitutability among the textile products. For the present, the textile products are classified together in one industry, apparel and other finished textiles. In the new classification of the textile industries, the processes of textile manufacture have been integrated to the finished raw material stage to form only one other industry, fabricated basic textiles. In the Bureau of Labor Statistics classification fabricated basic textiles are also treated as a single industry, but 5 classes of finished textiles are identified in separate industries. These are: special textile products (wool carpets, rugs, and carpet yarn; carpets, rugs, and mats from fiber [except wool]; felt goods; lace goods; paddings and upholstery filling; processed waste and recovered fibers; textile goods, n.e.c.); jute, linen cordage, and twine; canvas products; apparel; house furnishings and other non-apparel.

D. CONCLUSION

The discussion of the details of reclassification for three specific industry groups leads to the conclusion that in specific instances refinement of the 96-industry classification may not be readily achievable

because either the requisite data or appropriate theory does not exist. In general, however, great strides toward meeting the requirements of the theoretical scheme can be made simply by increasing the number of industries in order to rid the classification of some mixed-product industries whose value of product is substantial. In the new Bureau of Labor Statistics classification, a great deal has been accomplished in this direction. The extent to which this can be done depends on the number of industries permitted in the classification and the availability of data to determine the input structures of the industries. The treatment of the chemical industries is an example of reclassification for this purpose. In addition to chemicals, the 96-industry classification includes these industries for which similar treatment is needed: nonferrous metal mining, smelting and refining of nonferrous metals, nonferrous metal manufactures and alloys, industrial and household equipment, n.e.c., electrical equipment, n.e.c , iron and steel, n.e.c., nonmetallic mineral mining, nonmetallic mineral manufactures, and industries, n.e.c.

In the latest Bureau of Labor Statistics classification, each of these industries has been subdivided to obtain more homogeneous industries. For example, electrical equipment, n.e.c., has been split up to form the following 16 industries: wiring devices and graphite products, electrical measuring instruments, motors and generators, transformers, electrical control apparatus, electrical welding apparatus, electrical appliances, insulated wire and cable, engine electrical equipment, electric lamps, radio and related products, tubes, communication equipment, storage and primary batteries, and X-ray apparatus.

III. Improvement of Industrial Classification

The problem of improving industry classification without changing the fundamental assumptions about industry structure or utilizing new sources of data was discussed previously. This section deals with considerations involved in making such changes, focused by our unsuccessful attempt to establish a process-service industry [26] for machining.

In three of the chapters of this book, production functions which have been derived from technical data [27] are described. Each of the authors, particularly Mr. Chenery, adduce important theoretical advantages for this method of deriving production functions. Indeed, it is quite clear that since the interrelations between industries depend ultimately on technical relationships, the establishment of Leontief industry input

[26] See Section I, pp. 330-2.

[27] Chapters 8, 10 and 11. See particularly Chapter 8 for the general description of engineering production functions and Chapter 11 for an explanation of what constitute technical data and a list of types of sources of these data.

structures on the basis of engineering production functions would be superior to the practice of deriving them from the observed aggregate stocks and flows. Knowledge of technical production functions would make it possible to take account of input substitutability and scale effects where these cause non-linearity in technical relationships. These phenomena are widely recognized in economics and the advantage to input-output analysis in being able to deal with them is clear. In the construction of an industrial classification, knowledge of engineering production functions may also assist in handling the problems of process-mix and product multi-dimensionality.

A. 'PROCESS-MIX' IN INDUSTRIAL CLASSIFICATION

In the first section of this chapter, product-mix as a source of instability of technical coefficients was considered. Process-mix presents an analogous problem. If an industry is a vertical integration of separate processes, a change in technique in one process will invalidate the input structure of the industry in an unknown way, since the relation of the inputs to the separate processes is not explicit. If separate processes are identified, only the inputs for the process affected by the change will be invalidated, and since the processes are separately identified, the invalidated input structure can be corrected in light of the change in technique. Because the adoption by an entire industry of a new technique does not take place immediately, the correction of an industry input structure is not necessarily an *ex post* operation. To the extent that the rate of diffusion of an innovation in technique is known, the appropriate input structure for a particular future year can be determined as soon as the new technique has reached a stage of acceptance in the industry which means it will eventually be generally adopted.

In an economy in which innovation is rare, this consideration would not be important. However, the United States economy is characterized by a great deal of innovation in techniques of production. The industrial classification used in input-output analysis should be designed not only to take account of potential changes in demand for goods and services but, in addition, to take account of potential changes in the techniques by which they are produced.

If it is desired to refine the industrial classification with respect to a particular sector of the economy, an assessment of the relative importance of process-mix and product-mix as sources of technical coefficient instability should be made. If in the particular sector product-mix is more likely to be variable than are techniques of production, the industrial classification should identify separate products. If the sector is experi-

encing rapid technological change and its product-mix is not highly variable, the classification should identify separate processes.

In general, Census industries are defined so as to include more than one process;[28] therefore, in order to isolate separate processes in the industrial classification, input structures for these processes mut be obtained by constructing production functions from technical data.[29]

B. THE MULTI-DIMENSIONAL PRODUCT [30]

Thus far, it has tacitly been assumed that product-mix can always be eliminated from an industrial classification if the number of industries permitted is large enough. This is not quite the whole story—the exception is the truly multi-dimensional product. In the general case, product-mix exists because the industry in question is, in fact, an aggregate of separate industries. To extend this definition to cover the multi-dimensional product does violence to the technical and economic facts.

The technical fact is that the process by which the multi-dimensional product is made is such that it will generate a wide range of all the product dimensions and that different values of the dimensions affect costs of production differentially.[31] The economic fact is that a particular establishment does not confine itself to producing an output embodying one fixed combination of possible values of the output dimensions. Usually, an establishment, producing a multi-dimensional product, produces an array of products embodying different values of the same product dimensions.

In general, for reasons which will be apparent in the exposition which follows, the input structure for a multi-dimensional product must be obtained from the engineering production function for the product. It cannot be statistically inferred from aggregate stocks and flows, because the available data are not sufficient to yield the necessary relationships.

[28] For example, the metal fabricating industries each include the following processes (1) changes in the form of the metal (casting, forging); (2) cutting and shaping of the forms (machining, stamping); (3) joining of the forms (riveting, welding); (4) heat treating, (5) assembly. The series of processes which form the cotton textile industry are described in Chapter 10.

[29] Mrs. Grosse's production function for the textile industry (Chapter 10) gives explicitly the production functions for the separate processes in textile manufacture. Hence, if it were desired to identify, in the industrial classification, carding, spinning, weaving, etc., the necessary data for separate input structures would be available.

[30] By product dimension is meant any variable aspect of a product which must be specified to distinguish it from the general group of similar products to which it belongs. For instance, such a seemingly homogeneous group of products as eggs are not completely specified by quantity alone. A dozen eggs may be a dozen extra large, large, medium, or pullet eggs. They may be fresh or storage eggs. In order to pin down a particular quantity of eggs, it is necessary to specify the number, the size, and fresh or storage—a total of three dimensions.

[31] If different values of the dimensions do not give rise to different costs of production (amounts of processing), no product multi-dimensionality problem exists.

1. *Railroad transportation*

As an example of a multi-dimensional output, consider railroad transportation.[32] The minimal dimensions [33] of railroad transportation are tons and miles. If one wishes to find out the quantities of inputs necessary for some amount of the service, railroad transportation, the number of tons to be carried and the distance to be traversed must both be specified. Railroad transportation is clearly recognizable as a service which cannot be purchased unless the buyer specifies how much weight and what distance. A given railroad is equipped to transport a wide range of weights over a wide range of distances—any weight from an ounce to a great many tons can be carried a mile or a thousand miles.[34]

Since railroad transportation is minimally a two-dimensional output, and a wide range of both dimensions can be produced by any railroad company, railroad transportation, technically, is to be considered a multi-dimensional product. The economic fact with respect to the multi-dimensionality of railroad transportation is that one and the same railroad does produce transportation in a widely variable array of both dimensions. We are so accustomed to this variability, that it is hard to conceive of an economic situation which would vitiate the multi-dimensionality of railroad transportation. The process is such that the production function generally can be written:

$$\rho(t, d, i_1, i_2, \cdots, i_n) = 0$$

where

t = tons,
d = distance, and
$i_1, i_2, \cdots, i_n$ = inputs.

Only if economic conditions were such that for each separate railroad either t or d were a *constant* (not necessarily the same constant for all railroads) instead of a variable could railroad transportation sensibly be conceived as an instance of product-mix rather than product multi-dimensionality. In the former case, the valid procedure would be to subdivide the railroad industry into groups such that each member of a given group produced the same value of the constant dimension. Railroad

[32] In this case the output is a service rather than a product, but the argument is the same in either case.

[33] For simplicity, a number of considerations—refrigeration, warehousing, location—are neglected in this example. They do not affect the argument in any way except to make exposition more cumbersome.

[34] Tons and miles are also independently variable dimensions. Specifying a particular distance does not delimit a range of available weights or vice versa. If the two dimensions were not independently variable, it might be possible to consider one a function of the other and thereby reduce the two dimensional product to a one-dimensional product.

transportation is, however, a multi-dimensional product. It is a possible exception to the warning that the input structure for a multi-dimensional product must be obtained from an engineering production function. The industry appears to be one for which the variation in input requirement as a function of variation in the separate output dimensions can be determined by statistical inference.

First, because there are only two variable dimensions, the number of observations necessary to establish the separate effects of each on cost of service is small.[35] Second, the railroad industry is not a vertically integrated series of different processes so that cost data are relatively unambiguous. Furthermore, because of government regulation, cost accounting by railroads is uniform, and the relevant cost and operating data are available in published sources.

Input requirements as functions of variation in the separate output dimensions have not been calculated because the demand by Leontief industries for railroad transportation tends, for each industry compared with itself through time, to be for a fixed combination of the two dimensions. In other words, the product multi-dimensionality of railroad transportation is assumed not to give rise to instability of technical coefficients because the relationship of tons to miles of the railroad transportation used by each separate industry has not changed radically over the last twenty years.

2. *Machining*

Machining is a service which illustrates the difficulties attendant on the solution of the problem of product multi-dimensionality. Appropriate coefficients cannot be derived by statistical inference in this case. Despite the fact that it has proved possible to obtain the technical production function for machining, it cannot under present circumstances be directly utilized as the basis of the Leontief industry input structure. The machining production function and the problems in the way of utilizing it are discussed below.

a. *The production function for machining*[36]

The production function for machining is methodologically similar to the production functions of Chapters 8 and 11. The most significant difference lies in the product multi-dimensionality of machining as opposed to the product homogeneity of the other two cases.

[35] Suppose coefficients relating total input cost to each dimension were to be obtained by correlation. In a multiple correlation with two independent variables, providing statistical attributes of the observations are not unfavorable, 25 observations would be sufficient.

[36] I would like to acknowledge the helpful criticism of Hollis Chenery with respect to the discussion which follows.

The production function for the machining process consists of two sets of relationships; the first set contains the variables relevant to machining itself, the second set those relevant to the preparatory operations. These include all operations other than machining itself: setting up the machine, placing the work piece in position to be machined, and removing the work piece after it has been machined. (Preparatory operations will be discussed in (2) below.)

(1) THE MACHINING RELATIONSHIPS: The process of machining[37] can be split into three complementary subprocesses: turning, facing, and boring. Thus one of the output dimensions—shape of the piece[38]—defines the subprocess. For each subprocess the same variables must be included in the production function but the parameters will be different.

[37] In order to enable the reader who is unfamiliar with machine tools to judge the results which have been obtained, a cursory description of types of machine tools will be given.

The production function for machining defined in the text describes 3 processes into which all machining operations could be classified. Machine tools may also be classified in terms of the processes for which they may be used.

1. Lathes—lathes are of two main types—engine lathes and turret lathes. They differ principally in their flexibility and concomitantly in their suitability for jobs of varying lot size. The engine lathe is the most flexible of all machine tools; it will perform operations in all three subprocesses and will take pieces of very different sizes. However, the engine lathe will generally be more efficient for a particular operation only when the lot size is small.

The turret lathe (and the screw machine) is a modification of the engine lathe designed for the production of identical parts requiring multiple operations. In general, a turret lathe will do the same operations as an engine lathe, but will not have the flexibility.

2. Drilling machines will perform any of the boring operations, but will be specialized in terms of the size of the hole with which they are designed to work.

3. Grinding machines are machines employing an abrasive wheel for the purpose of removing metal. Grinders are used when requirements as to tolerance and finish are more stringent than can be met on ordinary machine tools. There are three types corresponding to the shape of the area to be machined—external cylindrical, internal cylindrical, and plane surface.

4 Milling machines—milling is a method of removing material from a surface by means of a rotary cutter. The milling machines' two most common operations are facing and key seating Milling is an operation particularly adapted to machining non-cylindrical shapes although it will do cylindrical shapes as well.

5. Cut-off machines are machines employing a saw to cut bar stock.

6. Polishing and buffing machines are used when very fine surfaces are desired.

7. Honing and lapping machines are used for work for which grinding would not give the required tolerances and finish.

8 Broaching machines are specialized machines for producing various types of regular- and irregular-shaped holes and for machining external surfaces. They could be used for very large lots as substitutes for milling machines.

[38] Shape of piece is not a completely adequate denotation of the dimension; perhaps more adequate would be shape of the area to be machined. An internal cylindrical surface will be bored (drilling is a type of boring), an external cylindrical surface will be turned, a plane surface will be faced.

The following symbols will be used to denote the variables included in the production function:

OUTPUT DIMENSION	SYMBOL
1. shape of the piece	Σ
2. size of the piece	z
3. machinability of the metal	u
4. tolerances	t
5. finish	g
6. amount of metal to be removed	b

PROCESS VARIABLES	SYMBOL
1. feed	f
2. speed	s
3. depth of cut	d
4. horsepower	hp
5. machine size	p

INTERMEDIATE VARIABLES	SYMBOL
1. rate of metal removal	n

Machining is the process of removing metal (from a work piece) in the form of chips by a power-driven machine. The first equation is:

$$b = \varphi_1(z, \Sigma)\,[39] \tag{9, 1}$$

The amount of metal to be removed depends on the size and shape of the piece. The second equation relates the rate of metal removal to the technical variables which determine this rate.

$$n = \varphi_2(f, s, d) \tag{9, 2}$$

In order to utilize equations (9, 1) and (9, 2), further specification of the variables is necessary.

$$u = \pi_1(f, s, d) \tag{9, 3}$$

$$hp = \pi_2(u, n) \tag{9, 4}$$

$$t = \pi_3(f, s, d) \tag{9, 5}$$

$$g = \pi_4(f, s, d) \tag{9, 6}$$

$$z \leq \pi_5(p) \tag{9, 7}$$

Equation (9, 3) is a relationship between the resistance to cutting of metal and the cutting variables — the cutting speed of the tool; the depth of cut for which the tool is set; and the feed, the distance the tool advances into the work or material at each revolution of the cutting tool or the piece being machined.

This relationship has been investigated by the A.S.M.E. Committee on

[39] See footnote 57, p. 354

Metal Cutting Data:[40] to derive, first, an equation for determining cutting speed,[41] and secondly, an equation for determining the economic life[42] of a tool.

Using the A.S.M.E. notation, the first equation[43] is:

$$V = \frac{K_t K_h K_d K_m K_f K_r K_c}{T^a L^{bT} M^n} \tag{9, 8}$$

where

V = cutting speed, feet per minute, measured on the uncut section of the work ahead of the tool;

L = total active length of the cutting edge of the tool, in contact with the work, inches;[44]

T = average thickness of chip, as cut from work piece;[44]

M = tool life, minutes, to complete failure[45] of the tool and includes only the time during which the tool is actively engaged in removing the chip from the work piece;

K_t = constant, determined experimentally, the value of which depends on the material of the tool;[46]

K_h = constant depending on the hardening treatment of a steel tool. For tools not given any heat treatment, such as Stelite and the cemented carbides, this term drops out;

K_d = constant depending on the tempering treatment of a steel tool. This term also drops out for Stelite and cemented carbide tools;

K_m = constant depending on machinability of the metal cut;

K_f = factor depending on kind and quantity of cutting fluid used;

[40] A.S.M E. Committee on Metal Cutting Data, *Manual of Cutting of Metals*, American Society of Mechanical Engineers, 1939 This equation is for the subprocesses, turning and boring, done with a single-point lathe tool.

[41] Although we have, for the sake of symmetry, grouped input dimensions on one side of equation (9, 3), it could also be $s = \pi'_2 (u, f, d)$.

[42] The input dimensions of this production function are process variables (horsepower, cutting speed, feed, etc.) rather than economic inputs (motor, cutting tool, machine base, etc.) and a further step relating the required process variables to available economic inputs is required. The feed, speed, depth of cut and machinability relationships determine the minimum requirements for a cutting tool; deciding the economic life of the cutting tool is part of the cost minimizing for the production function discussed below as part of the general problem of relating process variables to the economic inputs which will minimize cost.

[43] A.S M.E., op. cit. p. 253. This equation is an example of a synthetic design law. See ch. 8, p. 306.

[44] The variables L and T are functions of feed, depth of cut and the nose contour of the tool. All three of these latter variables affect chip proportions which in turn affect cutting speeds.

[45] Point of failure will vary according to the tolerances and finish required in a given machining operation.

[46] Tables of values of this and following variables are to be found in A.S.M.E., op. cit. pp 255-8. These tables contain experimentally obtained values for the tool materials, hardening treatment, tempering treatment, machinability of metal cut, etc., generally met in actual machining operations.

K_r = factor depending on rake angle of the tool;
K_c = factor depending on type of cut;
a = exponent determined experimentally which appears to depend on material cut;
b = exponent determined experimentally which appears to depend on the tool material; and
n = exponent determined experimentally which appears to depend on tool material, metal cut, and type of tool failure.

Equation (9, 8) can be transformed, given the contour and material of the cutting tool, the desired tool life, and providing the required horsepower [47] is available, into the equation (9, 3).

Equation (9, 4) is a technical relationship between energy requirement (horsepower) and the amount of work done per time unit, the rate of metal removal.

The A.S.M.E. Committee on Metal Cutting Data have derived empirically the following equation [48] for estimating horsepower requirement:

$$hp = \frac{PV}{33{,}000} \tag{9, 9}$$

where

hp = horsepower required at tool point [49] to overcome cutting resistance,
P = cutting pressure in pounds, and
V = speed of the work relative to the tool, in the direction of cutting, pressure, fpm.

[47] Equation (9, 3) contains cutting speed in feet per minute as a variable; whether the horsepower available is sufficient depends on the maximum spindle speed, revolutions per minute of a particular machine, available at a given feed and depth of cut. The equation relating spindle speed and cutting speed is:

$$rpm = \frac{12S}{3.1416D}$$

where

rpm = spindle speed in revolutions per minute,
S = cutting speed in feet per minute,
D = diameter of the piece being cut.

Source: By permission from *Machine Shop Estimating*, by W. A. Nordhoff. Copyright 1947, McGraw-Hill Book Company, Inc.

[48] A.S.M.E., op. cit. p. 274. This equation is a synthetic design law. See ch. 8.

[49] To calculate the power input to the machine tool making a given cut:

$$hp_i = K_w + \frac{P_t V}{33\,000E}$$

where

hp_i = horsepower input of machine,
P_t = tangential pressure in pounds,
K_w = horsepower required to run machine idle,
E = efficiency of the machine.

Although cutting pressure has a tangential, longitudinal, and normal component, the latter two may be neglected in estimating horsepower requirement. The equation[50] for the tangential component of cutting pressure is:

$$P_t = K_p K_a T^c L^d \tag{9, 10}$$

where

P_t = tangential component of chip (cutting) pressure in pounds, experimentally obtained,
K_p = constant, property of the metal being cut,
K_a = constant, depending on the true rake angle of the tool, in the direction of the chip flow,
T = average thickness of chip,
L = total active length of the cutting edge of the tool, in contact with the work, in inches,
c and d = exponents, determined experimentally, which appear to depend on the material cut.

Thus P_t depends, once the cutting tool is selected, on the machinability of the metal and the feed and depth of cut. (Feed and depth of cut, for a given cutting tool, determine T and L.) V together with feed and depth of cut determine the rate of metal removal; thus equation (9, 9) can be transformed into (9, 4).

Machinists' reference books contain numerous tables for this relationship—for example, Table 3:[51]

Equations (9, 5) and (9, 6) indicate limitations on the permissible values of feed, speed, and depth of cut depending on tolerances and finish required in the machined product. In achieving close tolerances and smooth finish, cutting speeds must be high and pressure on the work piece light so that it will not be deflected.

Equation (9, 7) states that machine tools have given capacities in terms of the size of piece which can be machined on them. Thus, once the size of the piece is given, a minimum size machine is determined.

When the appropriate values of the processing variables have been determined, the next step in the analysis is to relate these to the economic inputs. We can define input functions:

$$\epsilon_i = \delta_i(m_1 m_2, \cdots, m_k) \tag{9, 11}$$

where

ϵ_i = the i^{th} economic input, and

$m_1 m_2, \cdots, m_k$ = the process variables of the production function.

[50] A.S.M.E., op. cit.
[51] Oberg, Erik, and Jones, F. D., *Machinery's Handbook*, p 1648, Industrial Press, New York, 1944.

TABLE 3

Horsepower Required[1] to Take a Given Cut on a Lathe Using Round-nosed Tool

Metal (u)[2] (1)	Rate of Metal Removal (2)	Hp (3)
(1) Cast iron	1 cubic inch per minute	0.3 to 0.5
(2) Cast steel	1 cubic inch per minute	1.0 to 1.8
(3) Wrought iron	1 cubic inch per minute	0.5 to 0.6
(4) Machine steel	1 cubic inch per minute	0.5 to 0.6
(5) Steel, 50 per cent carbon	1 cubic inch per minute	1.00 to 1.25
(6) Brass and similar alloys	1 cubic inch per minute	0.20 to 0.25

[1]Horsepower required for drilling may be estimated by doubling the values for turning (per cubic inch of metal removed).

[2]The machinability of various metals has not been investigated to the extent which would permit formulation of a theoretical equation relating machinability to hardness, toughness, and other variable characteristics of metals which determine their machinability. Machinability indices are worked up by measuring, under given conditions, the maximum cutting speeds possible on various metals and indexing these. The following table gives the general order of metal machinability:

magnesium	100
aluminum	55
brass	45
iron	30
steel	20

Magnesium is the easiest metal to machine; thus the metals are listed in ascending order of difficulty in machining. Source: Nordhoff, W.A., op cit. p. 31.

The functions, δ_i, will in general be discontinuous.[52] Very little work has been done as yet on this phase of the analysis. Since the explanation of these relations requires special technical knowledge which this writer does not have, only a general indication of their form will be given here.

The most important of the economic inputs in machining are the motor; cutting tool; size of machine; design of the machine which affects its rigidity, proneness to vibration, flexibility, etc.; coolants and oils; skill of the labor required.[53]

The motor input function is:

$$H = \delta_1(hp, s) \tag{9, 12}$$

where

H = motor.

[52] See below.

[53] The term 'economic inputs' implies that these are specific variables which can be included in a cost function. Design of machine is not an adequate denotation of the required economic input, but will serve to indicate to the reader the structural characteristics of the machine which must be priced.

This function is obviously discontinuous; first, since motor horsepowers are discrete values and, secondly, because differences in speed requirements will make different types of motors appropriate. *Machinery's Handbook* lists [54] seven different types of motors used on the various machine tools:

1. Adjustable speed, shunt-wound, direct-current motor, wherever a number of speeds are essential.
2. Constant speed, shunt-wound, direct-current motor; when required speeds are obtainable by a gear-box or cone-pulley arrangement, or when only one speed is required.
3. Squirrel-cage induction motor, when direct current is not available.
4. Constant speed, compound-wound, direct-current motor (conditions as in 2).
5. Wound secondary or squirrel-cage induction motor with approximately 10 per cent slip (conditions as in 3).
6. Adjustable speed, compound-wound, direct-current motor.
7. Standard machine tool traverse motor.

Given the machinability of metal being processed, the input function for the cutting tool [55] is:

$$ct = \delta_2(s, f, d, hp) \tag{9, 13}$$

For single-pointed cutting tools for lathes, planers, shapers, turret lathes, and boring mills, the A.S.M.E. has worked out elaborate tables of this relationship. For example, for the rough turning of 0.40 carbon manganese steel, SAE TI340, hot rolled, at a feed of 0.002 inches per revolution and a depth of cut of $\frac{1}{32}$ inch, tool number 1 yields horsepower requirement of 0.3 and an available cutting speed of 286 *fpm* for a tool life of 60 minutes while tool number 2 requires 0.5 horsepower and yields a cutting speed of 470 *fpm* under the same conditions.

Summarizing the characteristics of the two tools: [56]

	TOOL NO. 1	TOOL NO. 2
Material	HSS 18-4-1	HSS 18-4-1
Nose radius	0 inches	$\frac{1}{16}$ inches
Side cutting edge angle	0 degrees	0 degrees
End cutting edge angle	6 degrees	6 degrees
Side rake	14 degrees	14 degrees
Back rake	8 degrees	8 degrees
Relief	6 degrees	6 degrees

[54] *Machinery's Handbook*, op. cit. p. 1640.

[55] Some characteristics of the cutting tool are directly determined by the shape of the piece Since the shape of the piece determines whether the machining will be a turning, facing, or boring operation, whether the cutting tool will be a milling cutter, drill, or lathing tool is already determined.

[56] Data are from A.S.M.E., op. cit. pp. 107 and 124.

Equations (9, 12) and (9, 13) indicate the nature of input functions. The economic inputs come in discrete sizes and types, each of which can be described by a set of process variables (the input function). For any combination of output variables, there are one or more combinations of motor, labor, cutting tool, etc., which will produce the required output. The actual combination to be used is determined by minimizing the cost function for the relevant combination of inputs. For example, cutting tool and energy and labor cost bear an inverse relationship to each other. A higher rate of output (therefore a lower unit labor cost) can be obtained by the use of a more expensive cutting tool which will withstand the increased heat of higher cutting speeds but require more horsepower to drive it.

Equations (9, 1) through (9, 7) are sufficient to specify the product and the amount of metal to be removed.[57] This set of equations limits to some extent the choice of machine tool upon which to perform the required machining operation. It specifies a minimum size of machine, a minimum necessary available horsepower input, and a minimum necessary available cutting speed. However, because the main process variables (f, s, d, hp) are not specific to any particular type of machine tool, the choice of technically suitable machine hours of different types per unit of product is wide. The most efficient type is found by cost minimization.

(2) THE PREPARATORY OPERATIONS RELATIONSHIPS: In order to establish these relationships, one further output dimension is necessary. This dimension is lot size. Lot size is not a variable which need affect machining time per unit of product, although in practice there is some tendency for automatic machine tools (tools for large lot sizes) to be more efficient in machining itself than general purpose tools. A machine tool designed to machine a single product can incorporate structural characteristics permitting the utilization of greater force than can a general purpose machine. This fact does not affect the analysis because it is insignificant in comparison with the primary effect of lot size. In machining there are significant economies of scale, i.e. economies dependent on the value of lot size. Associated with increases in lot size are potential decreases in minimum total unit cost due to utilization of less flexible (more automatic) machine tools. The use of more automatic machine tools decreases the amount of labor necessary per unit of output.

[57] This statement implies (1) that shape of piece (see p 348) has been used as an output dimension to determine the subprocess for complete product specification; and (2) that technologically inefficient combinations of machining and casting, machining and rolling, etc , are not included in the production function. This means that the work piece (casting or bar stock) is of such a size that the minimum amount of machining is required to produce the required output dimensions. This restriction is necessary in order that equation (9, 1) be determinable.

For machining

$$L_t = L_s + l(L_h + aL_m) \tag{9, 14}$$

where

L_t = total labor time required,
L_s = labor time for setting up the machine,[58]
l = lot size,
L_h = handling time for each piece,
L_m = machining time for each piece, and

$$a = \frac{1}{\text{no. of work stations.}^{59}}$$

It is reasonable to assume that L_m is independent of L_s. The term L_s and the terms L_h and a in equation (9, 14) are in general inversely related. To illustrate, consider the turning of screws. This operation could be done on an engine lathe with no set-up at all. In this case the term L_s would be 0, the value of the constant, a, would be 1, and L_h some number of minutes. The same operation could be done on a screw machine with an automatic feed. In this case, L_h drops out of the equation,[60] a becomes less than 1 and L_s takes on some value. Assume the same value for L_m in both cases. Depending on the costs of set-up, L_s, and operator labor, L_h and aL_m, and the relative costs other than labor associated with running an engine lathe and an automatic screw machine, there is some value of l above which cost per screw will be less on the automatic screw machine than on the engine lathe and below which the reverse will be true. Although this example is crude, it illustrates the balancing of L_s and $(L_h + aL_m)$, which is the essence of the problem of determining the appropriate degree of machine flexibility.

Carrying through the analysis at the same level of abstraction as that of the previous section,

$$y = c(L, l) \tag{9, 15}$$

where

l = lot size,
L = labor, and,
y = machine flexibility.

[58] The set-up is the installation of whatever jigs, fixtures, automatic feeds, etc., are to be used to machine the lot Set ups are of varying degrees of elaborateness and for varying degrees of permanence The cost varies from negligible amounts to thousands of dollars. Although in equation (9, 14) above, we are concerned only with the cost in labor time, an elaborate set-up occasions material costs also.

[59] Where automatic machines are employed, an operator may operate either two or more single work-station machines or more commonly a single multiple-station machine, e.g. turret lathe.

[60] This is not strictly true, since the bar stock must be loaded on the machine. But the time required for this can be neglected in our crude example.

and c denotes that equation (9, 15) is not a technical relationship but one which depends on cost considerations only. A lot of any size can be turned out, *ceteris paribus*, on a machine tool of any degree of flexibility.

It should be noted that machine flexibility is not a continuous variable but rather has discrete values.

The analytical procedure outlined above is applicable only to instances of new investment in machine tools. Where the production function is used to decide on the utilization of a fixed complement of tools, it is not always possible to minimize separately machining costs *per se* and preparatory costs in the manner implied above. It can be done when the flexibility of the general-purpose tool maximally efficient for the product is reduced by building automatic holding devices, etc., onto it in accordance with equation (9, 15).

Where use is being made of a complement of tools of fixed degrees of flexibility, it is necessary to find the joint minimum cost which may not equal the sum of the separate minimums.

If costs are split into two components, those associated with the machine set-up for the product and those associated with the actual machining of the product, then for each of the machines technically suited to the job (as determined from the first set of relationships) a cost equation can be written:

$$
\begin{aligned}
cM_1 &= S_1 + \alpha_1 X \\
cM_2 &= S_2 + \alpha_2 X \\
\cdot \quad & \quad \cdot \quad \cdot \\
cM_i &= S_i + \alpha_i X \\
\cdot \quad & \quad \cdot \quad \cdot \\
\cdot \quad & \quad \cdot \quad \cdot \\
cM_r &= S_r + \alpha_r X
\end{aligned}
$$

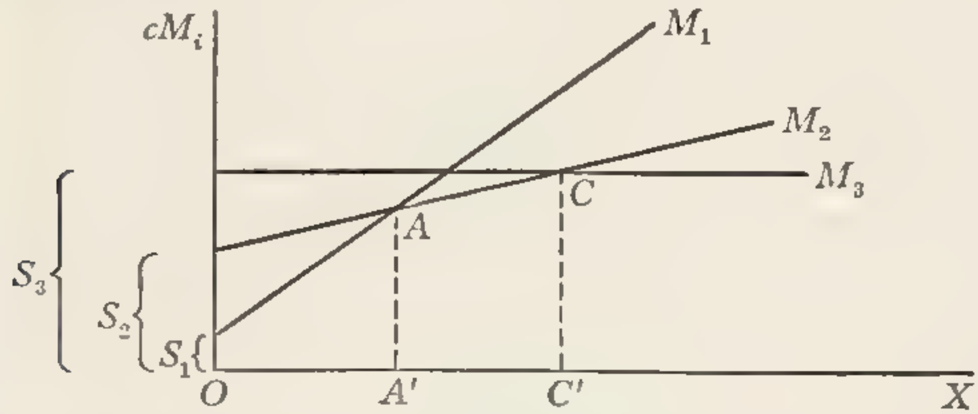
where

cM_1, cM_2, cM_i, cM_r = the total costs for the technically suitable machines,
S_1, S_2, S_i, S_r = the setup costs for each machine,
$\alpha_1, \alpha_2, \alpha_i, \alpha_r$ = the variable costs, and
X = the combination of output dimensions determining α_i and is a unit of product.[61]

[61] α_i is the minimum machining cost for the product *for the i^{th} technically suitable machine*, but the technically suitable machines are not necessarily equally efficient so that $\alpha_1 \neq \alpha_2 \neq \alpha_i \neq \cdots \neq \alpha_r$. If for instance $\alpha_i = \alpha_j$, then equation i or j should be suppressed depending on which has the higher set-up cost. Furthermore, since these equations are for a given group of tools, it is not necessarily the case that any one of them is the most efficient of all possible machine tools for the particular product, i.e. that any of the α's is the *minima minimorum*.

If these equations are plotted on a graph, one of whose axes represents values of X and the other values of cM_i, the range of values of X (therefore of different lot sizes) for which any machine is most efficient can be seen.

Graphically:



M_1 is the solution for values of output from 0 to A', M_2 from A' to C', and M_3 for values greater than C'.

b. *Problems in utilizing the production function*

The general problem in utilizing engineering production functions results from the hiatus between technical and economic data. To illustrate what is meant, consider the process of machining. The product dimensions which are necessary to the production function for the process are shape of the piece, machinability of the metal, amount of metal to be removed, tolerance, finish, size of part, and lot size. Machined products include such diverse items as tractors, electric toasters, and firearms. None of these products is specified by the Census in a manner which indicates the amount of machining necessary to produce it. Therefore, there is no direct way of associating machining costs with products. The missing link between technical data (the engineering production function for machining) and economic data (output of a Leontief metal-fabricating industry, e.g. electrical equipment) is a seven-way [62] frequency distribution summarizing the output dimensions of all the machined parts included in the final product of the industry. It is extremely doubtful whether any establishments in the metal-fabricating industries could give this information. In the absence of such direct information, the usual recourse is to attempt to infer the missing relationships from what is known. Our work in this connection has been of little avail. Statistical inference proved to be impractical for the metal-fabricating industries. First of all, the relationships which must be determined are extremely

[62] The engineering production function has seven output dimensions.

complex. The production function contains 15 variables in a non-linear, non-homogeneous relationship under rather weak constraints. Secondly, because the metal-fabricating industries are vertically integrated series of processes,[63] statistical data for Census industries on man-hours, installed horsepower, electric energy consumed, etc., relate to total processing and not to machining alone.

Because it has proved possible to obtain the engineering production function and yet be unable to utilize it, we are forced to conclude that the elimination of technical coefficient instability due to product multi-dimensionality is not always practical. The impracticality stems not from the difficulties of technical data, as such, but from the difference in the frame of reference of technical and economic data. The promising cases are those in which the frames of reference tend to coincide so that it is unnecessary to work out a transformation of engineering output dimensions into economic output dimensions. These are the cases where the output specifications which influence cost of production are used quite directly to designate the product. Actually, there are a goodly number of such cases. Most of the commodities which fall into the classification of finished raw materials[64] are priced on the basis of technical specifications and enumerated by the Census in separate homogeneous groups. These include the chemicals, petroleum products, coal, finished agricultural raw materials, primary metal products, and basic textile manufactures.

Since the analysis of the textile industry is available in this volume, this industry will be used to elucidate cases in which a production function derived from technical data could be used to establish the structure of economic inputs for a multi-dimensional product industry.

Although six dimensions are necessary to specify the output of the cotton textile industry, output of cotton textile products (gray goods) are specified by the Census in terms of the dimensions which affect processing inputs. Like metal fabricating, textile gray-goods manufacture is a vertical integration of separate processes. Unlike metal fabricating, the processes are related uniquely: first, because the material being processed is homogeneous; second, because there are no alternative processes for the same function; i.e. the only way to get cotton thread or yarn is by spinning it; third, because processes are not substitutable.

For metal fabrication there is considerable possible variation in the metal components of a fabricated metal product. For three of the series of functions (changes in form, cutting and shaping of the forms, joining, heat treating, assembly) in metal fabrication there are alternative processes. Changes in form may be accomplished by casting, forging, or rolling; cutting and shaping by machining or stamping; joining by riveting

[63] See footnote 42, p. 349.
[64] See Section I, p. 327.

or welding. Casting and machining are substitutable processes within limits. The more nearly the casting approximates the required dimensions for the part, the less metal will have to be removed by machining.

Further the textile production function is linear.

The foregoing discussion is not to be taken as an example of the only conditions for directly utilizing engineering production functions.[65] However, each departure from these conditions introduces problems of the sort manifest in machining.

With certain exceptions, notably some of the chemical (including metallurgical) products, textile manufacture, in terms of the considerations described above, is representative of the other finished raw material products. The extreme complications in metal fabricating, in turn, are probably confined to metal fabricating industries although fabricated products generally will present more complications than finished raw materials.

Finally, it should be stated explicitly that utilization of technically derived input structures for multi-dimensional products requires that the bill-of-goods demand be specified in terms of the output dimensions relevant to the technical production function. Insertion of the specific values of the output dimensions in the bill of goods is necessary to determine the specific technical coefficients to be incorporated in the input-output prediction matrix.

IV. Conclusion

In this chapter, three problems in devising a useful industrial classification for input-output analysis have been described. Product-mix is a well-known complication in many phases of economic analysis. It is the traditional index number problem of economics. Process-mix and product multi-dimensionality have the same effect as product-mix of introducing indeterminateness in input-output technical coefficients. They differ from product-mix in that they will generally require engineering production functions to cope with them. Statistical analysis alone is not a sufficient tool, although a combination of engineering and statistical methods works well in instances for which the requisite statistical data exist. Even when the production function can be derived entirely from technical material, its use requires that it be equipped not with hypothetical values of its parameters, but those obtaining in the aggregate in the United States' economy. In determining these values, statistical data, under present circumstances, have no substitute.

[65] When an engineering production function is to be used for a one-dimensional product industry, the difficulties are considerably reduced.

Chapter 10

THE TECHNOLOGICAL STRUCTURE OF THE COTTON TEXTILE INDUSTRY

Anne P. Grosse

I. The Use of Technical Data

The pure theory of input-output analysis suggests a method for explaining a wide range of economic phenomena in terms of a basic matrix (or set of matrices) of coefficients describing the technological interdependence of industries. The predictive value of this method depends on the reliability of the technical coefficients in the matrix, and there is general agreement that the realization of the full potential of the technique awaits substantial refinement in our description of technical structure. It is, therefore, important that the basic operational problems in the determination of the required production functions be studied explicitly.

Recently, a growing dissatisfaction with the traditional method of determining technical parameters for input-output purposes has prompted pilot investigations into the use of fields of information relatively unfamiliar to economists, particularly engineering and technical data, as a basis for improving the quality of the structural parameters.[1] The present study of the technological structure of the textile industry is one of these. It was undertaken expressly to test the feasibility of estimating economically meaningful production functions from engineering data.

The study begins with the derivation, in Section II, of a production function for the cotton yarn and cloth industry from such technical source materials. These engineering coefficients are compared with analogous coefficients estimated from economic statistics in Section III, and the use of both types of estimates simultaneously in the analysis of technological change is explored. These sections are introduced by a brief review of the limitations of the conventional statistical methods by which the matrices of coefficients for input-output analysis have been derived, and by a survey of the operational distinctions between coefficients obtainable from economic and from engineering data.

[1] Part II of this volume, 'Explorations in the Use of Technological Data,' is devoted to such pilot investigations of the problems involved in the use of engineering and technical data.

Thus far the matrices of input flow and capital stock coefficients in input-output analysis have been estimated statistically from 'economic' data. Linear homogeneous production functions are assumed and the technical restrictions governing input flows in each industry are described by a set of coefficients, a_{ij}, the quantity of the output of industry i technically required per unit of output of industry j. Since linear homogeneous production functions are assumed, these coefficients are equal to the relative outputs of the various industries in any given year. Similarly, if random variations are disregarded, a single set of observations of industrial outputs is sufficient to determine the matrix of capital coefficients, b_{ij}, the stock of goods produced by industry i required per unit of capacity of industry j. The capital coefficient matrix would be derived by observing, for each industry, the ratios of the stocks of the various kinds of goods held to its capacity.

An essential characteristic of such a procedure of estimating the technical parameters is that their values are determined from the same kind of information which they are designed to explain. Particularly where there is a limit imposed on the number of observations on which the estimates are to be based, it is difficult to achieve the degree of autonomy required for useful technical relationships.

On the other hand, a high degree of autonomy of coefficients is necessary to the scientific value of an input-output matrix. Since technological change is a slow process, it is reasonable to expect that a given technical matrix will be useful in explaining economic phenomena over a few years, at least. This requires that the coefficients exhibit reasonable stability with respect to changes in product-mix and that there be substantial independence among the individual columns in the matrix (i.e. stability of individual production functions with respect to changes in other production functions). The various methods of determining technical parameters must be evaluated with respect to these objectives. This problem of determining autonomous technical relationships subsumes at least three fundamental problems of input-output analysis:

1. the problem of dividing the economy into technologically homogeneous 'industries' (the classification problem); [2]
2. the problem of selecting technically significant attributes or input and output dimensions in each industry; [3]
3. the choice of appropriate types of functions in which these input and output variables are to be related in each industry.[4]

[2] The major criteria for evaluating industrial classification systems and various classification schemes are studied in detail in Chapter 9, Parts I and III.

[3] For a discussion of this problem, see ch. 8, especially pp. 301-6.

[4] The problem of choosing appropriate function types is treated extensively in Chapter 8. An illustration of the use of engineering materials in determining the forms of the production function is also given in Chapter 11.

It is formally possible, on the basis of an infinite sample of the set of all attributes of inputs, stocks, and outputs, to determine structural relationships of the required degree of autonomy by purely statistical inference. However, without a very large number of sets of observations, autonomous and confluent relationships could not be distinguished. Since the parameters are subject to technological change, the time span of the observations to be used in estimating the set of parameters is restricted, and with it, the size of the sample on which the estimates can be based. This limitation, coupled with the obvious practical deficiencies of available statistical data, makes it necessary to base classification and the choice of relevant variables and function types on *a priori* judgments. Thus, in practice, our schemes of industrial classification have never been determined by purely statistical procedures, but have been formulated directly on the basis of general knowledge of the technological and market characteristics of the various processes and products in our economy.

To a large extent, the classifications used in input-output analysis are based on those adopted by data collection agencies, particularly the *Census of Manufactures*. These, in turn, incorporate expert judgments concerning the technical and market characteristics of the various products and processes covered. On the other hand the assumption of linear homogeneous production functions and the adoption of a single unit or dimension for measuring each input and output are based on much more general theoretical grounds of plausibility and simplicity, rather than on any detailed direct technical knowledge. These latter two assumptions are, of course, also most convenient for the determination of the parameters by statistical methods.

The introduction of *a priori* restrictions on the technical relationships, however general, performs a function analogous to sampling stratifications, narrowing the role of pure statistical inference in the estimation of the parameters. Naturally, the task of assessing and refining these assumptions is of primary importance. There are two major types of approach to the testing of such assumptions, and of the parameters in general. The first consists of over-all checks on the predictive value of the matrix; the second, of comparisons of the individual production functions themselves with independent technical information.

With a large matrix of technical coefficients of uneven quality to be tested, little insight into the 'trouble spots' can be gained by over-all checks. It seems unlikely that further progress in the operational treatment of such issues as quality dimensions, substitution, and scale, as well as refinement in classification, can be achieved on a general or intuitive basis alone. For any substantial increase in the stability of the technical parameters, detailed factual information about individual technologies and industrial practices is required. Thus, resort to direct technological

information is warranted even within the framework of the orthodox statistical method of estimating the coefficients. Furthermore, for many purposes, there are advantages to using such information not only for the establishment of the basic classifications and function types, but for the estimation of the numerical values of the parameters as well.

Engineering and technical literature used in the planning and control of industrial production is a fertile source of the required technological information. This embraces studies of operations in individual plants and groups of plants, relevant scientific generalizations based on these studies or on auxiliary research, and the whole field of equipment design, plant and labor layout, and instructions and rules of thumb for organizing and productive process. Such teleological information is particularly useful because it often specifies input requirements in a form very convenient for the study of interindustrial relationships.[5] This type of technical data is readily available in textbooks for students of the industry's trades, technical manuals and handbooks, machinery catalogues, technical and trade journals, and special treatises on individual aspects of the industry's technology. Often the same kind of information can be obtained directly from engineers who specialize in organizational and managerial advice to individual firms. The most important feature of the use of this specifically technical information as opposed to economic statistics lies in the emphasis on developing the economic implications of given physical relationships rather than on inferring what the physical laws must have been from their economic manifestations. In this approach the danger of circularity is averted. At the same time it is then possible to incorporate the conclusions of a large volume of accumulated technological experience and even of controlled experiments into our explanations of economic phenomena.

Since the purely technical materials cover a range of theoretical and experimental experience in addition to the history of actual industrial operations, they can provide insight into many problems for which 'statistical' materials would be inadequate. For example: by changing the conditions of factor supply, technological change in one sector of the economy may result in change in the factor combinations used in another sector. The impact of a given technological change in a particular sector on the input-output ratios used in the other sectors will be predictable only if the production functions of these affected sectors reflect more information than could be inferred from past input-output ratios. In so far as these repercussions can be predicted on the basis of known technology at all, the predictions must be based on direct technical, rather than statistical, information.

The use of direct technical information in the estimation of production

[5] Chenery gives a thorough analytical discussion of the types of engineering design laws in the technical literature. See ch. 8, especially pp. 297-9.

functions brings up new problems of operational definition. So long as the technical parameters are inferred from the very facts which they are to explain, it is guaranteed automatically that they will be representative of the technology actually in use. When direct technical information is to be used, it is necessary to deal explicitly with the problem of distinguishing between those aspects of technology which are and are not relevant for any given economic problem.

The most difficult of such decisions are associated with technological change. Since innovation, in most instances, involves a change in the model of equipment in use, there will be, at any point of time, a family of economically relevant production functions, each associated with a different model or 'vintage' of installed equipment. The commitment of a plant to any given piece or set of equipment restricts the area on the production function in which it may operate. Thus plants with older equipment will generally behave in accordance with different technical restrictions from those with newer equipment. A production function which is to describe input-output relations in an industry with mixed new and old equipment must constitute an average of the individual production functions relevant to the various types of equipment installed in the industry. To determine appropriately weighted average parameters on the basis of technical information, a detailed equipment inventory must be known. On the other hand, parameters of this 'average' nature are achieved automatically by the conventional statistical method. Thus, when 'average' technical parameters are required, the influence of the direct technical information may be restricted to the choice of relevant variables and function types, and the numerical values of the parameters estimated from economic data.

In the analysis of industrial expansion and the effects of new equipment purchases on interindustrial relationships, it is useful to deal with an 'incremental' or 'best practice' production function, that is, with a production function describing operations with the latest models of equipment. The derivation of 'best practice' production functions involves the formulation of criteria for distinguishing between techniques which are feasible and those which are only theoretically possible at any given time. Actually, the appropriate criteria depend heavily on the period over which predictions based on the 'best practice' coefficients are to be made. For short-run problems, only those techniques are of interest which can be employed immediately, at least in some plants. Since the introduction of a technique generally awaits the availability of the appropriate equipment, we sometimes adopt the criterion that a technique be considered part of the production function if the relevant equipment is in use beyond the experimental stage, or (in industries where equipment is not all made to order) if the necessary equipment is available commercially. Natu-

rally, decisions of this type involve a large measure of economic judgment.

The role of economic judgment in the utilization of direct technical materials extends not only to the selection of economically relevant techniques, but also to the selection of appropriate formulations of the technical relationships. Before many of the more general technological laws can be used, they must be translated from variables denoting purely physical characteristics to those which are economically identifiable.[6] Technical laws are often formulated in terms of physical variables or properties which are characteristic of a number of economic commodities. The fact that a number of economically distinguishable commodities share a technically relevant physical property implies the presence of economic substitution.

This translation problem is less important in the use of managerial rules of thumb and the sort of information found in practical handbooks than in the use of purely scientific materials. Many of the translations from purely technical variables which are necessary for economic analysis are also necessary for business operations. It is a primary function of the engineer to translate the laws of pure science into convenient forms for the conduct of production operations. Nevertheless, the engineering relationships found in such sources must frequently be modified themselves, to make them more manageable for the analysis of interindustrial relationships. In the cotton textile production functions, for example, constants were substituted for certain of the engineering variables whose average industry-wide values tend to remain fixed over time. Such expedients as these make it possible substantially to decrease the number of variables in a technical restriction without seriously affecting its usefulness for the study of interindustrial relationships.

Frequently, a technical relationship, revised for economic purposes, will be so changed as to appear to the technician as a gross over-simplification, and even as a grotesque caricature of the true technical law. Generally, differences between the technical laws in their original form and their economic counterparts, production functions, are, at least in part, justifiable in terms of differences in the uses to which they are to be put. The value, for example, of keeping the number of technical variables at a minimum is obvious where as many as a hundred industries are to be studied simultaneously.

Since engineering relationships often require considerable modification before they can be used in the study of interindustrial relationships, the determination of production functions cannot, at least at this stage, be relegated wholly to engineers. On the other hand, the technical complexity and specialization of the engineering materials make the task an awk-

[6] For further discussion of the problem of translation from 'engineering' to 'economic' production functions, see ch. 8.

ward one for economists. Ideally, therefore, the investigation of technical production functions calls for close co-operation between the two groups.

If technical relationships are to be used for economic analysis, the economist must first establish very precisely the types of relationships required before engineering assistance can be enlisted. The derivation of the textile production function described in this chapter represents for a single industry, the sort of preliminary work which is required as a basis for enlisting such further technical assistance.

II. The Cotton Textile Production Function

Engineering and technical source materials are used in the estimation of the cotton-textile production function, in two related capacities: as a basis for the selection of appropriate variables and function types, and as a source of numerical estimates of the parameters. The first of these functions is prerequisite to the determination of the numerical coefficients either from statistical or from engineering data. The study of the production function for the cotton yarn and cloth industry begins, therefore, with a survey of the general characteristics of the technical relationships. This is followed by the estimation of the parameters, from engineering data and then from economic statistics.

A. General Characteristics of Technical Relationships

It has already been pointed out that the general characteristics of the technical relationships are dictated both by engineering considerations and by the nature of the economic investigation. This is easily apparent in the formulation of the relations to be determined here for the cotton yarn and cloth industry.

In accordance with the general requirements of the dynamic input-output model, the technical parameters are divided into two major categories: those describing technical conditions governing the relationships between 'flows' or time rates of input and time rates of output, and those describing the technical relationships between 'stocks' of inputs and time rates of output.

On the basis of a preliminary survey of the technology of the industry, the relationships among input and output flows are tentatively divided into: (1) relations between raw materials and output; (2) relations between labor and output; (3) relations between power and output; and (4) relations among maintenance, replacement, and output. The relations between stocks and output are, in turn, divided into those governing equipment and those governing inventories. All of these relationships except the maintenance, replacement, and the inventory functions are described below.

The production organization of a textile plant suggests a stagewise subdivision of the technical relationships within the industry. Characteristically, the sequence of operations by which ginned cotton is converted into woven cloth (exclusive of finishing) is carried on in a single plant, which is divided into departments corresponding to successive operations or stages in the processing of cotton fiber. Each stage is associated with a major type of direct processing equipment. The principal stages in the manufacture of carded cottons are, successively:[7]

1. Opening: Raw cotton arrives at the mill in large bales, each containing about 500 pounds of cotton in a highly compressed form. The 'opening' operation generally involves the use of a series of machines, beginning with a bale opener and followed by a series of 'openers' whose function is to tear apart and fluff up the compressed mass of raw cotton and to begin the cleaning process.

2. Picking: This operation continues the cleaning process and puts the cotton into 'laps' of uniform density. The 'picker' machine, used for this purpose, passes the fluffed cotton between a pair of rollers to a set of beaters which revolve at high speed and propel the cotton against a series of grid bars through which a portion of the loosened foreign matter falls out. The cotton then moves onto a cylindrical wire cage or screen where an additional portion of the dirt is removed. Following this, the cotton again passes between sets of rollers and emerges at the front of the machine in loosely matted layers or 'laps.'

3. Carding: From the 'picker' the laps are fed into the 'card.' This is a machine with a set of rollers which deliver the cotton onto a large cylinder covered with short vertical wire teeth. This cylinder revolves very close to a series of flats also covered with wire teeth. The cotton passes from the large cylinder onto a smaller cylinder from which it is removed and gathered into rope or sliver form and loosely placed, by means of a coiling device, in a tall can.

4. Drawing: Four to six strands of sliver are fed into drawing rolls and are combined or drawn out into a single sliver of the same diameter or weight as those entering the machine.

The principal mechanical feature of a drawing frame consists of pairs of rollers which draw out the sliver into an increasingly longer, and at the same time thinner, cotton strand. This is achieved by regulating the speed of the rollers, each succeeding pair moving more rapidly than the preceding. This reduction of the size of the sliver is called 'drafting.' Drafting is also used in the subsequent roving and spinning operations for reducing the sliver to the required cotton yarn. The drawn sliver is wound into tall cans.

[7] The following general description of the stages of textile manufacture is adapted from Stern, Boris, *Mechanical Changes in the Cotton Textile Industry*, 1926-36, pp. 6-21.

5. Roving: This term is used to denote the cotton sliver after it has been drafted and reduced in size, twisted slightly, and wound on double bobbins. The term 'roving' is also applied to the process of making the roving. The finished drawing sliver is very coarse and must be reduced in size before it can be put into the spinning frame. This is done on a series of roving frames which are similar in principle and vary only in the size of the bobbins made on them. The standard roving frames, in ascending order of fineness of output, are known as slubbers, intermediates, fly-frames, and jacks. For very coarse yarns, only one roving process is required and only slubbers are used, while for finer yarns, two, three, or four roving processes may be required. In recent years the so-called 'long-draft' roving frames have been introduced in many plants. Long-draft roving frames have increased greatly the amount of draft that can be achieved at each roving process, and thus have reduced the number of roving processes required to make most kinds of yarn. With today's equipment, all but the finest yarns can be made with only one roving process.

The roving spindles put sufficient twist into the roving to permit further processing without stretching.

6. Combing: Only the finer yarns are combed. In the production of combed yarn the drawing sliver is prepared for combing by winding 24 ends together on a 'sliver machine' without drafting. Then the sliver laps are further processed by combining 4 or more on a ribbon machine and are drafted in proportion to the number of sliver laps used. The ribbon laps are then fed to the combing machine, which consists of from 6 to 8 heads, where the short staple in the fiber is removed. The cotton from these heads is combined into one strand and put into tall cans similar to those used in the drawing operation.

7. Spinning: The roving delivered to the spinning frame must be further drafted to the required size. In order to give the yarn maximum strength, considerable twist is imparted to it by the spinning frame. The spinning frame now generally used in the manufacture of the majority of cotton yarns is of the ring type. It consists essentially of a series of drafting rolls, similar to those used on drawing and roving frames, a number of steel rings upon which a small wire or 'traveler' revolves, and the same number of spindles. Twist is imparted to the spun yarn by the wire traveler revolving around the bobbin which is carried on the spindle on which the finished yarn is wound.

The yarn designed for use in 'filling' (weft or crosswise threads) is ready to go to the weaving department directly after it passes the spinning process, if spun on bobbins suitable for direct loom use. Frequently additional processing known as 'filling winding' is used to rewind the fill-

ing yarn onto suitable bobbins and to eliminate weak spots in the yarn prior to the weaving process.

8. Winding: Warp threads, however, must still undergo several processing operations before reaching the weaving stage. The first of these is winding. It consists of winding the cotton yarn from a large number of the small spinning bobbins onto a large package. This is necessary in order to obviate the constant replenishing of the supply of small bobbins on the warper. It may be performed on either the 'spooler' or the automatic winding machine.

9. Warping: In this operation between 300 and 400 warp 'ends' or threads are wound on a large section beam. An expansion comb is used to keep the individual ends spaced from each other. The machine is equipped with a stop-motion attachment which automatically stops the machine whenever an end breaks.

10. Slashing: Before the warp yarn is ready for weaving it must be treated with a combination of starch and softeners, known as 'size,' which is used to strengthen the warp. This operation is performed by a machine known as a 'slasher,' which sizes and dries the warp and winds it on the loom beam.

11. Weaving: Prior to putting the loom beam into the loom it is necessary to separate the ends of the warp and draw them through the drop wires, the harnesses, and the reed, according to a predetermined design. This may be done by hand, or by a stationary or a portable tying-in machine.

When the threads have been arranged in proper order, the ends tied in, the warp beam placed in the loom, and the necessary adjustments made, the process of weaving the cloth begins. The loom mechanism raises part of the warp threads and lowers the rest, making a v-shaped opening through which the 'shuttle,' carrying the filling, passes. Then the position of the warp threads is reversed and the filling left by the shuttle is locked into the warp by the forward motion of the reed. The operation of the traveling of the filling through the warp is known as a 'pick.'

12. Cloth room. Finally the cloth rolls from the looms are automatically sewed together, sheared of loose threads, brushed, and inspected.

For all but the last stage, the output of each is the raw-material input of the next, and for all but the first stage, the raw-material input is the output of the previous stage.

With a given type or 'quality' of product, each of the stages can be described in terms of a linear homogeneous production function with fixed coefficients. The adoption of this type of production function is justified not merely by its convenience in the theoretical input-output model, but also by engineering considerations. Fixed coefficients are adopted be-

cause of the absence of evidence of significant economic substitution in the engineering literature. This does not imply that inputs per pound of cotton cloth produced are the same throughout the industry. Certainly the contrary is true. However, most short-run variations in input-output ratios in the industry are to be explained, not by changes in input prices, but, for the most part, either:

1. in terms of quality differences in the product. The major quality differences can be taken into account by the introduction of multidimensional outputs;
2. in terms of differences in technical opinion about what the parameters actually are. These differences constitute a range of indeterminacy in the production functions; [8] or,
3. in terms of differences in the 'vintage' of equipment in use. This aspect of variation is elaborated later in this chapter.

Homogeneous functions are appropriate because the size of the machine unit is virtually constant at most stages, changes in capacity involving increases in the number rather than in the size of the machine units.[9]

For each kind of cotton cloth, four basic structural relationships are to be described at each stage; [10]

Let

k_{1i} = the fiber input required per unit of intermediate output at each stage, i,

k_{2i} = the machinery stock required per unit of intermediate output at each stage, i,

k_{3i} = the labor input required per machine unit at each stage, i,

k_{4i} = the power input required per machine unit at each stage, i,

x_i = the time rate of intermediate output of each stage, i,

x_{i-1} = the time rate of output of the previous stage (or the time rate of fiber input at stage i),

m_i = the number (stock) of active machine units at stage i,

l_i = the labor input at stage i, and,

p_i = the horsepower of active machinery at stage i.

[8] This indeterminacy is consistent with the concept of the production function as a technical 'horizon.'

[9] Machines in the industry do vary in size, but it is generally possible to choose a machine unit which will yield a homogeneous function. For example, spinning machines may have fewer or more spindles, but production is generally figured on a per spindle basis, independently of machine size.

[10] Combing and related operations are omitted from this study. This should not constitute a serious limitation on its usefulness, since combed cottons constitute only a small portion of total cotton production.

Then these relationships are:

$$x_{i-1} = k_{1i}x_i \tag{10, 1}$$

$$m_i = k_{2i}x_i \tag{10, 2}$$

$$l_i = k_{3i}m_i \tag{10, 3}$$

$$p_i = k_{4i}m_i \tag{10, 4}$$

Since (10, 2), (10, 3) and (10, 4) are valid simultaneously, (10, 3) and (10, 4) can be written:[11]

$$l_i = k_{2i}k_{3i}x_i = k'_{3i}x_i \tag{10, 3a}$$

$$p_i = k_{2i}k_{4i}x_i = k'_{4i}x_i \tag{10, 4a}$$

Given $k_{11}, k_{12} \cdots k_{1n}$, where n is the final stage of production, the inputs of all factors at each stage can be expressed as linear functions of x_n, final output:

$$x_{i-1} = k_{1i}k_{1i+1} \cdots k_{1n}x_n = k''_{1i}x_n \tag{10, 1b}$$

$$m_i = k_{2i}k_{1i+1} \cdots k_{1n}x_n = k''_{2i}x_n \tag{10, 2b}$$

$$l_i = k'_{3i}k_{1i+1} \cdots k_{1n}x_n = k''_{3i}x_n \tag{10, 3b}$$

$$p_i = k'_{4i}k_{1i+1} \cdots k_{1n}x_n = k''_{4i}x_n \tag{10, 4b}$$

When the k_{1i}'s are known, the variable x_o can replace x_{i-1} in the set (10, 1b), eliminating the intermediate forms of fiber input (or output) $x_1 \cdots x_{n-1}$. Thus raw cotton input, x_o, is related to final output through the function

$$x_o = k_{11}k_{12} \cdots k_{1n}x_n = K_1x_n \tag{10, 1c}$$

Total labor and power inputs for the process as a whole $\bar{L}$, and $\bar{P}$, are related to final output by summing the (10, 3b)'s and (10, 4b)'s over the various stages.

$$\bar{L} = \sum_{i=1}^{i=n} l_i = \sum_{i=1}^{i=n} k'_{3i}k_{1i+1} \cdots k_{1n}x_n = K_3x_n \tag{10, 3c}$$

$$\bar{P} = \sum_{i=1}^{i=n} p_i = \sum_{i=1}^{i=n} k'_{4i}k_{1i+1} \cdots k_{1n}x_n = K_4x_n \tag{10, 4c}$$

[11] Since (10, 3a) and (10, 4a) combine, respectively, (10, 2) and (10, 3), and (10, 2) and (10, 4), they have a lower degree of autonomy than (10, 2), (10, 3), and (10, 4). This becomes a disadvantage in the study of technological change which affects (10, 2) and (10, 3) and (10, 2) and (10, 4) independently. If k_{3i}, k_{4i} and the technologically changed k_{2i} are known, k'_{3i} and k'_{4i} can be deduced. The prediction of the effect of a change on (10, 2), (10, 3a) or (10, 4a) requires this knowledge of (10, 2) and (10, 3) or (10, 4). For this reason, the more autonomous relationships (10, 1) through (10, 4) are specified wherever possible in this study.

Machinery inputs are measured in technical units which are not directly comparable among stages. For present purposes, it is most convenient to treat the machinery inputs at the various stages as distinct factors. Equations (10, 1c), (10, 3c), (10, 4c) and the set (10, 2b) constitute the consolidated production function for the process as a whole.

$$x_o = k_{11}k_{12} \cdots k_{1n}x_n = K_1x_n \qquad (10,\ 1c)$$

$$m_i = k_{2i}k_{1i+1} \cdots k_{1n}x_n = k''_{2i}x_n \qquad (10,\ 2b)$$ [12]

$$\overline{L} = \sum_{i=1}^{i=n} l_i = \sum_{i=1}^{i=n} k'_{3i}k_{1i+1} \cdots k_{1n}x_n = K_3x_n \qquad (10,\ 3c)$$

$$\overline{P} = \sum_{i=1}^{i=n} p_i = \sum_{i=1}^{i=n} k'_{4i}k_{1i+1} \cdots k_{1n}x_n = K_4x_n \qquad (10,\ 4c)$$

Since there are wide ranges of variation in the types of cotton cloth produced, and infinite gradations within these ranges, an unmanageably large number of separate production functions of this type is required for a full description of the technical structure of the industry. The effect of product variation on factor requirements is most marked with respect to machinery. Fortunately, it is possible to take these variations in machinery requirements with product 'quality' into account by introducing a number of output dimensions besides weight into the machinery-output functions. The number of product dimensions which have a significant effect on the rates of machine output varies from stage to stage. At the opening and picking stages, for example, variations in machinery requirements with product quality can be disregarded altogether without serious effect on the stability of the machinery-output function. At the remaining stages, one or more of the following variables is introduced:

1. c, yarn count: the number of hanks of 840 yards each per pound of yarn. This is a measure of the weight per yard, or fineness of the yarn. While the counts of warp and filling used in a given cloth generally differ, an average of warp and filling counts is used here to limit the number of quality dimensions to be handled;
2. t, the 'twist multiplier,' or the number of turns of twist per inch of yarn, divided by the square root of the yarn count;
3. b, the width of cloth in inches;
4. e, the number of warp ends per inch of cloth;

[12] For reasons similar to those already cited on p. 370, it is necessary to know the more autonomous relationships, (10, 1) through (10, 4) or (10, 1), (10, 2), (10, 3a), and (10, 4a), to predict the effect of a possible change in technology at any particular stage on (10, 1b), (10, 2b), (10, 3b), and (10, 4b) and on the consolidated relationships (10, 1c) and (10, 3c) and (10, 4c).

5. p, the number of 'picks,' or crosswise threads per inch of cloth; and
6. average degree of intricacy of pattern. This affects machinery requirements only at the weaving stage and will be handled by introducing different loom functions for plain and patterned goods.

Functions of these dimensions are substituted for the constants, k_{2i}, in the set of equations (10, 2):

$$k_{2i} = f_{2i}(c, t, b, e, p) \tag{10, 2d}$$

Not all of these product dimensions are significant for output (in pounds per hour) at all stages, and one or more of $f_{2i}(c)$, $f_{2i}(t)$, etc., will be constant for some values of i. In the opening, picking, and drawing stages, for example, $f_{2i}(c, t, b, e, p) = k_{2i}$ will be treated as constants.

It is important to note that c, t, b, e, and p are dimensions of the final product and not necessarily the dimensions in which the outputs of the intermediate stages are measured. Machinery required per unit of output of any given stage may also be expressed as a function of one or more dimensions, d_{ji}, of the immediate product, x_i, of that stage:

$$k_{2i} = f'_{2i}(d_{ji}) \tag{10, 2c}$$

These intermediate dimensions are uniquely related to the dimensions of final output and therefore the former can be eliminated from the production function for the process as a whole.

The introduction of these 'quality' dimensions performs the analytical task of consolidating the production functions for a large number of types of product into a single invariance. This really amounts to a rudimentary 'process service' scheme, with a single, general production function subsuming many special production functions for particular products.[13] The quantity of 'spinning service,' 'weaving service,' etc., required per pound of product depends on the product characteristics as measured by the supplementary product dimensions.[14] Given the necessary product description, machinery-output functions of the form (10, 2) are determined, that is, the linear homogeneity of the production function is retained.

While the linear homogeneous form of the production function is adopted with respect to output in pounds, no general assumption is made as to the linearity or homogeneity of the production function with respect to other output dimensions. Thus the multi-dimensional machinery-output functions will all be linear with respect to weight of output, but not neces-

[13] For a discussion of the logic of the process service industry classification, see ch. 9, especially pp. 330-2.

[14] Since different staple lengths of cottons are required for different types of product (see below, Table 3), these functions can be construed as relating various different raw materials to their respective outputs.

sarily linear with respect to other output characteristics. To determine the form of the machinery-output function appropriate to each stage, a detailed study of the technical characteristics of the equipment or operation to be performed is desirable. A brief survey of the procedures used in formulating these functions at each stage will serve to illustrate the roles of technical information and of economic factors in setting them up.

B. Machinery-output Functions

1. *Basic function types*

a. *Spinning and roving*

Spinning and roving machines are mechanically very similar, both being designed to draft and twist the fibers as they pass between the spindles and rollers, from which the product is delivered. The greater the amount of drafting and twisting to be done, and the finer the roving being processed, the smaller will be the rate of output per spindle, in pounds per hour, of the machine. On the other hand, the rate of output in pounds tends to vary directly with the speed at which the spindles revolve. These mechanical characteristics of the process are summarized in the general technical formula used in the scheduling of actual production operations:

$$\text{pounds per spindle hour} = \frac{\begin{array}{c}\text{revolutions}\\ \text{per minute}\\ \text{of spindle}\end{array}}{\begin{array}{c}\text{twist per}\\ \text{inch}\end{array}} \times \underset{\substack{\text{(inches}\\ \text{per yard)}}}{\overset{\substack{\text{(minutes}\\ \text{per hour)}}}{\frac{60}{36}}} \times \underset{\substack{\text{(yards}\\ \text{per hank)}}}{\frac{1}{840}} \times \frac{1}{c'} \text{ or } \frac{1}{c} \qquad (10, 7)$$

where

c = yarn count in the case of spinning, and
c' = 'hank roving,' in the case of roving.[16]

Twist per inch is equal to t, the 'twist multiplier,' times the square root of the yarn count, c, or hank number, c'.

While the twist multiplier and yarn count are final product dimensions, c' and the number of revolutions per minute of the spindle are not, and it remains to substitute constants for them, or to translate from intermediate to final product dimensions.

The spindle speeds prescribed by textile machinery catalogues vary with the count and twist of the yarn to be produced. Chart 1 shows the variation of prescribed spindle speed with count and twist. Given sufficient data, it should be possible to express prescribed spindle speed as an

[16] Hank roving is defined in the same way as yarn count, i.e. the number of hanks of 840 yards weighing one pound.

exact function of count and twist. Two factors have discouraged the attempt to do this thus far:

1. lack of easily available technical information explaining the form of the relationship among spindle speed, twist and count; and
2. a decision (to be discussed later, see below, p. 405) to assume a stable twist multiplier for all industry-wide applications of the production function.

Since for a given twist multiplier and within the common range of yarn produced (i.e. 15's to 50's) variations in spindle speed with yarn count are not very great, spindle speeds will, for the time being, be treated as constant.

For present purposes, then, (10, 7) can be rewritten:

$$\text{lbs. per spindle hour} = \frac{\lambda_4}{tc^{3/2}} \qquad (10, 7a)$$

where

λ_4 = a constant.

Roving spindle speed does not vary with the fineness of product produced on a given roving frame and hence is also treated as a constant. Furthermore, hank roving is not a final product dimension. To convert (10, 7) into final product dimensions for roving, it is necessary to know the relation between hank roving to be spun and yarn count of the final product. The ideal relation between roving and yarn size is not specified in the technical literature.[16]

A ratio of hank roving to yarn count of .12 is assumed here. This ratio was suggested by a correlation of prescribed roving and spinning sizes given in trade-journal articles, to be used in various plants, and its appropriateness is confirmed by the fact that the typical draft, divided by the number of doublings, of long-draft spinning frames is approximately 8. This implies that the roving used is generally approximately one eighth of the yarn count spun.

For very fine yarns (above 60's), a second roving process is sometimes necessary. In view of the relative unimportance of this type of product, the second roving process can be neglected without serious distortion. Finally, since a constant twist multiplier is to be adopted for finished yarn, a constant twist multiplier (1.2) is also assumed for long-draft roving frames.

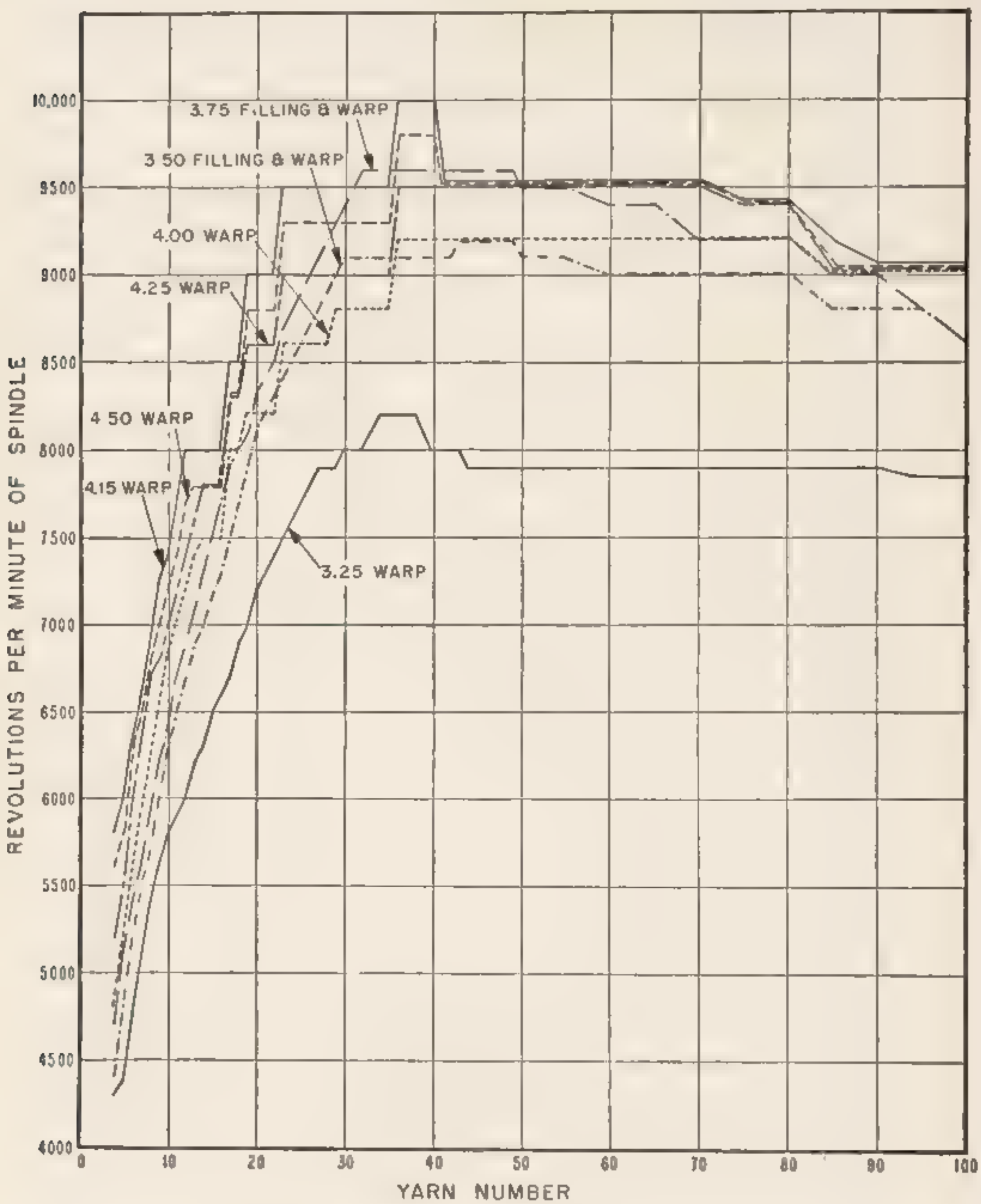
For roving, then, (10, 7) becomes:

$$\text{lbs. per spindle hour} = \frac{\lambda_5}{c^{1/2}} \qquad (10, 7b)$$

[16] See Merrill, G. R., 'Cotton Organizations,' *Textile World*, May 1936, p. 1062.

CHART 1

VARIATION OF PRESCRIBED SPINDLE SPEEDS WITH YARN COUNT AND TWIST



b. *Carding*

The finer the count of yarn to be produced, the finer the lap to be produced in the carding process, and hence the smaller the rate of output per card. The best carding speed to be used for any given product is considered a matter of some discretion by technicians. Machinery catalogues generally prescribe a recommended range of carding speeds appropriate

to the production of the various ranges of yarn counts.[17] Since these recommendations are made in terms of yarn count, a final product dimension, no intermediate product dimensions need be introduced.

c. *Winding, warping, and slashing*

Output per warper per creel end and per winder spindle are prescribed by machinery catalogues in units of yards of warp yarn per hour. To convert into final product dimensions, it is necessary to multiply warp yarn yardage by the number of pounds of cloth derived per yard of warp yarn, exclusive of fiber waste:

$$\frac{1}{c \times 840} \times \frac{e + p}{e} \times \frac{36}{b}$$

where

$\frac{1}{c \times 840}$ = the weight of warp yarn per yard of warp yarn,

$\frac{e + p}{e}$ = the ratio of total cloth weight to warp weight per square inch of cloth,

$\frac{36}{b}$ = the ratio of length to width per yard of cloth.

Output per winder and warper unit per hour can therefore be written:

$$\frac{\lambda_7(p + e)}{bec} \qquad (10, 8)$$

$$\frac{\lambda_8(p + e)}{bec} \qquad (10, 9)$$

Output per slasher is prescribed by machine catalogues in units of warp yardage. This is equivalent to the number of linear yards of cloth, minus cloth contraction. Slasher output is converted into units of final cloth poundage by multiplying slasher output in yards by:

$$\frac{e \times b}{c \times 764} \times \frac{(e + p)}{e} \times \frac{36}{b}$$

where

$\frac{e \times b}{c \times 764}$ = the weight of warp yarn per yard times the number of threads in the cloth,

$\frac{e + p}{e}$ = the ratio of total cloth weight to warp weight per square inch of cloth,

$\frac{36}{b}$ = the ratio of length to width per yard of cloth, and

[17] See below, Table 2, p. 386.

764 = the 'cloth constant.' It is based on a hank of 840 yards and represents a length of yarn, as measured in the cloth, that is equivalent in weight to a hank of the average warp and filling counts before sizing or weaving. Cloth constants range from less than 700 to over 800, but 764 is a fairly typical value.[18]

Output per slasher hour can be written:

$$\frac{\lambda_9(p+e)}{c} \qquad (10, 10)$$

d. *Looms*

Rate of loom output is prescribed by machine catalogues in number of shuttle crossings or 'picks' per minute. These loom speeds vary with the kind of pattern to be woven, the weight of filling, and the width of fabric.

Cam looms are used for plain weaves, while dobbies and jacquard machines are used with the looms for the weaving of patterned goods. Box looms are used when different colors of filling must be combined. The number of picks per minute of loom operation tends to be lower for the latter three types than for simple cam looms, and three alternative machinery-output functions are required at this stage. The appropriate range of prescribed loom speeds can be determined only after the description of the product is given.

Recommended loom speeds are quoted most often for the most common loom width, producing a 40-inch fabric. Loom speeds must be decreased by roughly two picks per minute for every additional inch of cloth width, b', where $b' = b - 40$.

Loom output is expressed in yards per hour by dividing the number of picks per inch of cloth by the number of picks per minute, and multiplying by $^{60}\!/_{36}$. To convert into pounds, it is necessary to know the number of pounds per yard of cloth,

$$\frac{1}{c \times 764} \times b(e+p)$$

where

$\dfrac{1}{c \times 764}$ = the number of pounds per yard of yarn, and

$b(e + p)$ = the number of yards of yarn contained in each linear yard of cloth.

[18] See Clark, William A., *Weave Room Calculations*, 1926, 2nd ed., p. 50.

Output per loom hour can be written, then,

$$\frac{\lambda_{10} b(p + e)}{c} \tag{10, 11}$$

In summary, then, the following types of expressions relating pounds of output per machine hour to the various quality dimensions seem appropriate from a survey of the general technical characteristics of the respective processes or stages:

1. opening, picking, and drawing

$$k_{21} = \lambda_1 \tag{10, 12}$$

$$k_{22} = \lambda_2 \tag{10, 13}$$

$$k_{23} = \lambda_3 \tag{10, 14}$$

2. carding: see Table 2
3. roving: $$k_{25} = \frac{\lambda_5}{c^{3/4}} \tag{10, 7b}$$
4. spinning: $$k_{26} = \frac{\lambda_6}{tc^{3/2}} \tag{10, 7a}$$
5. winding: $$k_{27} = \frac{\lambda_7(p + e)}{bec} \tag{10, 8}$$
6. warping: $$k_{28} = \frac{\lambda_8(p + e)}{bec} \tag{10, 9}$$
7. slashing: $$k_{29} = \frac{\lambda_9(p + e)}{c} \tag{10, 10}$$
8. weaving: $$k_{2(10)} = \frac{\lambda_{10} b(p + e)}{c} \tag{10, 11}$$

These are the specific forms of the general function (10, 2d).

2. *Determination of the numerical values of the technical parameters from engineering data*

Once the general characteristics of the technical relationships are established, the numerical values of the parameters can be determined from the same kinds of technical materials used to establish the basic function types. Technical information describing the latest types of equipment is usually the most readily available, making this method most suitable to the derivation of 'best practice' coefficients. The derivation of these is described below.

For any given model of machinery, the value of the equipment coefficient can be estimated from catalogue descriptions of recommended operating speeds and percentage allowances for normally necessary machine stoppages. These catalogue recommendations are generally based on several types of considerations: knowledge of the structural characteristics of the equipment; experimentation under controlled conditions in model plants; and surveys of performance records of the equipment after it has been installed in customers' plants. Thus, even though there may be wide variation in machinery performance from plant to plant, catalogue recommendations tend to be fairly reliable as a measure of average performance of new equipment. If anything, they tend to be conservative, under- rather than over-estimating machine productivity. Naturally, however, the estimates by individual technical experts of expected machine performances will vary with the individual experiences and policies, and with certain implicit assumptions about the nature of the plant in which the equipment will be installed. While the role of such judgment may be important in the managerial decisions of individual plants, the range of variation in the estimates is generally not large enough to jeopardize their value for the study of interindustrial relationships.

Differences in technical judgment arise not only in the estimation of the performance characteristics of particular models of machinery, but also in the selection of the most efficient of a series of models of machinery, available from different manufacturers. In the textile industry, this problem is simplified, first, by similarities in the rates of output of different models of machinery and, second, by the fact that equipment for some stages is made by only one or two major producers. Nevertheless, differences in engineering estimates of the best practice technical parameters are bound to arise, and it is important for the interpretation of any given estimate to recognize the role of individual judgment in arriving at it. This can be appraised from the comparison of sets of such estimates by technical experts, given on the following pages. The reconciliation of remaining differences among these estimates should constitute one of the most important aspects of further work in the refinement and revision of our technical parameters for the textile industry.

The first set of parameters of the machinery-output functions was derived, one by one, from individual machinery catalogues or from direct estimates by technical experts.[19] The steps followed in deriving these parameters can be traced through Table 1. First the relevant rates of intermediate output (i.e. output of the product of each stage, as opposed to final output) or machine speeds were determined.[20] Expressions for

[19] For sources of each of the individual coefficients, see the source note to Table 1.

[20] For the exact kind of technical information required for the estimate in each case, see above, pp. 374-9.

determining the rates of output of machines at each stage, based on the assumption of continuous operation, are presented in Column 2.

Second, percentage allowances for normally necessary machine stoppages are specified in Column 4 and subtracted from the expressions in Column 2. Stoppage estimates are obtainable from the same kinds of sources as the other machine-speed information. Such a set of stoppage percentages will, of course, be only roughly representative of actual stoppages in individual plants. The bulk of stopped machine time results from such interruptions as breakages in yarn, which must be repaired before operations on the machine can be resumed. The actual amount of stopped machine time depends, among other things, on the skill of workers in handling the stops, and on the labor load, i.e. the number of machines tended per worker. It has been suggested that the percentage of stopped machine time varies with the degree of utilization of machinery, labor loads being greater at low than at high levels of operation. This tendency is not recognized by all technical experts, others claiming that optimum labor loads will always be such as to keep machine stoppage time within certain prescribed limits related to the time required to make the actual repairs. At any rate, it seems unlikely that variations in average machine stoppage time for the industry will be enough to introduce serious error into our estimates of these parameters.

The derivations of the relations between dimensions of intermediate and of final output (where these dimensions differ) are explained in the text above. The expressions in Column 5 are converted into final product units, and listed in Column 6 of Table 1.

The expressions in Column 6 have the same dimensions as the final product, but do not take into account the loss in weight due to fiber waste at each stage. These are multiplied by the ratios of weight of final output to weight of intermediate output at each stage, $\frac{x_n}{x_i}$, to give final output per machine unit at each stage. The ratios $\frac{x_n}{x_i}$ are derived by taking the reciprocals of the ratios in Table 1, Column 3, lagged one stage.[21] They appear in Column 7 of Table 1. Final output per machine unit appears in Column 8.

The reciprocals of the expressions in Column 6 are the required estimates of machinery required per pound of final output, f_{2i}, given in Column 9.

It is interesting to contrast these estimates of the technical equipment parameters with comparable information derived from other engineering sources. Table 3 compares technical estimates of machine speeds with

[21] A detailed explanation of the derivation of these input weight ratios is given in C, below.

TABLE 1

Machine Speeds and Machinery Requirements per Pound of Output at Each Stage of Carded Cotton Textile Production with Modern Equipment, 1946-9

Stage	Machine Unit	Intermediate Output per Machine Hour 100%	Intermediate Output Units
	(1)	(2)	(3)
(1) Opening			
bale breaker	breaker	2,000	pounds per hour
first opener	opener	750	pounds per hour
second opener	opener	750	pounds per hour
(2) Picking	picker	370(maximum)	pounds per hour
(3) Carding	40" flat card	[1]	pounds per hour
(4) Drawing	drawing delivery	14.2[2]	pounds per hour
(5) Roving	long draft slubber spindle	$332c'^{(-3/2)}$ (1,050 RPM)	pounds–hank roving per hour
(6) Spinning	spindle	$\frac{18.25}{tc^{3/2}}$ (9,200 RPM)	pounds–yarn per hour
(7) Winding	spindle	42,000–54,000	yards per hour
(8) Warping	end in creel	45,000–54,000	yards per hour
(9) Slashing	slasher	3,600	yards per hour
(10) Weaving	loom	1–plain; (183 to 223)–$2b'$ 2–dobby and jacquard; (173 to 213)–$2b'$ 3–box; 185–$2b'$	picks per minute

[1] *See below*, pp. 384-90

[2] 55 grain silver is assumed

Sources: Opening, drawing, spinning: Saco-Lowell Shops, *Saco-Lowell Handbook: Engineering and Technical Data*, Vol. 4, Boston, Massachusetts.

Picking, winding, warping, slashing: Max J. Weyl of Textile Industry Research.

TABLE 1 (Continued)

Machine Speeds and Machinery Requirements per Pound of Output at Each Stage of Carded Cotton Textile Production with Modern Equipment, 1946-9

Per Cent Stops	Intermediate Output per Machine Hour Including Stops $1/f'_{2i}$	Pounds Intermediate Output per Machine Hour-final Units $x_i/m_i = 1/f_{2i}$
(4)	(5)	(6)
10	1,800	1,800
5	712	712
5	712	712
5	350	350
5	1	1
10	13.2	13.2
15	$290c'^{-3/2}$	$32.8c^{-3/2}$
8	$\frac{16.80}{tc^{3/2}}$	$\frac{16.80}{tc^{3/2}}$
20	33,600–43,200	$\frac{33{,}600\text{–}43{,}200}{840c} \times \frac{p+e}{e} \times \frac{36}{b}$
15	38,200–46,000	$38{,}200 \text{ to } 46{,}000 \times \frac{1}{c \times 840} \times \frac{p+e}{e} \times \frac{36}{b}$
25	2,700	$\frac{2{,}700 \times 36 \times (e+p)}{840c}$
10	1–(165 to 200)–$2b'$	
	2–(156 to 192)–$2b'$	$\frac{\text{Col. 5}}{p} \times \frac{60}{36} \times \frac{b(e+p)}{c \times 764}$
	3–(167–$2b'$)	

Roving: H & B American Machinery Company, Catalog 75, *Technical Data, High Draft Roving Frames.*

Weaving: Single shuttle plain and dobby; Howard L. Smith of Draper Corporation.

Box looms: Max J. Weyl of Textile Industry Research.

TABLE 1 (Concluded) — Machine Speeds and Machinery Requirements per Pound of Output at Each State of Carded Cotton Textile Production with Modern Equipment, 1946-9.

	Ratio Pounds Final Output to Output this Stage x_n/x_i	Pounds Final Output per Machine Hour x_n/m_i	Machine Hours per Pound of Final Output $m_i/x_n = f_{2i}$
	(7)	(8)	(9)
(1) Opening			
bale breaker	.87	1,570	.00064
first opener	.88	626	.00176
second opener	.88	626	.00176
(2) Picking	.89	312 (maximum)	.00338 (average)
(3) Carding	.95	1	1
(4) Drawing	.96	12.8	.078
(5) Roving	.97	$31.8c^{-3/2}$	$.0315c^{3/2}$
(6) Spinning	.98	$\frac{16.48}{tc^{3/2}}$	$.061tc^{3/2}$
(7) Winding	.99	$\frac{33{,}200-42{,}700}{840c} \times \frac{p+e}{e} \times \frac{36}{b}$	$.0002$ to $.0003 \times \frac{bec}{36(p+e)}$
(8) Warping	.99	$37{,}800$ to $45{,}000 \times \frac{1}{c \times 840} \times \frac{p+e}{e} + \frac{36}{b}$	$(.000022$ to $.000026) \times \frac{840ceb}{36\,(p+e)}$
(9) Slashing	.99	$\frac{2{,}680 \times 36(e+p)}{764c}$	$\frac{.0004 \times 764c}{36(e+p)}$
(10) Weaving	1.00	*See* Col. (6)	$\frac{p}{\text{Col. 5}} \times \frac{36}{60} \times \frac{c \times 764}{b(e+p)}$

latest equipment available in 1936, with our first set of estimates, given in Table 1, Column 2. While the former are based on prewar practice, they are sufficiently recent to be comparable with postwar technology, technological change having been negligible during the war period. Examination of the table will reveal a fairly close agreement between the two sets of estimates.

Two more sets of estimates, both based on 1947 technology, are available for comparison with those in Table 1. The first was published in connection with a study of labor costs by a textile cost accountant,[22] while the second was published in connection with a comparison of machinery requirements in 1947 and 1910, based on information supplied by Whitin Machine Works.[23] For neither of these plants is the equipment described in as great detail as in the Barnes study (see source reference in Table 3), and hence it is necessary to base the comparison of the machines on very rough estimates of their productivities.

The machinery inventories of these two plants are summarized in Table 4. Plant I is designed for the production of sheeting, while Plant II is designed for the production of print cloth.

These are both mills containing approximately 1,000 looms and approximately 50,000 yarn spindles. Since, however, Plant I produces a somewhat coarser product than Plant II, direct comparisons of machine productivities at each stage are not too significant. To compare the productivities of these two hypothetical plants with each other and with the estimates presented in Table 1, the following procedure is used:

1. Estimates of output at the weaving stage for Plants I and II are made. Since the details of cloth to be woven in these plants are not given, it is necessary to introduce assumptions as to the exact specifications of the cloth produced, to determine loom speed in picks per inch. The assumed cloth and yarn specifications of the two plants are presented in Table 5.

Since both of these cloths are plain weaves of 40-inch width, the loom-speed estimates for the Draper X2 40-inch loom are used in the estimation of cloth output. These looms are run at speeds ranging from 184 to 223 picks per minute, depending on fabric type, yarn quality, and size of shuttle. A loom speed of 200 picks per minute is assumed for Plant I, making the coarser cloth, and of 210 picks for Plant II, making the finer cloth.

Estimates of loom output per hour are found by combining these assumptions as follows:

[22] Gallier, Ted H., 'Indirect Labor Can be Accurately Controlled,' *Textile World*, April 1947, p. 106.

[23] 'Comparison of Machinery in a 1910 Print Cloth Mill and 1947 Mill of Equal Capacity,' *Textile Industries*, July 1948, p. 217.

$$\text{loom output in yards per hour} = \frac{\text{picks per minute}}{\text{picks per inch}} \times 1.667 \times .90,$$

$$\text{loom output in pounds per hour} = \frac{\text{yards per hour}}{\text{yards per pound}};$$

for Plant I:

$$\text{yards per hour} = \frac{200}{60} \times 1.667 \times .90 = 5.6 \times .90 = 5.00,$$

$$\text{pounds per hour} = \frac{5.56}{3.60} \times .90 = 1.40;$$

for Plant II:

$$\text{yards per hour} = \frac{210}{92} \times 1.667 \times .90 = 3.43,$$

$$\text{pounds per hour} = \frac{3.81}{3.50} \times .90 = .98.$$

TABLE 2

Card Production Guide

Grade and Staple of Cotton	Kind of Goods	Counts of Yarn	Card Production in Pounds per Hour	
			Average Work	Quality Work
	(1)	(2)	(3)	(4)
(1) 3/4" to 1" Strict Low Middling Strict Good Ordinary	Twines, coarse yarns, carpet warps, ducks and canaburgs, insulating yarns	4 to 12	11 to 14	9 to 11
(2) 7/8" to 1" Middling to Strict Low Middling	Heavy sheetings and drills, coarse knitting yarns, ordinary industrial fabrics, coarse tire yarn	12 to 20	10 to 12	8 to 10
(3) 1" to 1-1/8" Low Middling to Strict Middling	Knitting yarns, converters' goods, print cloths, fine twills, medium tire yarn	20 to 30	8 to 10	6 to 8
(4) 1-5/32" to 1-3/8" Middling to Strict Middling	Carded shirtings, fine convertibles, shade goods	30 to 50	7 to 10	5 to 8
(5) 1-5/32" to 1-1/4" Middling and Better	Combed yarns for knitting, mercerizing, and weaving	30 to 70	5 to 8	4 to 6

Source. Saco-Lowell Shops, *Saco-Lowell Handbook: Engineering and Technical Data*, Vol. 4, Boston, Massachusetts, p. 82.

TABLE 3

Comparison of Independent Engineering Estimates of Machine Speeds with Latest Equipment, 1936-47

Operation	Machine Unit	Speed Dimension	Best Practice Machine Speeds, Barnes Estimates: Sheeting	Carded Broadcloth	Combed Broadcloth	Carded Filling Sateen	Canton Flannel	Our Estimates (See Table 1, Col. 2)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) Picking	picker	pounds/hr. (100%)	307	300	300	325	367	370 (maximum)
(2) Carding	card	pounds/hr. (100%)	8.9	8.3	8.2	8.2	12.9	6–12[1]
(3) Drawing	delivery	pounds/hr. (100%)	17.9	14.7	16.0	14.7	20.8	14.2
(4) Roving	spindle	revolutions per minute/spindle	1,075	1,100	1,100	1,100	700	1,050
(5) Spinning	warp spindle	revolutions per minute/spindle	9,500	9,800	9,800	9,900	8,200	9,200
	filling spindle		8,500	9,000	8,800	9,000	6,500	
(6) Spooling	spindle	yards per hour	72,000	72,000	72,000	72,000	54,000	42,000–54,000
(7) Warping	end in creel	yards per hour	54,000	54,000	54,000	54,000	54,000	45,000–54,000
(8) Slashing	slasher	yards per hour	4,200	4,200	4,200	4,200	2,400	3,600
(9) Weaving	loom	picks/minute	200	192	192	192	200	183–223[2]

[1] This is an estimate of the range of recommended speeds for ordinary yarn counts, based on Table 2.

[2] Estimate for plain weaves.

Source: Barnes Textile Associates, *Survey of Technological Improvements and Developments in the Cotton Textile Industry*, 1910-36, Columns 3 through 8 and Table 1, Column 8.

TABLE 4

Equipment to be Used in Two Hypothetical 1947 Cotton Mills

Stage of Production	Number of Machine Units	
	Plant I	Plant II
(1) Picking	4	3
(2) Carding	105	106
(3) Drawing	198	144
(4) Roving	2,880	5,576
(5) Spinning	45,880	46,056
(6) Winding	380 spindles	324 spindles
(7) Warping	2,468 end creels	2,468 end creels
(8) Slashing	4	2
(9) Weaving	1,000	1,075

Source: See text, p. 385.

TABLE 5

Assumed Cloth and Yarn Specifications for Hypothetical Plants Described in Table 4

Plant	Width	Yards per Pound	Ends x Picks	Warp Yarn Count	Filling Yarn Count
	(1)	(2)	(3)	(4)	(5)
I	40	3.60	60 x 60	21.5	25
II	40	3.50	80 x 92	30	38

2. Given these estimates of output of woven cloth, the number of machine units per pound or per yard of woven cloth or warp yarn is computed for each stage. The results for the two plants are directly comparable with each other and with the corresponding figures in Table 1, Column 8, for those stages of textile production where rate of output is virtually independent of the supplementary product dimensions (i.e. of product dimensions other than weight). At those stages where rate of output depends on other product dimensions, a comparison can be made between the machinery productivity estimates made for each of the plants and estimates of machine productivity for products of these same descriptions, based on the expressions given in Table 1. The estimates for Plants I and II can then be compared indirectly in terms of their respective percentage deviations from the standards based on Table 1.

The comparisons of the three sets of estimates of machinery requirements per unit of output are presented in Table 6. The machine capacity at each stage is obtained by dividing estimated final output by the number of machine units in the hypothetical plant. In the warping and slashing stages, however, figures so obtained are likely to under-estimate capacity because of indivisibilities. At least two warpers (and, likewise, two slashers) are required so long as the output of the plant requires any-

TABLE 6

Comparison of Three Independent Estimates Based on 1947 Standards
of Machinery Requirements in Carded Cotton Textile Production

Operation	Machine Units Required Plant I	Plant II	Pounds of Final Output per Machine Unit[1,2] Plant I	Plant II	Pounds of Final Output per Machine Unit, Based on Table 1[2] Plant I	Plant II	Per Cent Deviation Plant I from Standard $\frac{(3)-(5)}{(5)} \times 100$	Per Cent Deviation Plant II from Standard $\frac{(4)-(6)}{(6)} \times 100$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) Picking	4	3	350	326	312	312	12	4
(2) Carding	105	106	13.33	9.24	10	9	25	3
(3) Drawing[3]	198	144	7.08	6.80	12.80	12.80	45	47
(4) Roving	2,880	5,576	.486	.176	.270	.160	80	10
(5) Spinning	45,880	46,056	.031	.021	.029	.023	6	9
(6) Winding[4]	380 spindles	324 spindles	31,600	33,840	34,000 – 43,700	33,200–42,700	7 to 28	2 to 21
(7) Warping[4]	2,468 end creels	2,468 end creels	25,700	23,450	37,800 – 45,500	37,800–45,500	–	–
(8) Slashing	4	2	1,250	1,715	2,680	2,680	–	–
(9) Weaving	1,000	1,075	1.4	.98	–	–	–	–

[1] Total hourly output is assumed to be 1,400 pounds or 5,000 yards for Plant 1, and 980 pounds or 3,430 yards for Plant 11.

[2] Output of all stages except winding, and warping, and slashing is measured in pounds per hour. Slashing is in linear yards of warp, and winding and warping are in linear yards of yarn in warp.

[3] Two drawing stages are assumed

[4] Total winding and warping output is derived as follows:

Yards of warp thread, Plant I = 5,000 (yards of cloth) x 60 (ends per inch) x 40 (width) = 12,000,000
Plant II = 3,430 (yards of cloth) x 80 (ends per inch) x 40 (width) = 11,050,000.

Source: See footnotes 22 and 23, p. 385 and Table 1.

thing more than one. For this reason, only the general orders of magnitude of the ratios of output to number of machines are comparable in these stages, and no percentage difference figures are furnished.

The three independent estimates of machine capacity are in very close agreement for the spinning, picking, and winding stages. The large discrepancies between the drawing-equipment capacity estimate for Plants I and II, and that based on Table 1, can be accounted for by the failure to consider the direct variation of the fineness of drawing sliver with yarn count, in the last estimate. It is interesting to note that discrepancies between the Barnes estimates of machine speeds in drawing and our own tend to be in the opposite direction from the discrepancies shown in Table 6 (see Table 3).

Major discrepancies also appear among the estimates of carding equipment requirements and roving-equipment requirements. The card capacity assumed for Plant I is 25 per cent in excess of the maximum output suggested by the Saco-Lowell card production guide,[24] for the appropriate range of thread counts. The estimate of roving-spindle capacity of Plant I is 82 per cent in excess of the normal output per roving spindle for that thread count suggested in the H & B Catalogue for High Draft Roving Frames. Since the estimate for Plant II and also the Barnes estimates are much closer to our own, it is probably justifiable to conclude that the estimate of roving-frame productivity for Plant I is too high.

C. FIBER INPUT COEFFICIENTS

While machinery requirements vary significantly with product quality, the weights of fiber inputs per pound of output are virtually the same for the common types of cloth. In each case, a small percentage of the weight of cotton supplied to each stage from the preceding one is lost because of cleaning and through waste. Given estimates of the percentage weight loss at each stage, a complete set of fiber input coefficients for the various stages of the process can be estimated. At any stage, i, fiber input from the previous stage required per unit of output is determined by:

$$x_i = (1 - a_i)x_{i-1} \tag{10, 12}$$

where

a_i = the waste percentage at any stage, i, and

$$k_{1i} = \frac{1}{1 - a_i} \tag{10, 13}$$

By this principle, the output of any stage is related to the initial raw cotton input, x_o, as follows:

$$x_i = (1 - a_1)(1 - a_2) \cdots (1 - a_{i-1})(1 - a_i)x_o \tag{10, 14}$$

[24] See above, Table 2.

and the output of any stage is related to final output, x_n, as follows:

$$x_i = \frac{x_n}{(1 - a_n)(1 - a_{n-1}) \cdots (1 - a_{i+1})} \qquad (10, 15)$$

(10, 1b) can now be rewritten:

$$x_{i-1} = \frac{1}{(1 - a_n)(1 - a_{n-1}) \cdots (1 - a_1)} \cdot x_n \qquad (10, 1b')$$

where

x_{i-1} = the amount of fiber to be processed at any stage, i. (10, 1c) can now be rewritten:

$$x_o = \frac{1}{(1 - a_n)(1 - a_{n-1}) \cdots (1 - a_1)} \cdot x_n \qquad (10, 1c')$$

Normal values for the waste percentages, a_i, are estimated for each type of machine by equipment manufacturers, and published in standard production handbooks. A sample set of such waste percentages is given in Table 7, Column 1. Expected waste percentages at each stage are small, and tend to change very slowly over time. Therefore it is reason-

TABLE 7

Per Cent Waste, Raw and Intermediate Fiber Requirements per Unit of Output at Each Stage and Fiber Processed at Each Stage per Unit of Final Output in Carded Cotton Cloth Manufacture

Stage	Per Cent Waste (a_i) (1)	Fiber Input Each Stage per Unit Output Each Stage (k_{1i}) (2)	Ratio Input This Stage to Final Output (Weight) (k''_{1i}) (3)	Ratio Weight This Stage to Raw Stock (4)
(1) Opening				
bale breaker	5.0	1.050	1.226	.950
first opener	2.0	1.020	1.165	.931
second opener	0.5	1.005	1.141	.926
(2) Picking	1.5	1.015	1.136	.912
(3) Carding	6.0	1.060	1.119	.857
(4) Drawing	0.5	1.005	1.052	.853
(5) Roving (per stage)	1.0	1.010	1.046	.844
(6) Spinning	2.0	1.020	1.036	.827
(7) Spooling	1.0	1.010	1.025	.819
(8) Warping	0.5	1.005	1.015	.815
(9) Slashing	1.0	1.010	1.010	.807
(10) Weaving	0.75	1.0075	1.0075	.801

Source: Adapted from Merrill, G. R., 'Print Cloth,' *Textile World*, April 1937, p. 950; and National Association of Cotton Manufacturers, *Yearbook*, 1931, p. 173

able to believe that the estimates quoted for the 1930's in Table 7 are approximately valid for postwar practice.

The fiber input coefficient for the process as a whole was computed from the data in Table 7:

$$K_1 = \frac{1}{(1 - a_1) \cdots (1 - a_n)} = 1.226$$

Column 2 of Table 7 contains the estimates, based on the given waste percentages of individual fiber coefficients for each stage, k_{1i}; and Column 3, the estimates of fiber inputs at each stage per pound of final output, k''_{1i}.

D. LABOR COEFFICIENTS

To estimate labor requirements in a textile mill, it is necessary to specify man-hours required in a large number of different tasks, each requiring specialized skills. In each department these jobs are divided, roughly, into two types, comprising 'direct' and 'indirect' labor. Direct labor consists of tending jobs associated with direct processing equipment, while indirect labor includes mainly supervisory and maintenance tasks. It is reasonable to expect that direct labor requirements will be a function of both the quantities and types of equipment installed in the mill, while supervisory labor inputs will tend to be constant for a mill of a given order of magnitude.[25] This is demonstrated by a survey of technical estimates of labor requirements in plants producing various types of cloth in 1910 and 1936.[26] Table 8 shows the number of workers in a supervisory capacity, as compared with the total number of workers in each department in five hypothetical cotton mills, producing different types of fabric, in 1910 and 1936. It is readily apparent from the table that employment in supervisory capacities tends to be stable as compared with other types of labor inputs, which vary over time, and with the type of product being made.

To explain direct labor input requirements in a given plant, then, the type of product should be taken into account. Ideally, labor inputs, like machinery requirements, should be expressed as functions of more than one dimension of product. However, it is more difficult to estimate such labor input functions than it is to estimate machinery input functions directly from engineering literature. The reason for this is that, unlike machinery design laws, the basic theory of labor layout, required for determining the appropriate function types, is not formulated explicitly in

[25] Maintenance requirements are not treated in this chapter.

[26] The estimates of labor requirements used in this chapter are based on Barnes Textile Associates, op. cit This study estimates the quantities of labor that would be required to produce a given amount of cotton cloth of various kinds in plants equipped with the most modern machinery available in 1910 and 1936.

available published materials. The study of labor requirements is complicated further by heterogeneity of labor, since labor loads vary with the skill of the operators, by such institutional factors as the setting of labor loads by custom or in union contract,[27] and by the fact that labor

[27] In so far as labor loads are established by union contract, the task of estimating labor coefficients is simplified, in that they can be determined directly from the texts of union agreements.

TABLE 8

Variations in Supervisory and in Other Types of Labor Input Required to Produce a Given Quantity of Cloth with Type of Fabric Produced, 1910 and 1936

	Number of Workers Required									
	Canton Flannel		Sheeting		Carded Broadcloth		Combed Broadcloth		Carded Filling Sateen	
	1910 (1)	1936 (2)	1910 (3)	1936 (4)	1910 (5)	1936 (6)	1910 (7)	1936 (8)	1910 (9)	1936 (10)
(1) Carding Department										
supervisory	6	6	12	10	10	8	8	8	6	6
nonsupervisory	284	142	226	102	164	86	210	100	150	80
Total	290	148	238	112	174	94	218	108	156	86
(2) Spinning Department										
supervisory	14	14	12	12	12	12	12	12	12	12
nonsupervisory	206	152	188	122	144	106	138	102	142	104
Total	220	166	200	144	156	118	150	114	154	116
(3) Spooling and Warping Department										
supervisory	2	2	2	2	2	2	2	2	2	2
nonsupervisory	124	50	68	24	58	22	70	24	42	18
Total	126	52	70	26	60	24	72	26	44	20
(4) Slashing and Drawing-in Department										
supervisory	2	2	2	2	2	2	2	2	2	2
nonsupervisory	42	26	30	18	22	14	20	14	22	12
Total	44	28	32	20	24	16	22	16	24	14
(5) Weaving Department										
supervisory	6	6	6	6	4	4	4	4	6	6
nonsupervisory	268	176	232	168	186	124	172	106	212	142
Total	274	182	238	174	190	128	176	110	218	148
(6) Cloth Room										
supervisory	4	2	4	2	4	2	4	2	4	2
nonsupervisory	40	34	40	34	34	32	26	24	28	26
Total	44	36	44	36	38	34	30	26	32	28

Source: U.S. National Research Project, *Mechanical Changes in the Cotton Textile Industry, 1910 to 1936*, pp. 23-6.

This study is, in turn, based on the technical estimates of the Barnes Textile Associates (see above, footnote on p. 392).

CHART 2

Basic 1-side, 40 hr. FRAME LOAD 200 spindles SPINNING 34s filling Spin and Clean

Sides per Spinner 1 Spindles per side 100 Total spindles ---

Ends down per M 35 (per side 100 spindles per hr.=3.5 RPM.front roll 135

Lb. per spindle (%) 90% (0.99 lb.) Wt. of roving 12-oz. (0.75 or 3/4 lb.)
1.5 oz.
Size of roving bobbins 8 x 4 Wt. of yarn (0.09375 lb.) Lb. per spinner 99 lb.
Doffs per spindle - 40 hr. 10.5 at 7 min. per side doff.
Ends pieced - 40 hr. 40x3.4 =140 per side Robing bobbins creeled-40 hr. 132, 1 side

MINUTES PER SPINNER

Piece ends 140 x 21 sec. = 2940 ÷ 60 = 49 min. (49)
Creeling 132 x 15 sec. = 1980 ÷ 60 = 33 min. (33)
Doffing —
Cleaning 95 min. per side (pro-rated) (95)
Rest (10% of 40 hr.)
TOTAL Min. per side per 40 hr. 177 min.

Sides per spinner
Piece ends at 21 sec. Total minutes Actual – no rest – 177 min.
Change roving at 15 sec. Sides possible 10% – –12.2

Reference: Fig. 1 MINUTES PER SPINNER 34s filling Spin only
Piece ends 49 min.
Creeling 33 min.
Doffing
Cleaning
Rest (10% of 40 hr.)
TOTAL 82 min.

Sides per spinner 1
Total minutes 82 (actual)
Sides possible 10% rest – – 26.34 (2634 spindles)

Reference: Fig. 1 MINUTES PER SPINNER 34s filling Spin and Doff
Piece ends 49 min.
Creeling 33 min.
Doffing 73.5 min.
Cleaning -
Rest (10% of 40 hr.)
TOTAL 155.5 min.

Sides per spinner 1
Total minutes 155.5 (actual)
Sides possible 10% rest – – 13.9 (1390 spindles)

Reference: Fig. 1 MINUTES PER SPINNER 34s filling Spin,Doff,Clean
Piece ends 49 min.
Creeling 33 min.
Doffing 10.5 doffs 1 side—40 hr. at 7 min. per doff– 73.5 min.
Cleaning 95 min.
Rest (10% of 40 hr.)
TOTAL 250.5 min.

Sides per spinner 1
Total minutes 250.5 (actual)
Sides possible 20% rest– – 8.62 (862 spindles)

Explanation to Chart 2:

1. 'The basic task analysis of a spinner's work load includes all the information necessary to quick calculation of the number of sides which can be assigned to the spinner. Many overseers already have much of this information and need only know how to apply it. If this is the case, the reader can begin with the actual application of the data, as shown in the latter part of Fig. 1 and labeled "Minutes per Spinner." Note that data is on a 1-side basis. Data in the upper half of Fig. 1 may be readily obtained. Ends down per 1000 spindle hours, for example, may be obtained by a minimum 4-hour check of ends down in the spinning room. Weight of roving and yarn bobbins may be obtained by actual weighing (minus tare weights) or by calculation from the standard hanks-per-doff. Time studies, or very close estimates, must be used to determine the amount of time required by piecing ends, changing roving (creeling), doffing and cleaning. Under "Minutes per Spinner," the actual calculations begin. On this particular layout of 34s filling, the spinner pieces ends, creels, and cleans the frame. Consequently, the total time for the three functions is obtained. This total is 177 minutes of actual work on 1 side for 40 hours. To allow 10 per cent rest, the number of minutes in 40 hours, 2400, is multiplied by 0.90. This results in 2160 actual working minutes in 40 hours. This figure divided by 177 gives the number of sides in the work load, 12.2. Number of sides assigned were 12.

2. 'Applying the same basic data as shown in Fig. 1, the overseer of spinning can adjust the spinner's work load quickly for a special assignment which includes only piecing ends and creeling, with no cleaning duties.

3. 'Occasionally the spinning room has special assignments which require the spinner to doff the frames, as well as spin. The calculation above shows the number of spindles which the spinner can operate on 34s filling.

4. 'The above spinner's task is worked out for an assignment which includes piecing ends, creeling, cleaning and doffing. The number of spindles possible to run on this basis is 862.'

Source: Gunther, F. H., and Gross, Marcus, 'Spinner's Task Can Be Set Quickly,' *Textile World*, April 1947, pp. 108-9.

loads in tasks which require patrolling of a large number of machines are dependent on the layout of the individual plant.

These factors are taken into account automatically in the use of time-study techniques for the setting of labor loads in individual plants. In time-study analysis, each worker's job is broken down into component operations and labor loads are determined on the basis of intra-plant studies of the time required to perform each operation. Chart 2 illustrates the type of time-study techniques determining labor loads in a specific job, spinning, recommended in a trade-journal discussion. Sample forms to be used in this procedure are shown, and their use is explained in the text below them.

Model labor layouts, based on such time-study techniques as these, are sometimes drawn up by textile engineering firms. These are, or can be, standardized on the basis of expert judgment as to average degree of skill available and typical plant layout, and used in the derivation of technical labor coefficients for the study of interindustrial relationships. Such a set of labor coefficients for principal occupations is derived from the Barnes Textile Associates study of labor requirements for various types of products with new (1936) machinery [28] and presented in Table 9

[28] See above, p. 392, footnote 26.

TABLE 9

Labor Requirement Standards per Machine Hour and per Pound of Output with Latest Equipment, 1936, and Approximate Percentage Change in Labor Productivity, 1936–49

Type of Operator	Machine Unit	Canton Flannel: Machine Units Per Man	Canton Flannel: Cotton Consumed per Man-hour(lbs)	Carded Filling Sateen: Machine Units per Man	Carded Filling Sateen: Cotton Consumed per Man-hour(lbs)	Carded Broadcloth: Machine Units per Man	Carded Broadcloth: Cotton Consumed per Man-hour (lbs)	Combed Broadcloth: Machine Units per Man	Combed Broadcloth: Cotton Consumed per Man-hour(lbs)	Sheeting: Machine Units per Man	Sheeting: Cotton Consumed per Man-hour(lbs)	Approximate Per Cent Increase In Labor Productivity 1936–49
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
(1) Picker tender	picker	2.75	1,070	2.50	1,028	4.0	1,254	4.0	1,271	3.0	985	0
(2) Card tender	card	20.25	268	25.00	206	25.00	409	25.50	212	21.50	107	50
(3) Drawing-frame tender	delivery	33.0	535	26.6	206	32.0	252	25.3	254	24.0	282	0
(4) Slubber tender	spindle	144	268	240	49	240	52	240	53	240	104	0
(5) Spinner	spindle	1,542	148	2,462	54	2,353	63	2,462	67	2,046	86	0
(6) Doffer	spindle	3,727	357	5,198	114	5,228	139	5,848	159	3,361	141	0
(7) Slasher tender	slasher	1	1,070	1	514	1	627	1	636	1	979	0
(8) Weaver	loom	47.4	172	58.4	46.7	58.7	66	35.9	61	58.4	89	0
(9) Loom fixer	loom	98.6	357	98.9	79	101.5	114	107.8	182	98.7	151	0

Source: Barnes Textile Associates, op. cit.

together with suggested percentage adjustments which bring them up to postwar standards. In most departments, no appreciable change in labor requirements per machine unit has taken place since 1936.

Labor coefficients are given both on a 'per machine unit' and a 'per pound of cotton consumed' basis.[20] As would be expected from the 'tending' character of the operations, labor inputs per machine vary less with the type of product than do labor inputs per pound of cotton consumed. Variations in the latter reflect changes in both machine units per man-hour and output per machine hour. Table 10 presents the coefficients of variation (relative dispersions) of the two measures among the various types of fabric for each kind of job.

While, for the most part, the variabilities of the machine units per man-hour ratios are not very great, they might be explained in terms of variations in the quality dimensions of the product. A preliminary survey raises hopes that some such systematic relationships between labor requirements and product characteristics can be determined with further study. As an illustration, Chart 3 shows the variations of the machine units per man ratios in spinning, doffing, slubber tending, and weaving with selected product dimensions. The product dimensions were selected on the basis of general knowledge of the job characteristics. At the spinning and roving stages, for example, the finer the product, the slower the rate of machine output, and hence the smaller the rates of creeling and doffing required. Thus it is to be expected that machine units per man in spinning and doffing will vary directly with yarn count. In weaving, the amount of thread repairs required might be expected to vary with the number of threads per inch, and also with the count of yarn, both very fine and very coarse yarns tending to be somewhat weaker than intermediate counts, and, hence, requiring greater labor inputs.

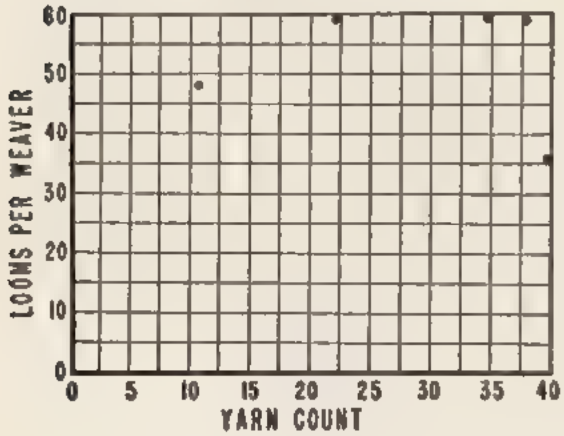
Unfortunately, available technical information is still not sufficient to confirm the choice of relevant product dimensions and to dictate the choice of function types for the establishment of multidimensional labor input functions. For the time being, therefore, the sheeting labor coefficients, selected because of the representativeness of the sheeting quality dimensions, are used as labor coefficients for the industry's production function.

The list of occupations in Table 9 comprises a selected group of principal jobs. Coefficients for the large number of other tasks can be computed by a similar method from the same source. Since, however, the total number of tasks described is so large as to make the list unwieldy for use in economic analysis, an 'all other' or 'miscellaneous' coefficient

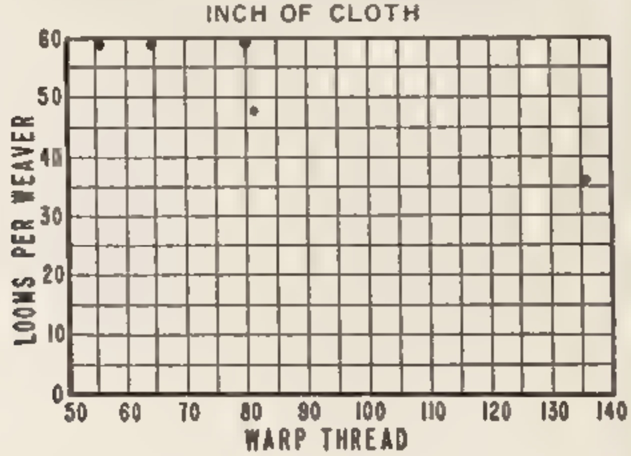
[20] Since fiber inputs per pound of output do not vary very greatly with the type of cloth, labor per pound of cotton consumed is approximately proportional to labor per pound of output.

CHART 3

VARIATIONS IN LOOMS PER WEAVER
WITH YARN COUNT



VARIATIONS IN LOOMS PER WEAVER
WITH NUMBER OF WARP THREADS PER
INCH OF CLOTH



VARIATIONS IN SPINDLES PER SPINNER
WITH YARN COUNT

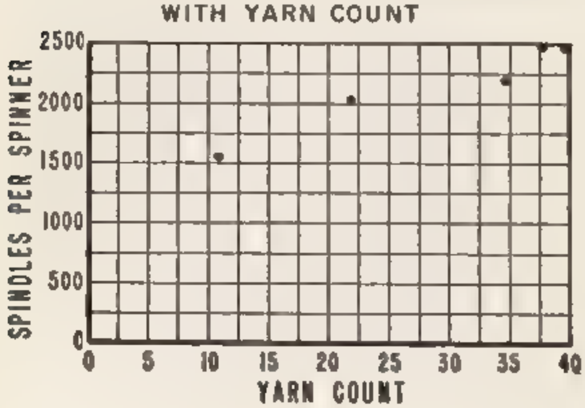
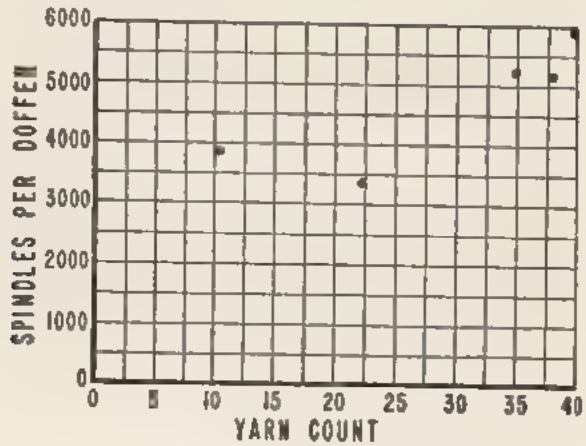


CHART 3

VARIATIONS IN SPINDLES PER DOFFER



VARIATIONS IN ROVING SPINDLES PER SLUBBER TENDER WITH YARN COUNT

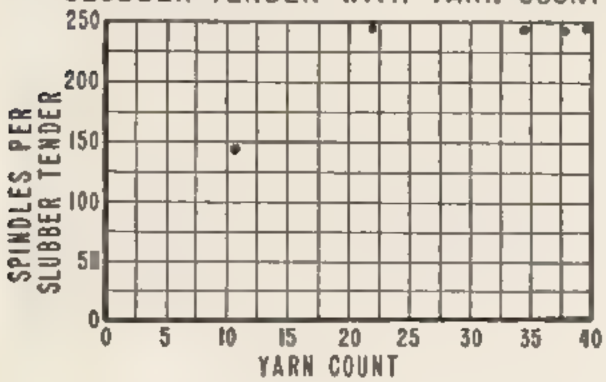


TABLE 10

Relative Dispersion of Labor Input Coefficients Based on Machine Inputs and on Cotton Consumption for Various Occupations in the Production of Five Types of Cotton Cloth

Occupation	Coefficient of Variation of Coefficients, $\frac{\text{Standard Deviation}}{\text{Mean}}$	
	Machine Units per Man (1)	Pounds of Cotton Consumed per Man-hour (2)
(1) Picker tenders	.1946	.1054
(2) Card tenders	.0916	.2596
(3) Drawing frame tenders	.1290	.3832
(4) Slubber tenders	.1739	.7970
(5) Spinners	.1612	.4049
(6) Doffers	.2049	.4872
(7) Slasher tenders	0	.2848
(8) Weavers	.1743	.5136
(9) Loom fixers	.0348	.5470

TABLE 11

Coefficients for Miscellaneous Labor Inputs[1] by Departments in a Cotton Sheeting Mill

Department	Total Employment Man-hours	Employment Covered by Principal Occupations (man-hours)	Miscellaneous Labor Input (man-hours)	Cotton Consumed per Man-hour of Miscellaneous Labor
	(1)	(2)	(3)	(4)
(1) Carding	4,480	2,880	1,600	97.9
(2) Spinning	5,760	2,960	2,800	56.0
(3) Spooling and warping	1,040	480	560	279.8
(4) Slashing and drawing	800	320	450	326.4
(5) Weaving	6,960	2,800	4,160	37.7

[1] Miscellaneous labor inputs are defined as those labor inputs not covered in the list of principal jobs described in Table 9.

Source: Barnes Textile Associates, op. cit.

is used to summarize these miscellaneous labor inputs in each department. It is, of course, to be understood that the sizes of the 'miscellaneous' labor coefficients can be reduced to any desired degree by increasing the detail of the list of specified jobs.

The miscellaneous labor coefficients are derived by dividing cotton consumption per 80 hours by the difference between the total number of man-hours in each department and the number covered by the principal occupations described in Table 9. These miscellaneous labor coefficients are presented in Table 11. These, together with the coefficients in Table 9, can be used to estimate roughly the total labor requirements per pound of cotton consumed in textile manufacture.

E. POWER INPUT COEFFICIENTS

The number of horsepower required to run each particular type of machine is generally specified by the machine-makers' catalogues. Horsepower requirements, for running major machinery at each of the stages discussed above, are given in Table 12.

Given these horsepower requirements, kilowatts used per hour in the running of any machine can be estimated from the expression:

$$\text{kilowatts per hour} = \frac{\text{motor horsepower} \times \text{kilowatts per horsepower}}{\text{motor efficiency}} \quad (10, 16)$$

where

kilowatts per horsepower = .746.

TABLE 12

Power Requirements for Various Kinds of Textile Machinery

Machine	Required Horsepower	Horsepower per Machine Unit
	(1)	(2)
(1) Bale opener	3-5	3-5
(2) Cleaning and blending feeder	2	2
(3) Vertical opener	5	5
(4) No. 12 opener	7.5	7.5
(5) Picker	5-7.5	5-7.5
(6) Card	1.5-2	1.5-2
(7) Drawing frame	.25 per delivery	.25 per delivery
(8) Roving frame	3-5	.03 per spindle
(9) Spinning frame	10-15	.069-.073
(10) Winder	5	5
(11) Warper	2	2
(12) Slasher	2	2
(13) Loom	.25-1	.25-1

Source Adapted from Saco Lowell Shops, *Saco-Lowell Handbook: Engineering and Technical Data*, Vol. 4, Boston, Massachusetts; and *Textile World Yearbook*, 1946-7.

Total power requirements per unit of final output can, of course, be obtained by multiplying the estimates of power requirements per machine unit by machine units per unit of final output, for the respective stages, and summing the power requirements for all stages.

F. SUMMARY

The technical relationships described thus far are sufficient to determine required inputs of direct processing machinery, power, labor, and fiber for the production of a specified type of cotton cloth with equipment of recent vintage. It may be helpful to review these 'best practice' relationships before going on to describe the derivation of the 'average practice' parameters.

Machinery requirements depend on the type of product to be produced. To estimate these, the product must be described in terms of six relevant product dimensions: yarn count, width, picks, ends, twist, and degree of pattern intricacy. The number of machine units required at each stage can be determined by substituting the appropriate numerical value for those dimensions into the expressions in Column 9, Table 1. Thus, for example, for fabric made of 20's yarn, about

$$S = .0315c^{3/2} = .0315(20)^{3/2} = 2.82$$

spindle hours would be required per pound of cloth.

Given machinery requirements, power required to run machinery can

be estimated by multiplying the requisite number of units of equipment by the horsepower per machine-unit coefficients, given in Table 12. Thus, for example, horsepower required per pound of output per hour in spinning is about:

$$.070 \times 2.82 = .197.$$

The horsepower requirement at each stage can be converted to kilowatts through the equation on page 400.

Similarly, approximate estimates of labor input per pound per hour can be made by dividing the number of machine units per unit of output by the number of machine units per man-hour, given in Table 9, Column 10. For example, selected labor inputs in the spinning department would be:

$$\text{spinners: } \frac{\text{2.82 spindles per pound of output per hour}}{\text{2046 spindle units per spinner}} = \text{.00138 spinner hours per pound of output}$$

$$\text{doffers: } \frac{\text{2.82 spindles per pound of output per hour}}{\text{3361 spindles per doffer}} = \text{.00084 doffer hours per pound of output}$$

III. Average and Best-Practice Production Functions

The production function described in the last section was defined operationally in terms of current catalogue information and technical experts' opinions of best practice currently feasible. This is known as a 'best practice' production function, and represents the technology associated with the latest models of equipment. In the cotton textile industry there are few (if any) plants completely equipped with the latest models of machinery although there are many with the latest equipment in a single department. Since, in general, older [30] equipment constitutes a considerable portion of the stock, the best-practice production function will not be representative of the technology of the industry as a whole at any given time.

The 'average' production function is an appropriately weighted mean [31] of the production functions associated with each of the models of equipment in use in the industry. Since, in the cotton textile industry, technological change tends to take the form of increases in output per unit of each input, the coefficients of the average production function tend to be lower than those of the best-practice production function at any given time. Given assumptions as to the relevant variables and function types, the parameters of the average production function can be inferred from

[30] It is important to note that the aspect of age of equipment stressed here is not its length of service, since changes in productivity with age are themselves part of the technical production function for a process. While this makes the analysis more cumbersome, it can easily be taken into account. The relevant feature of age, here, is associated with technological change, that is, its vintage.

[31] I.e. an output-weighted mean, or, for some purposes, a capacity-weighted mean.

industry-wide statistics of inputs and outputs. Particularly where radical changes in the technique of production are taking place, the relevant variables in the average production function will not necessarily be the same as in any one of its components, and as in, particularly, the best-practice production function. This makes the task of selecting variables and function types for the average production function more difficult than would otherwise be expected. However, in the cotton textile industry, recent technological change has been characterized mainly by gradual increases in machine speeds and drafts and decreases in tending requirements, and hence, this problem can be handled relatively easily. With the exception of the roving stages, appropriate parameters for the average production function are determined by substituting observed values of inputs and the various dimensions of output into the same types of functions used for the best-practice production function.

A comparison of average and best-practice parameters is of interest in the study of technological change and diffusion. However, often it is most useful not to compare the parameters directly, but to compare actual productivity, in pounds of output per unit of input in any given year, with productivity for products of the same dimensions produced with best-practice coefficients. This type of comparison of average and best-practice technology reveals the order of magnitude of the differences in performance at the dimensions of current production most clearly. To make such a comparison, average quality dimensions must first be computed for the relevant product. With the exception of t, the twist multiplier, these quality dimensions of current product in the textile industry can be determined from information published in the *Census of Manufactures*. Unfortunately, the latest Census available for this study is 1939, and dimensions characteristic of postwar operations are not yet available. However, inasmuch as the basic characteristics of the average product have been fairly stable over time, and the technical coefficients themselves have not changed much since 1939, comparisons of best practice and average performance based on 1939 product dimensions are significant These average dimensions are derived as follows:

1. Average yarn count. Yarn production statistics, both 'for own use' and for sale, are grouped into yarn count classes, and mean yarn count can be calculated from these frequency distributions of yarn counts.[32]

[32] Since the relationship between yarn count and machinery requirements, $m = .246c^{3/2}$ where m is spindles per pound of output, is not linear, there will be a bias (in this case, a downward bias) in using the mean yarn count to estimate total machinery requirements. Since the number of class intervals is only 4, and the distribution has open ends, there would seem to be only spurious accuracy in using the individual class midpoints instead of the distribution mean. Furthermore, the fact that the distribution of yarn counts is positively skewed tends to impart an upward bias to the use of the mean, which tends to counterbalance the downward bias already described.

The mean yarn counts produced in 5 years during the period 1929 and 1939 were:

TABLE 13

Average Counts of Yarn Spun, 1929-39

Year	Average Yarn Count Spun
1929	21.8
1931	21.9
1935	20.7
1937	20.8
1939	20.8

Source: Computed from Department of Commerce, *Biennial Census of Manufactures.*

Apparently the average yarn count spun tends to remain fairly stable over time, as compared with the range (about 1 to 150) of counts spun in any given year. The computed average is also very close to 20's, the technical standard for a 'typical' yarn count.

2. Average width. Average width of woven goods is obtained by dividing production in square yards by linear yards produced in 1939. The average width of cloth produced in 1939 was 38.8 inches. (No Census statistics for linear yardage were published for 1931, 1933, 1935, or 1937.) This width—38.8 inches—is also in close agreement with the industry standard of 38½ inches for the width of a typical print cloth.[33]

3. Average number of picks and ends per square inch. The total number of threads (picks plus ends) per square inch is estimated from the technical formula:

$$\frac{b(e+p)}{840c} = w \qquad (10, 17)$$

The average values for b, width, and c, yarn count, were already determined. For the year 1939, $b = 38.8$, and $c = 20.8$. w is calculated by dividing total pounds of cloth woven by number of linear yards woven in the given year. For 1939, $w = .294$ pounds per linear yard, and hence $p + e = 132.4$. The numbers of picks and ends are approximately equal for most cloths, the number of picks being generally just slightly smaller than the number of ends. Thus the average number of picks per inch for cloth woven in 1939 was probably about 64, and the number of ends, 68. This estimate resembles closely the industrial standard of 60×64 for a typical print cloth.[34]

4. Average degree of intricacy of pattern. The degree of intricacy of pattern affects machinery requirements primarily in weaving, through its effect on the number of picks per minute at which a loom can be run. The relative importance of plain, dobby, jacquard, and box loom opera-

[33] See Clark, William A., op. cit. p. 14.
[34] Ibid.

tions for over-all productivity is measured by the relative numbers of loom hours of operation in each category. The appropriate loom-hour statistics are not available for prewar years, but are published currently. They are given in Table 14 for the year 1947. In view of the general stability of product-mix in the cotton goods industry, these proportions of the various types of loom operations are probably fairly representative of the 1939 operations as well.

The percentages in Column 2 can be used as weights in averaging the loom speeds appropriate to these different types of looms.

5. The twist multiplier. There is no published statistical information on which an estimate of the average twist multiplier for the industry

TABLE 14

Plain, Dobby, Jacquard, and Box Loom[1] Hours in Cotton Production in the United States, 1947

Type of Loom	Thousands of Loom Hours (1)	Per Cent of Total Loom Hours (2)
(1) Plain	1,708,458	85.3
(2) Dobby	146,224	7.3
(3) Box	113,597	5.7
(4) Jacquard	34,207	1.7
Total	2,002,486	100.0

[1] It is, of course, possible for a box loom to be equipped with dobbies or jacquard harnesses. Apparently the categories used in this tabulation have been so defined as to be mutually exclusive, since the total of loom hours in each category is equal to aggregate loom hours in the industry.

Source: Department of Commerce, Bureau of the Census, Facts for Industry, Series M15 A-07, *Cotton Broad Woven Goods, Summary for* 1947; p. 11.

can be based. A value of 4 appears on the basis of general evidence of the construction of common cloths to be fairly representative of the product of the industry.[35]

[35] The twist multipliers characterizing the types of cloth described in the Barnes study (see source note to Table 3, p. 387) are as follows:

FABRIC	TYPE OF YARN	TWIST MULTIPLIER
Sheeting	Warp	4.50
	Filling	4.00
Carded Broadcloth	Warp	3.85
	Filling	3.70
Carded Filling Sateen	Warp	4.50
	Filling	4.00
Combed Broadcloth	Warp	3.80
	Filling	3.50
Canton Flannel	Warp	4.75
	Filling	3.50
	Average (equal weights)	4.01

Given these average product dimensions, it is possible to estimate what output per machine hour (in pounds of this product) would have been with best-practice coefficients. In Table 15 these estimates are compared with statistical estimates of actual average output per machine hour at several stages of cotton production in selected recent years.

Similar comparisons are also made for the fiber and labor input coefficients. In so far as any of the quality dimensions of the product affect the values of these best-practice coefficients, approximate average values for these dimensions are already implicit in them. Hence the fiber and labor coefficients are already in a form directly comparable with the average input-output ratios in the industry.

The technical best-practice ratio of cotton yarn output to raw cotton input has changed little over the past few decades. As is shown in Table 1, the best-practice ratio of output to input is .827, while the actual ratios, in those years for which we know them, have been:

YEAR	RATIO OF YARN OUTPUT TO COTTON CONSUMED
1927	.834
1929	.814
1931	.834
1935	.845
1937	.786
1939	.920
Mean	.839

These ratios were obtained by dividing statistics of 'yarn spun on the cotton system' by cotton consumed.[36] The slight excess of the average ratio above technical expectations may be explained by the spinning of fibers other than cotton on the cotton system.

In Table 16, 1946 best-practice labor coefficients are compared with 1946 output per man-hour in selected occupations. Raw cotton consumed is used as a measure of output both in the best-practice and in the average productivity measures. Average output per man-hour in 1946 is estimated on the basis of a sample percentage distribution of man-hours by type of job in the cotton textile industry.[37] The number of man-hours spent at each job in the industry is found by multiplying the proportion

[36] Department of Commerce, Bureau of the Census, *Census of Manufactures*, 1939; ibid. Annual Report, *Cotton Consumption and Distribution*.

[37] Department of Labor, Bureau of Labor Statistics Industry Wage Studies, Series 2, #37, *Cotton Textiles* 1946.

TABLE 15

Output per Active Machine Unit per Hour, 1939, 1942, 1946, and Best Practice Standards Based on Technical Data, Cotton Textile Industry

Machine Unit	Year	Output per Active Machine Hour (pounds per hour)	Best Practice Standard (pounds per hour)	Per Cent Difference of Standard
	(1)	(2)	(3)	(4)
(1) Card	1939	9.9	8 to 10	....
	1942	8.3		....
	1946	11.1		....
(2) Drawing delivery	1942	9.0	12.8	29.7
(3) Roving spindle	1942	.22	1.06	336
(4) Spinning spindle	1939	.0323		35.3
	1942	.0352	.0437	19.4
	1946	.0360		17.6
(5) Loom	1939	1.64		5.8
	1942	1.55	1.74	10.9
	1946	1.86		6.9

Sources. 1946 machine productivities (except in spinning) are the quotient of pounds of yarn consumed to number of machine hours (Department of Commerce, Bureau of the Census, Facts for Industry Series M15 A-07, *Cotton Broad Woven Goods*), multiplied by the appropriate technical ratio from Table 7 above to convert from 'yarns consumed' to final output.

Estimates of output per spindle hour in 1946 were constructed by multiplying the ratio of cotton consumed to active cotton spindle hours (Department of Commerce, Bureau of the Census, Annual Reports, *Cotton Production and Distribution*) by the technical best-practice ratio of yarn output to cotton consumption, Table 7.

Output per loom hour in 1942 was found by multiplying 1942 spindle hours (ibid.) by the ratio of loom hours to spindle hours for the period April 1944 to March 1945 (Department of Commerce, Bureau of the Census, Facts for Industry Series M15 A-07, op. cit.), dividing this estimate of 1942 loom hours into cotton consumed in 1942 (Department of Commerce, Bureau of the Census, Annual Reports, op. cit.) and multiplying by .801, the technical ratio of loom output to raw cotton consumed.

Output per card hour in 1942 was derived by computing the mean from Department of Commerce, Bureau of the Census, Facts for Industry Series 32-16-1, *Cotton and Rayon Mill Machinery in the United States*, 1942, Table 8

Outputs per roving-spindle hour and per drawing-delivery hour in 1942 were found by multiplying the number of machine units in place (ibid.) by the number of operating hours per spinning spindle in 1942, dividing cotton consumption by this product and multiplying by the relevant ratios in Table 7 to convert from cotton consumption to final product.

Output per card hour in 1939 was derived by the same method as output per card in 1942, the 1939 *Census of Manufactures* being the source of the statistics of cards in place.

Output per loom hour in 1939 was found by dividing cotton broad goods over 12 inches in width woven in 1939 (ibid.) by the product of the number of broad looms in place (ibid.) and the number of hours operated per spinning spindle in place.

in each occupation in the sample by an estimate of total man-hours.[38] Cotton consumption per man-hour in each job is the quotient of the cotton consumption [39] divided by the respective man-hours per job estimates.

Similarly best-practice labor coefficients for 1936 are compared with average productivity estimates for 1937, the closest year to 1936 for which a sample percentage man-hour distribution by occupation is available.[40]

The comparisons of the coefficients for machinery and for labor reveal the expected relationships between best practice and average productivity: with few exceptions our statistical estimates of productivity per man-hour and per machine-hour are lower than those based on best-practice coefficients. The discrepancies between average and best-practice coefficients tend, also, to be smallest in those operations which have undergone little technological change in recent years (i.e. output per man-hour and per machine-hour in carding and drawing).[41] Presumably this is so because there already has been sufficient time for a greater degree of diffusion of the best-practice techniques at those stages.

It is also significant that the values of the average coefficients tend to rise over time. This is explained by two factors: increases in the best-practice coefficients, and changes in the proportions of new and old equipment in the stock. These two factors are central in the traditional economic analysis of the problem of technological change. Changes in best-practice coefficients constitute changes in technological possibilities, that is, extensions of the technological horizon. It is generally recognized that technical research and invention are not independent of economic conditions, and there is undoubtedly much that economists could contribute to an explanation of this aspect of technological change. Traditionally, however, changes in the best-practice production function have been viewed as data. It is consistent with this tradition that, in empirical research, we define the best-practice parameters operationally in terms of accepted engineering standards.

[38] Man-hours worked in the cotton goods industry in 1946 was estimated by multiplying the ratio of man-hours worked in cotton textiles in 1947 (Department of Commerce, Bureau of the Census, 'Rayon and Related Manufactures,' *Census of Manufactures*, 1947, p. 4) to man-hours worked in all textile plants in 1947 (*Statistical Abstract of the United States*, 1949) by the quotient of the all textile wage bill (National Income Supplement, *Survey of Current Business*, 1946) divided by average hourly earnings in textiles in 1946 (*Statistical Abstract of the United States*). All the sources above are publications of the Department of Commerce.

[39] Department of Commerce, *Cotton Production and Distribution.*

[40] The percentage distribution of man hours by job in 1937 is taken from Department of Labor, Bureau of Labor Statistics, Bulletin #663 (Henricks, A. F.), *Wages in Cotton Goods Manufacturing*, p. 97. For description of method of estimating total man-hours in 1937, see below, p. 413

[41] While it is true that best-practice standards of card-tending requirements changed considerably between 1936 and 1946, it was possible to take advantage of this change by installation of new parts and minor alterations and the rise in carding labor productivity was not conditional on completely new equipment installations.

On the other hand, changes in the proportions of new and old equipment in the industry's stock constitute the problem of innovation. Given the technical parameters appropriate to all vintages represented in the equipment stock, the explanation of the value of the average coefficient at any given time can be viewed as a problem of establishing appropriate weights in the average, and the problem of innovation can be treated as that of explaining changes in the weights assigned to the various types of equipment in the average production function. As new equipment units are added, the weight assigned to the best-practice production function is increased. Thus the best-practice capital stock coefficients are sometimes called 'incremental.' Similarly, as older equipment is scrapped, the weight assigned to the 'older' production functions tends to decrease.

Given initial values for the average coefficients, the best-practice and the 'oldest' coefficients covering a given period and the corresponding scrappage and new-machine purchase statistics, it should be possible to predict the coefficients of the average production function at the end of the period.

With fixed coefficients of production,[42] the averaging process can be performed separately for each coefficient.

Let

$b_{ij}(t)$ = the output of j per unit of input of i at time t,
$b_{ij}(t_o)$ = the output of j per unit of input of i at some initial time t_o,
b'_{ij} = the relevant coefficient of the production function for equipment units taken out of service,
$\hat{b}_{ij}$ = the output of j per unit of input of i under conditions of best practice,
I = the number of new equipment units purchased between t_o and t,
S = the number of equipment units taken out of service between t_o and t, and,
T = the total number of equipment units in use at time t_o.

[42] The assumptions of fixed coefficients and technological change are compatible provided that the older process does not remain an economic substitute for the newer one. If it is assumed that no technological information is forgotten, then the old coefficients sometimes do remain as economic alternatives to the new ones. There is no economic substitution, however, if each of the input requirements per unit of output is equal to or less than the old after technological change, or if the new process will involve lower costs than the old under all conditions which can reasonably be expected to govern the rest of the economic system.

Where new equipment is involved, this cannot be determined without comparing the old and new machine units in terms of respective inputs required to produce them. Only if the machine-making input requirements per unit of textile output are reduced can it be said that machinery requirements per unit of output have been lowered. Similarly, if new textile machines must be produced by different machine tools, etc., it may be necessary to analyze additional industries to settle this question. For practical purposes, the coefficients associated with old models of machinery can be disregarded as alternatives when machinery producers themselves discontinue the old models. This is generally the case in the textile industry.

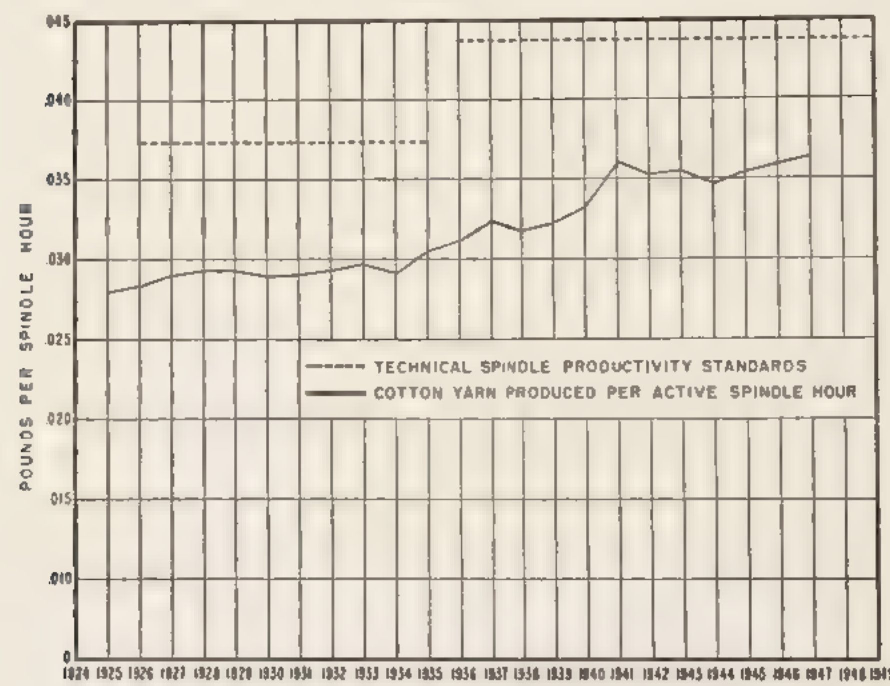
TABLE 16

Best Practice Labor Input Coefficients 1936 and 1946, and Average Labor Productivity 1937 and 1946, in Selected Occupations in the Cotton Yarn and Cloth Industry

Occupation	Cotton Consumed per Man-hour			
	1936 Best Practice	1937 Average Practice	1946 Best Practice	1946 Average Practice
	(1)	(2)	(3)	(4)
(1) Picking	985	646	985	575
(2) Card tending	197	212	296	272
(3) Drawing frame tending	493	323	493	461
(4) Roving frame tending	104	76	104	150
(5) Spinning	86	43	86	53
(6) Doffing	141	98	141	115
(7) Slasher tending	979	715	979	545
(8) Weaving	89	47	89	56
(9) Loom fixing	151	151	151	143

Sources: Best-practice coefficients: Barnes Textile Associates, *Survey of Technological Improvements and Developments in the Cotton Textile Industry*, 1910-36.
Average-practice coefficients: see text preceding this table.

CHART 4



Then the average coefficient is related to the coefficients of the component parts of the stock as follows:

$$b_{ij}(t) = b_{ij}(t_o) + \frac{I}{T}[\bar{b}_{ij} - b_{ij}(t_o)] + \frac{S}{T}[b_{ij}(t_o) - b'_{ij}] \qquad (10, 18)$$

For purposes of illustration this relationship has been applied in the explanation of changes in average output per spindle hour in the cotton textile industry.[43] For this particular coefficient it has been possible to trace both the average and the best-practice coefficients over the period 1925-47. The values of the two sets of coefficients are given in Table 17 and Chart 4.

Predictions of the average coefficient on the basis of equation (10, 18) are made for two periods, 1925-36, and 1936-40. The separation between the two periods is made on the basis of the shift in the best-practice coefficient about 1936.[44] War and postwar periods were omitted for lack of new-machine production data. The values of productivity per active spindle hour predicted on the basis of equation (10, 18) are compared with actual productivity in Table 18. I is measured by new spindles shipped for installation, S by estimated scrappage, and T by the number of spindles in place. The difference between the average coefficient at t_0 and the coefficient for discarded equipment was estimated at .005 in both cases.[45] (This is about equal to the size of the difference between the best-practice coefficients for the two periods.)

In view of the fact that the assumed productivities of scrapped equipment are only approximate, the very close agreement between actual and predicted productivity should not be considered too significant. Nevertheless, the fact that they are of the same general order of magnitude serves to confirm the contention that technical coefficients of direct economic relevance can be derived from purely technical sources.

Since direct labor input requirements depend on the type of equipment in use, it should also be possible to predict changes in the average labor coefficients by the same procedure as that used to explain changes in the average machinery coefficients.

[43] The effects of changes in average product dimensions on productivity were not taken into account in these computations. The necessary data describing the average product at the beginning of the earlier period covered were not available, while in the later period, changes in average product dimensions were small enough to be neglected.

[44] The actual productivity series shows some annual fluctuations which can probably be explained in terms of year-to-year quality variations. Since information as to average yarn count produced is available only for a few years, these irregularities could not be eliminated from the series These variations were considered small enough to be neglected in the comparisons made over longer periods of time.

[45] Because of a lack of reliable descriptive information our estimate of the productivity of discarded equipment can only be very rough It is estimated (see Henricks, A F, op. cit.) that mule spindles constituted roughly half of the discarded spindles during the period 1923-37. These probably had a productivity of less than half the 1910 standard for ring spindles, and would account for a difference of about .003 of the assumed difference between average productivity and that of the discarded spindles during the first interval. The remainder of the discarded spindles was most likely of the vintage of around 1890, and could easily account for an additional discrepancy of .002 between the productivities of average and discarded spindles. Similarly, it is assumed that the average spindle discarded during the period 1936-40 had a productivity slightly higher than that of the ring spindles discarded in the earlier period.

TABLE 17

Cotton Spindles in Place, New Cotton Spindles Shipped for Installation, Estimated Spindle Scrappage, Cotton Yarn Produced per Active Spindle Hour, and Best-Practice Cotton Yarn Spinning Productivity, 1925-40

	Year	Spindles in Place (thousands)	Average Spindle Productivity (pounds per active spindle hour)	Best Practice Coefficient (pounds per active spindle hour)	Cotton Spindles Shipped for Installation (thousands of spindles)	Estimated Spindles Scrapped (thousands of spindles)
		(1)	(2)	(3)	(4)	(5)
(1)	1925	37929	.0281	.0371	335	341
(2)	1926	27568	.0284	•	217	696
(3)	1927	36696	.0290	•	496	1089
(4)	1928	35540	.0293	•	256	1652
(5)	1929	34820	.0294	•	321	976
(6)	1930	34025	.0289	•	252	1116
(7)	1931	32673	.0289	•	205	1604
(8)	1932	31709	.0293	•	144	1169
(9)	1933	30893	.0298	•	349	960
(10)	1934	30942	.0293	•	530	300
(11)	1935	30093	.0306	•	215	1379
(12)	1936	28147	.0313	.0437	469	2161
(13)	1937	26982	.0325	•	773	1634
(14)	1938	26372	.0317	•		1383
(15)	1939	25261	.0323	•	1620	2469
(16)	1940	24750	.0332	•		

Sources: Spindles in place is quoted from the Department of Commerce. Bureau of the Census. *Statistical Abstract of the United States* The description of the method used for computing yarn production per active spindle hour is given in the source note to Table 15. Best-practice coefficients are based on Saco-Lowell technical data. Cotton spindles shipped for installation is from Henricks. A. F., *Wages in Cotton Goods Manufacturing*, Department of Labor Bureau of Labor Statistics, Bulletin #663.

Total new spindles installed in 1938, 1939, and 1940 were estimated by dividing the number of new spinning machines (including machines for spinning all kinds of fibers) produced in 1939 (Department of Commerce Bureau of the Census, *Census of Manufactures*, 1939) by two, to eliminate (very roughly) wool and rayon spinning machines produced, and multiplying by 260, an approximate average of the number of spindles per frame. The resulting figure was multiplied by 3, on the assumption that the estimate of new spindles purchased in 1939 is an average for the three years.

Scrappage in any given year was taken to be the number of spindles in place the year before, plus the number of new spindles installed during the given year, minus the number in place in the given year.

To demonstrate this, it was first necessary to estimate the values of the average labor coefficients over a period of time. Time series of average output per hour in various occupations were constructed by a

TABLE 18

Actual and Predicted Values of $b(t)$, Output per Active Cotton Spindle Hour, and Proportions of New and Scrapped Spindles to Initial Stock of Spindles in Place, 1925-35, and 1936-40

Beginning of Period (t_0)	End of Period (t)	Proportion New Spindles to Initial Stock $\frac{I}{T}$	Proportion Scrapped Spindles to Initial Stock $\frac{S}{T}$	Predicted Productivity ($b_{ij}(t)$) (pounds per active spindle hour)	Actual Productivity (pounds per active spindle hour)
	(1)	(2)	(3)	(4)	(5)
1925	1935	.0876	.297	.0304	.0306
1936	1940	.1020	.272	.0339	.0332

procedure similar to that described on page 411. For each year percentage distributions of man-hours, based on sample studies, were multiplied by total man-hours in the industry to yield estimates of man-hours spent in each job. The quotient of cotton consumption divided by man-hours in each job measures labor productivity in that particular operation.

The time series of cotton consumption per man-hour in selected occupations are compared with the Barnes' technical standards for 1910, 1936, and 1946 in Table 19 and this information is shown graphically in Chart 5.

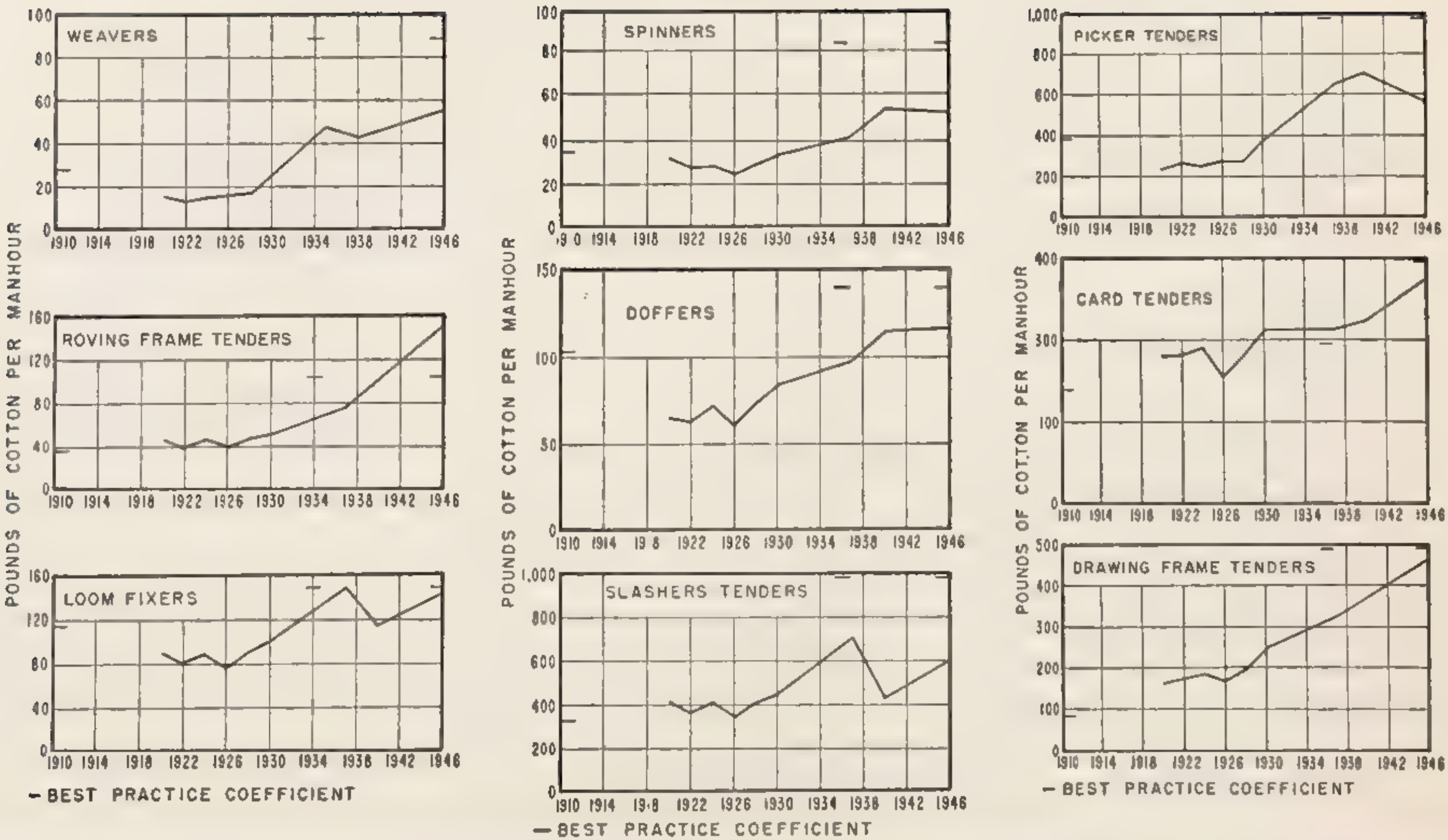
Despite the indirect method by which the average labor productivity estimates were derived, they are of the same order of magnitude as the technical standards, and with two exceptions (in carding and roving[46]) the average labor productivity estimates are below the best-practice coefficients but tend to approach them over time. Thus it is reasonable to expect that average labor productivity can be explained in terms of a relationship such as equation (10, 18), where the b's are redefined as labor, rather than equipment, coefficients.

Unfortunately, as in the explanation of the equipment coefficients, the only reasonably satisfactory quantitative information on new equipment purchases deals with spinning machines. Furthermore, there is even less information concerning the labor coefficients relevant to discarded equipment than there was concerning these machines' productivities. In the case of spinners, the 1910 best-practice coefficient is very close to the average productivity of 1920, which suggests that there was no great difference between average labor productivity and that associated with discarded equipment in 1920.

Assuming this difference to be negligible, cotton consumption per man-

[46] These exceptions can most likely be accounted for in terms of statistical errors in the average productivity estimates.

CHART 5



WEAVERS
SPINNERS
PICKER TENDERS
ROVING FRAME TENDERS
DOFFERS
CARD TENDERS
LOOM FIXERS
SLASHERS TENDERS
DRAWING FRAME TENDERS
POUNDS OF COTTON PER MANHOUR
— BEST PRACTICE COEFFICIENT

TABLE 19

Pounds of Cotton Consumed per Man-hour, 1920-46, and Technical Best Practice 1910 and 1936, for Principal Occupations in the Cotton Yarn and Cloth Industry

Year	Picker Tenders		Card Tenders		Drawing Frame Tenders		Spinners		Doffers	
	Cotton Consumed per Man-hour	Technical Standard	Cotton Consumed per Man-hour	Technical Standard	Cotton Consumed per Man-hour	Technical Standard	Cotton Consumed per Man-hour	Technical Standard	Cotton Consumed per Man-hour	Technical Standard
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1) 1910		394.0		140.7		98.5		35.2		103.8
(2) 1920	229.0		180.4		168.5		30.8		64.4	
(3) 1922	261.1		182.9		173.1		27.6		63.6	
(4) 1924	246.6		189.5		184.3		27.9		71.8	
(5) 1926	265.0		155.6		171.8		24.8		61.8	
(6) 1928	268.0		179.5		191.8		28.8		72.6	
(7) 1930	368.4		213.5		247.8		33.6		83.8	
(8) 1936		985.0		197.0		493.0		85.6		140.8
(9) 1937	646.4		212.1		323.2		43.4		97.9	
(10) 1940	709.1		223.6		370.7		54.4		114.1	
(11) 1946	575.1	985.0	727.4	296.0	461.1	493.0	53.0	85.6	115.3	140.8

Sources: Technical standards: see Table 16.

Cotton consumed per man-hour: for description of the procedure used in deriving, see text.

Percentage distribution of man-hours by occupation: Department of Labor, Bureau of Labor Statistics, Bulletin #492, *Wages and Hours of Labor in Cotton Goods Manufacturing*, 1910-28; *Monthly Labor Review*, December 1941, pp. 1506-7, Table 10; ibid. 'Wages and Hours of Labor in the Cotton Goods Industry,'

TABLE 19 (Continued)

Pounds of Cotton Consumed per Man-hour, 1920-46, and Technical Best Practice 1910 and 1936, for Principal Occupations in the Cotton Yarn and Cloth Industry

	Slasher Tenders		Weavers		Roving Frame Tenders		Fixers	
	Cotton Consumed per Man-hour	Technical Standard	Cotton Consumed per Man-hour	Technical Standard	Cotton Consumed per Man-hour	Technical Standard	Cotton Consumed per Man-hour	Technical Standard
	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
(1) 1910		328.0		28.2		36.4		116.0
(2) 1920	420.3		15.7		45.0		89.3	
(3) 1922	372.1		13.7		41.9		82.6	
(4) 1924	414.0		15.2		45.7		88.7	
(5) 1926	347.5		16.3		39.2		77.8	
(6) 1928	409.4		17.5		45.5		90.0	
(7) 1930	452.5		24.7		52.6		101.0	
(8) 1936		979.0		89.0		103.8		150.5
(9) 1937	715.3		47.3		76.0		150.3	
(10) 1940	434.9		43.3		102.8		113.0	
(11) 1946	599.4	979.0	56.0	89.0	149.8	103.8	142.7	150.5

hour in 1937 is predicted on the basis of equation (10, 18). The relevant machinery purchase data, together with the average productivity in the initial year, the best-practice coefficient for the period, and the predicted and actual 1937 productivities are given in Table 20.

Since doffers also work at spinning machines, output per doffer hour should be predictable on the basis of the same machinery-purchase information as output per spinner hour. The predicted and actual outputs per doffer hour are also given in Table 20. These are based on the assumption that the difference between the average productivity per doffer hour in 1920 and that of doffers working with discarded equipment is zero. In view of the sizable excess of the 1910 coefficient over the 1920 average productivity,[47] this assumption is probably unrealistic for doffing. The omission of this factor may account for the discrepancy between the actual and predicted productivities in doffing for 1937.

Since both labor and machinery coefficients with scrapped equipment were estimated very crudely, the fact that agreement between actual and predicted productivities is extremely close should not be taken too seriously. Nevertheless, the fact that, in each case, actual and predicted values are of the same general order of magnitude should encourage further investigation in this approach to the analysis of technological change.

Where an age distribution of equipment is available, it should also be possible to reconstruct the average technical coefficients for any given year by weighting the coefficients relevant to each type of machinery by proportion of that machinery to total stock For such a rough check on the relations between best-practice and average labor coefficients, we can take advantage of certain indirect evidence of the age distribution of equipment, published in a 1942 inventory of cotton and rayon mill machinery.[48] The proportions of equipment in the various age categories can be used to weight the best-practice coefficients associated with their respective vintages of equipment to reconstruct the average coefficient at any given time.

[47] See above, Table 19.

[48] Department of Commerce, Bureau of the Census, Facts for Industry Series 32-16-1, *Cotton and Rayon Mill Machinery in the United States,* 1942.

November 1930, p. 166; Henricks, A. F., *Wages in Cotton Goods Manufacturing,* Department of Labor, Bureau of Labor Statistics, Bulletin #663; Department of Labor, Bureau of Labor Statistics, Industry Wage Studies Series 2, #37, *Cotton Textiles,* 1946.

The man-hours series for the years 1920-40 are computed from an index published in Department of Labor, Bureau of Labor Statistics, Employment and Occupational Outlook Branch, Productivity and Technological Development Division, *Productivity in Selected Manufacturing Industries,* 1919-40, p. 34, and from an estimate of man-hours worked in 1935, published in Department of Commerce, Bureau of the Census, *Man-hours Statistics for 32 Selected Industries,* November 1935.

TABLE 20

New Spindles Shipped for Installation, Best-Practice Coefficient, and Predicted and Actual Cotton Consumption per Man-hour in Spinning and Doffing, 1937

Occupation	Cotton Consumed per Man-hour 1920 (pounds)	Best-Practice Coefficient	New Spindles Shipped for Installation 1920-37 (000)	Spindles in Place 1920 (000)	Predicted Productivity 1937 (pounds of cotton consumed per man-hour)	Actual Productivity 1937 (pounds of cotton consumed per man-hour)
	(1)	(2)	(3)	(4)	(5)	(6)
Spinning	30.8	85.6	7816	35,834	42.8	43.4
Doffing	64.4	140.8			81.1	97.9

Sources: Spindles in place and shipped for installation: see Table 17. Other data: see Table 16.

In this equipment inventory, spinning spindles are classified into those having long and regular draft, and into tape- and band-driven categories.[49] Long-draft spinning has been available since the 1920's, and it can safely be presumed that the regular draft, band-driven spindles are, on the average, of the vintage described by the 1910 labor coefficients. On the other hand, the bulk of the long-draft spindles was probably installed somewhere between the late 1920's and 1942, and their operations will be described, at least roughly, by the 1936 coefficients.

The productivities of both spinners and doffers in 1940 are close to the values predicted on the basis of the 1910 and 1936 best-practice coefficients. The fact that the predicted values of the coefficients are in excess of the actual productivities may be explained in part by the fact that some new machinery was probably purchased between 1940 and 1942.

In this final section it has been pointed out that, given the best-practice technical parameters, the initial values of the average technical coefficients and the technical coefficients associated with discarded equipment, the explanation of this second aspect of technological change is a matter of determining the level of new machine purchases and retirements[50] within the framework of given technical alternatives. Thus innovation becomes a special aspect of the investment problem: at least a minimum retirement figure is technologically determined while, in an open system, the bill of goods places a limit on the possible changes in capacities of industries.[51] The remainder of the explanation can be re-

[49] Since all long-draft spinning frames are tape-driven, and since tape-driven frames of regular draft are easily converted to long draft, the proportion of tape-driven frames in the stock is just about equal to the proportion of frames with long draft.

[50] If older than average equipment is reserved for peak loads, the average production function will also vary with the level of employment.

[51] Assuming fixed coefficients, the bill of goods determines the minimum allowable capacity for each industry.

TABLE 21

Comparison of 1940 Average Labor Productivity with Weighted Mean of Best-Practice Coefficients, 1910, 1926

Occupation	1910 Best Practice Coefficient (cotton consumed per man-hour)	Proportion of Band-driven Spindles	1936 Best Practice Coefficient (cotton consumed per Man-hour)	Proportion of Tape-driven Spindles in 1942	Weighted Mean of 1910 and 1936 Coefficients	Cotton Consumed per Man-hour 1940
	(1)	(2)	(3)	(4)	(5)	(6)
Spinning	35.2	.505	85.6	.495	60.2	54.4
Doffing	103.8	.505	140.8	.495	122.1	114.1

Sources: Best-practice coefficients: Barnes Textile Associates, op. cit.
Proportion of tape- and band-driven spindles: Department of Commerce, Bureau of the Census, Facts for Industry Series 32-16-1, *Cotton and Rayon Mill Machinery in the United States,* 1942.

duced to the more familiar problems of cost minimization and substitution. On the scale of empirical general equilibrium analysis, of course, even these are by no means simple questions to handle.

The explanation of changes in the average technology in use in terms of the best-practice production function does not constitute a complete solution to the innovation problem, in that changes in the weights assigned to the various component production functions are not explained. These changes depend on investment decisions. In so far as investment decisions are conditioned by cost differentials, it is important to know the production function describing any individual technique introduced, in order to test hypotheses explaining its introduction. Thus, while determination of best-practice and average production functions does not in itself explain investment, it is an important prerequisite to necessary studies of empirical investment.

The present study has tested the feasibility of deriving economically meaningful technical coefficients, particularly best-practice coefficients, from technical source materials. Comparisons of the best-practice and average coefficients have demonstrated that this type of procedure can produce economically significant results. This is not to deny the seriousness of the data problems encountered. Even a cursory review of the technical findings will bring to the reader's attention discrepancies among technical estimates of equipment requirements, while available information on labor-input requirements has proved insufficient for the determination of the desired types of multidimensional labor-output functions. It is most important, however, that these deficiencies are not inherent in the approach to the problem, but are due to very specific gaps in our technical knowledge. Given sufficient resources, these gaps can be filled in through direct consultation with technical specialists.

The selection of significant input and output dimensions on a technical basis, and the reformulation of the innovation problem in terms of best-practice and average coefficients point up needs for specific new kinds of economic statistics. Thus, the increase in stability of the production function with respect to changes in product-mix cannot be realized unless our output statistics describe each year's product in terms of the appropriate quality dimensions. In this particular respect, the Census description of the cotton textile industry's product has been fairly satisfactory in recent years. On the other hand, conclusive tests of the relation between our best practice and average production functions must await the development of detailed equipment inventory and age distribution statistics.

At the close of a highly detailed empirical study, then, a need for even more detailed data becomes evident. This is to be expected at the conclusion of a pilot study. While it is always possible to establish the general nature of the data needed, *a priori*, certain specific informational requirements will never be discovered before the empirical investigation is launched. As the work progresses, there will be a constant interaction between empirical findings and the development of further data requirements. As is so often the case, one of our most valuable achievements is to have singled out those things about which we want to know more.

Chapter 11

COMMERCIAL AIR TRANSPORTATION IN THE UNITED STATES

Allen R. Ferguson

THE OBJECT of the present paper is to ascertain and explain the amounts of current and capital inputs in the production of air transportation in the United States in the years 1939 and 1947. One might construe the 'air transportation' industry to include airports and airways. However, they are entirely different from the airlines with respect both to types of equipment used and to institutional organization. Further, airports and airways produce services which are used by groups other than the commercial air carriers. Therefore it is better to consider the airport and airway services as being produced by distinct industries.

For lack of data all air transportation performed in private, military, and non-scheduled commercial flying is excluded from the computations made below. Similarly because their problems are significantly different from those faced in domestic air transportation the computations also exclude international carriers.

One type of material input, fuel, the most important, and one group of laborers, the flight crews, will be considered. All capital requirements will be analyzed.

The pertinent data are shown in Table 1.

The inputs will be taken up in this order: capital, flight-crew labor, fuel. First it is necessary to explain and estimate output.

I. The Measure of Output

Although no single measure of output is entirely satisfactory, available ton miles produced in a year will be used in computing input coefficients. 'Available ton miles produced' exceed the amount of service *sold* because aircraft typically fly with less than full loads. Since costs vary primarily with capacity available, rather than with capacity sold, the former is the preferable unit. Also a ton-mile unit provides a uniform physical measure for all types of traffic and takes account of the basic dimensions of any transport service.

Output for the years 1939 and 1947 was, respectively, 133.4 and 1,220.8 millions of available ton miles.

TABLE 1

Input and Output Coefficients, United States Domestic Air Carriers,[1] 1939 and 1947

	A. Capital							
	Inputs in Dollars				Physical Inputs			
	Absolute Inputs (millions $)		Input Coefficient[2] ($ per ton mile)		Absolute Inputs		Input Coefficient[2]	
	1947	1939	1947	1939	1947	1939	1947	1939
					(airplanes)		(airplanes per million ton miles)	
(1) Aircraft	155.6	N.A.[5]	0.1275	N.A.	676.0[4]	251.7[4]	0.5537	1.887
(2) Other flight equipment	116.4	N.A.	0.0953	N.A.	–	–	–	–
Total flight equipment	272.0	N.A.	0.2228	N.A.	–	–	–	–
(3) Other property and equipment	77.5	N.A.	0.0635	N.A.	–	–	–	–
Total capital	349.5	38.4	0.2863	0.288				
	B. Labor							
					(men)		(men per million ton miles)	
(4) Pilots and co-pilots	35.9	6.8	0.0054	0.051	–	–	–	–
(5) Stewardesses	6.6	0.7	0.0054	0.005	–	–	–	–
(6) Other flight personnel	0.6	–	0.0005	–	–	–	–	–
Total flight crew	43.1	7.5	0.0353	0.056	8272	1948	6.776	14.60
	C. Materials							
					(millions of pounds)		(pounds per ton mile)	
(7) Fuel	34.8[3]	5.1	0.0285[3]	0.038	1765	283	1.446	2.12

[1] Scheduled airlines only.
[2] Ton mile means available ton mile.
[3] Fuel and oil.
[4] Average number of airplanes in use during year.
[5] Data not available.

Sources: Civil Aeronautics Board, *Annual Airline Statistics*, 1947, pp. 32-53, 69, 82, and Table VII; 1938-42, pp 77, 99, 139, and Table VIII.
Civil Aeronautics Administration, *Statistical Handbook of Civil Aviation*, 1948, pp. 63, 65.

For 1947 the datum is obtainable in the desired form directly from government sources,[1] and is more reliable than the figure for 1939.

For 1939 the Civil Aeronautics Board did not publish available ton miles. Available seat miles and actual cargo ton miles were published.[2] Available seat miles were converted to ton miles and to this were added the ton miles of cargo actually carried. Since no method of estimating the degree of under-utilization of cargo capacity is available, this estimate of output seems to be best.

II. Capital

Table 1 shows the inputs of capital, fuel, and flight-crew labor of the domestic trunkline airlines for the years 1939 and 1947. Input data for several other factors of production can also be obtained, but Table 1 is limited to those analyzed below.

The preponderance of aircraft in total airline investment is clear from Table 1-A and is magnified by the fact that several other types of investment stand in a relatively rigid complementary relationship to aircraft. Therefore an analysis of aircraft investment will go a long way toward explaining airline capital requirements.

The problem of ascertaining how many airplanes are required is amenable to quantitative analysis of a rather satisfactory type. The method employed by the airlines appears [3] to proceed from an estimate of demand through steps which embody, more or less systematically, the analysis presented below. The expected level of output is taken as given, and we shall proceed to ascertain the amount of capital required for any particular level. For this purpose it is useful to make explicit the following truistic relationship:

$$x = H_H V_b W'_q \tag{11, 1}$$

where

H_H = total block-to-block flying time, airplane hours per month,
V_b = block speed, miles per hour,
W'_q = payload capacity per airplane, tons,
x = monthly output, ton miles.

Thus output, in ton miles per month, equals capacity (in tons) per airplane times airplane miles flown; the latter equals speed times airplane hours.

[1] Civil Aeronautics Board, *Annual Airline Statistics* (hereafter *AALS*), Washington, 1947, p. 32.

[2] Ibid. 1938-42, p. 1.

[3] No detailed and systematic study of the airlines' method has been made so far as the writer is aware; the statement above is based upon a few *ad hoc* studies which have come to the writer's attention.

Define utilization in hours per month per airplane as

$$H'_H = \frac{H_H}{A} \tag{11, 2}$$

where

A = number of airplanes in use.

Then

$$x = A(W'_q H'_H V_b) \tag{11, 3}$$

and

$$A = \frac{x}{W'_q H'_H V_b} \tag{11, 4}$$

or

$$\frac{A}{x} = \frac{1}{W'_q H'_H V_b} \tag{11, 5}$$

Thus the physical capital coefficient (airplanes per ton mile) depends only upon utilization and payload capacity per airplane and block speed. Given these, capital varies linearly with output. The determinants of these three factors are now to be ascertained.

Consider first *utilization*. The number of aircraft required varies in inverse proportion to utilization, and actual utilization of a fleet depends upon the number of available airplane hours and the time distribution of demand.

Since demand oscillates widely over time in air transportation, aircraft are often idle simply because of lack of demand for their services. Thus, *ceteris paribus*, the *marginal* capital coefficient depends on the time pattern of the new output, whether it conforms with, aggravates, or mitigates the current fluctuations in output.

Determining fleet availability is a maximization problem corresponding closely to the simple theory of the firm.[4] Many airplane hours are spent in maintenance, ground operations (loading and unloading, and servicing), and in training and reserve status. The cost of maintenance and ground operations appears to be a U-shaped function of the time spent in them. Thus, maximizing aircraft availability may increase the cost of maintenance and ground operations above the minimum which could otherwise be obtained. Further, maintenance, ground operations, and the holding of planes in reserve to protect schedules all affect product quality and, hence, revenue.

Thus the problem of availability involves a maximization in which the pertinent elements are the cost of the ground and maintenance operations as a function of the speed of those operations; (relative to) the

[4] This is true except in the unlikely case where the hours of aircraft idleness imposed by the demand pattern happen to coincide with the optimum time for performing the non-flying operations which engage airplanes.

current costs of aircraft; and the revenue resulting from the cleanliness and safety of the airplanes, from promptness in servicing them, and from the protection of schedules with reserve equipment.

The second element in equation (11. 5) is *block speed*, with which aircraft requirements vary, again, inversely. Block speed is defined as the average speed between the loading ramp at the airport of departure and the ramp at destination. Block speed is less than air speed because of time spent in taking-off and landing, taxiing, engine run-up, awaiting take-off clearance, and in maneuvering preparatory to landing.

It is difficult to systematize the determinants of these time losses. Actual take-offs and landings consume little time. The time spent in taxiing varies with the size and layout of airports. Time spent in maneuvering for landing and awaiting take-off clearance depends upon traffic density and the weather. In clear weather and light traffic little time is lost, but in dense traffic and instrument weather, delays may be very substantial. The number and complexity of the engines determines the time spent in engine warm-up. In the interest of economical operation the management has a clear-cut motive for minimizing the time spent in these operations.[5]

The speed in the air is subject to managerial control within technical limits which are rather narrowly defined, given the particular type of aircraft.[6] The flight altitude influences the spread between maximum and minimum speeds as well as the absolute levels of these limits. The levels of the limits are also influenced by wind conditions. In deciding upon the airspeed, management is faced with a complex minimization problem; for increased speed not only decreases the capital requirements and flight-crew labor costs but also very rapidly increases fuel costs per hour.[7] Further, since speed is one of the chief product characteristics of air transportation it significantly influences revenue. The problem is complicated by the fact that speed interacts with the scheduling of flights, another important product characteristic as well as a major determinant of some costs. Lastly, block speed tends to increase as the length of individual hops increases, because of the decreased relative importance of the time losses just discussed.

The third determinant of aircraft requirements is *payload capacity per airplane*. Capacity is measured in tons.[8] The payload capacity is determined by three factors: the construction of the aircraft (including the

[5] Although fuel consumed in taxiing does increase with speed this effect appears to be negligible within the limits set by safety considerations.

[6] The balance of speed and other desirable attributes is one of the major problems in aircraft design, however.

[7] Cf. the discussion below, pp. 433-8.

[8] Capacity is sometimes measured in seats, which can be readily converted to weight by assuming an average weight per passenger. Occasionally interior volume and shape are significant, but historically these dimensions have been of minor importance.

power plant and details of interior arrangement); the regulatory limits set upon maximum take-off and landing weights; and the weight of fuel (and oil) aboard.[9]

The weight of fuel aboard increases and, hence, payload decreases with the distance between fueling stops. There is no satisfactory rule of thumb for relating maximum payload capacity to the gross weight of the airplane; indeed, the problems of determining the desirable value for this ratio and for the gross weight itself are among the more difficult aspects of airplane design.[10]

For purposes of determining the investment in aircraft it would be desirable to have a precise relationship or, at least, some rule of thumb relating payload capacity and unit price of equipment. There is, however, no such rule. One of the reasons for this is that the aircraft industry is typified by very rapidly decreasing costs. Therefore the size and timing of purchases may influence the price of individual units. More important, government purchases may entirely dominate the pricing situation since they may make the difference between mass production and handicraft techniques.

Then, to summarize, the required number of aircraft varies inversely with block speed, utilization, and payload capacity per airplane. None of these entities is rigidly fixed, but each is variable within technical limits. Since to increase any of them (at least beyond some point) increases some elements of cost, management is faced with a minimization problem with ramifications running into many departments of the firm. Even given the aircraft capacity required, it is not easy to predict the money investment because there is no simple relation between payload capacity and money price per unit.

We turn now from the investment in aircraft to the determinants of total airline investment. Because of the rigid complementarity between aircraft and other flight equipment there is a firm theoretical justification for expecting investment in both categories of flight equipment to vary together. Rather than pursuing the difficult (if possible) task of establishing the determinants of the investment other than aircraft, it was deemed satisfactory merely to relate such investment to the investment in aircraft. 'Other flight equipment' largely consists of aircraft engines, spare parts, radio equipment, and propellers (in that order).[11] Except for spares and spare parts there is a virtually rigid relation between the number of aircraft of any type and the number of units of 'other flight equipment.'

In regard to the rest of the investment this procedure is clearly less defensible but since there is some degree of complementarity between air-

[9] Assuming the weight of other supplies and the operating crew as constant.

[10] For the details of weight regulations the reader is referred to Civil Aeronautics Board, *Civil Air Regulations*, Part 61.

[11] *AALS*, 1947, p. 69.

craft and non-flight property and equipment,[12] and since such investment is small and the coefficient of correlation high (.911), the procedure appeared justifiable.

The results of the correlation appear in Table 2.

TABLE 2

Correlation of Other Airline Investments with Aircraft for the Years 1943-48, U.S. Domestic Trunk Lines

Year	Coefficients of Correlation		Line of Regression (000 omitted in the constant)	
	$R_{x.y}$	$R_{x.z}$	$y =$	$z =$
All Data 1943-8	0.917	0.911	$86.767 + 0.618x$	$69.935 + 0.565x$
1943	0.987	0.912	$5.657 + 0.686x$	$-10.636 + 1.371x$
1945	0.900	0.878	$51.942 + 0.676x$	$68.379 + 0.666x$
1946	0.958	0.907	$2.183 + 0.982x$	$20.423 + 0.711x$
1948	0.923	0.897	$174.769 + 0.549x$	$54.091 + 0.531x$

where

x = dollar value of aircraft, original cost,
y = dollar value of other flight equipment, original cost,
z = dollar value of property and equipment other than flight equipment, original cost.

The statistical analysis consisted of plotting the values for non-aircraft flight equipment and for non-flight property and equipment against aircraft for each of the 16 domestic trunklines for the years 1943 and 1948,[13] the years at the extremes of the sample. Visual examination indicated that there was enough correlation in all four cases to justify computation of lines of regression and coefficients of correlation. Lest a single correlation analysis of all the data should obscure internal inconsistencies it was decided to perform the computations for several years as well as for the whole sample. Since there is a significant break in the data for most of the investment categories for many firms between the years 1945 and 1946, it was considered inadvisable merely to compute values for the extreme years and one middle year, as was the original plan. Therefore the computations were made for the two extreme and the two middle years, 1943, 1945, 1946, and 1948. The results are shown in Table 2.

In all cases the coefficient of correlation is found to be very high, the lowest being that of non-flight property versus aircraft for the year 1946;

[12] Largely ramp, shop and hangar equipment, furniture, etc., investment in buildings, property improvements and a very small investment in land. For details see the sources of Table 1.

[13] For 1948, figures for the 12 months ending 30 September 1948 were employed because they were the most recent available at the time of writing. No data were available for the period prior to 1943. The sources of the data are *AALS*, 1943, Table VI, 1944, 1945, 1946 Gotch and Crawford, *Air Carrier Analysis*, December 31, 1947, and September 30, 1948, Table VII.

even here a value of 0.878 was found. Therefore it seems entirely defensible to take the lines of regression as indicating the relation between the aircraft investment and the other two types of investment.

With the exception of the 1943 line for non-flying property and the 1946 line for flight equipment, the lines of regression have substantially consistent characteristics, especially in regard to slopes. Therefore it appears that the line for the entire sample does in fact represent the data rather well. Hence it is defensible to assert that the dollar value of investment in flight equipment other than aircraft equals about six tenths (0.618) of the value of aircraft plus a constant, $868,000, and that the value of non-flight property and equipment equals a little more than half (0.565) of the value of aircraft plus a constant, $699,000. Hence we can say (summing the equations for the two lines of regression and adding x to both sides) that the total investment of an airline equals about $1,570,000 plus 2.18 times the value of the aircraft employed.[14]

III. Labor

The present discussion of labor inputs will be restricted to flight-crew labor, composed of pilots, co-pilots, stewards (or stewardesses), and sometimes flight engineers.

The physical labor coefficients (man-hours per ton mile) are determined partly by institutional and partly by technical factors. The ratio of labor to capital (man-hours to airplane hours) is determined primarily by government regulation. The pilot complement on commercial aircraft is prescribed by the Civil Aeronautics Board and is usually 2.[15] On unusually long flights (more than 8 hours uninterrupted) a reserve crew is required. Furthermore, when the aircraft is multi-engined (larger than the DC-4), a flight engineer is also required. One or two hostesses usually fly on passenger flights.

The design of the airplanes also, of course, influences the labor-capital ratio, but for the most part aircraft are designed to conform to the legal requirements rather than having the adjustment in the other direction. The labor unions have apparently influenced the size of the pilot complement very little, but they have exerted some influence toward employing more than one stewardess on large equipment types.

Thus, with the ratio of man-hours to plane hours rather rigidly fixed at 5 and 3 for 4- and 2-engine airplanes respectively, the physical labor input coefficient in man-hours per ton mile depends upon the number of ton miles of output per airplane hour. This in turn depends (as has been discussed) upon the speed and payload capacity of the equipment.

The labor force required, the number of crews rather than the number

[14] Because the equations are lines of regression, not 'true' equations, this is not precise.
[15] Civil Aeronautics Board, *Civil Air Regulations*, Part 61.

of man-hours, for any particular level of output, depends upon the average productivity of the crews. Maximum productivity (in hours per month) is fixed rigidly by law for pilots and co-pilots at 85 hours of flying per month (with minor exceptions), and this limit has been taken over in collective bargaining agreements covering the other crew members. The actual hours flown average considerably less than the maximum because of the necessity of maintaining a reserve of personnel.

From questionnaires sent to the major trunklines to ascertain the reserve factor used in determining personnel inputs, it was found that the labor inputs average about 1 crewman of each type per 75.6 airplane hours. Thus for 4-engine equipment the coefficient (men/plane hour) is[16]

$$\frac{5}{75.6} = .066$$

and for 2-engine equipment it is

$$\frac{3}{75.6} = .040$$

Hence

$$\frac{L'}{H_H} = K \qquad (11, 6)$$

where

K = .040 for 2-engine aircraft, and
K = .066 for 4-engine aircraft,
L' = number of aircrew laborers.

Substituting for H_H its equivalent from equation (11, 1) yields

$$\frac{L'}{x} = \frac{K}{V_t W'_q} \qquad (11, 7)$$

the labor coefficient in men per ton mile. The required labor force depends, then, upon output, x, the institutional requirements and reserve crew practices which determine K, the payload capacity and the ground speed of the aircraft.

It may be noted that multiplying equation (11, 7) by 75.6 gives the coefficient in terms of man-hours per ton mile.

The determination of the money cost of flight-crew labor is a little more involved. From the collective bargaining agreements between the airlines and flying personnel it is possible to obtain a general formula indicating the determinants of money wages.

The formula[17] presented below was developed by taking from all the

[16] Cf. p. 428 above.

[17] Although derived independently this equation is similar to that presented by Mentzer, W. C., and Nourse, Hal E., in 'Some Economic Aspects of Transport Airplane Performance,' *Journal of Aeronautical Sciences*, April and May 1940, pp. 227-34 and 302-8.

bargaining agreements the most typical clauses covering each type of wage payment. Since the agreements are very similar in all cases this procedure gives a modal formula which is a rather reliable representation of the wage determinants in the industry as of 6 July 1949.[18] Although the money rates of pay vary considerably over a period of years the wage formula is very stable, because Decision 83 of the National Labor Board has defined the legal minimum for pilots' wages since 1934.

The total flight-crew wage bill is heterogeneous, being composed of the salaries of the co-pilots, stewards (or stewardesses), and engineers plus the base pay, hourly, mileage, and gross-weight pay of the pilots. The modal formula for total monthly flight-crew wages is

$$L = H_H \left\{ \frac{1}{H_A} \left[C_p + C_G + C_B + C_E + a_g\left(\frac{D_1 H_A}{H_H} - 100 H_H\right) + a_j\left(\frac{D_2 H_A}{H_H} - 10{,}000\right)\right] + 2gW + H_{dn}(a_a + a_n V_b) \right\} \quad (11, 8)$$

where

D_1 = the distance flown in the month per pilot up to 10,000 miles,
D_2 = the distance flown in the month per pilot provided it exceeds 10,000 miles,
H_A = the number of hours flown in the month per crew member,
H_{dn} = the so-called day-night factor which equals 1.5 times the number of hours flown at night plus the number of hours flown in the daytime, which sum is divided by the total number of hours,
H_H = total block-to-block flying time, airplane hours per month,
L = the monthly wage bill of the flight crews of an airline, dollars,
V_b = block speed, miles per hour,
W = gross weight.

The C terms stand for the base pay of the pilot and the salaries of the stewardess, co-pilot, and engineer, in that order, in dollars

The lower case letters are constants determined in the bargaining agreements; as of 6 July 1949 they had the modal values:

$$a_a = 3.4$$
$$a_g = .02$$
$$a_j = .015$$
$$a_n = .008$$
$$g = .0175$$

[18] The last date for which complete information on the status of contracts was available The information was taken from the collective bargaining agreements as reproduced by the Airlines' Personnel Relations Conference and its predecessor, the Airlines' Negotiating Conference.

The values of a_g and a_j have been essentially constant since 1943; a_a and a_n as well as the C factors have been changed (increased) historically through collective bargaining, especially since the recent war; and g has been added since 1946.[19]

The C terms, salaries and base pay, vary for each crewman with seniority and, hence, for an entire firm or for the industry depend solely upon the number of crewmen of each type and their average seniority.

The long expression

$$a_g\left(\frac{D_1H_A}{H_H} - 100H_H\right) + a_j\left(\frac{D_2H_A}{H_H} - 10{,}000\right)$$

represents the pilots' mileage pay. This term indicates that the pilots receive a pay of 2 cents [20] per mile for every mile less than 10,000 flown at speeds in excess of 100 miles per hour, and $1\frac{1}{2}$ cents [20] per mile for every mile in excess of 10,000 flown in a single month. The W term indicates that each pilot gets 1.75 cents [20] per hour for each 1,000 pounds gross weight of the aircraft he flies. The last term in the major bracket indicates the hourly pay of pilots. It shows that beyond some point (155 miles per hour) [20] pilots' hourly pay increases in proportion to the (block) speed of the airplane. The total monthly value of the terms outside the minor bracket are both a function of hours flown; hence they are multiplied by H_H. The total monthly value of those within the minor brackets is a function of the number of crewmen; hence they are multiplied by the quotient, total hours divided by the average number of hours expected [21] per crewman each month.

IV. Fuel Consumption [22]

The final input to be analyzed is the fuel used in flying operations. The determinants of fuel consumption per month in flight are represented in equation (11, 9):[23]

$$F = \frac{3600H\left[\frac{b_1s_1}{2}\rho V_e^3 + \frac{2W^2}{b_2s_1\rho V_e} - T\right]}{cee_t} + BM_b + E \qquad (11, 9)$$

[19] Northrup, Herbert R., 'Collective Bargaining by Airline Pilots,' *Quarterly Journal of Economics*, August 1947, p. 561.

[20] The current values of a_g, a_j, g, a_a.

[21] The expected flying time—not the actual, *ex poste*, time—is pertinent in determining the number employed.

[22] The material presented in this section is worked out in more detail in 'Empirical Determination of a Multidimensional Marginal Cost Function,' *Econometrica*, July 1950.

[23] The fractional part of equation (11, 9) was developed from the discussion of power required and power available in von Mises, Richard, *Theory of Flight*, New York, 1945, p. 421 and ch. XIII.

where

b_1 = coefficient of drag of the airplane for zero lift,
b_2 = a factor in the coefficient of induced drag, equal to approximately 2.5 times Span^2/s_1,
B = number of landings in a month,
c = combustion energy per pound of fuel, foot pounds,
e = propulsive efficiency,
e_t = thermal efficiency,
E = fuel consumed in miscellaneous ground operations, pounds,
F = total fuel consumed per month, pounds,
H = flying hours, in equilibrium flight per month,
M_b = fuel consumed per landing, pounds,
s_1 = wing area, square feet,
T = power equivalent of exhaust thrust net of intake drag,
V_e = true airspeed, feet per second,
W = gross weight, pounds,
ρ = air density, slugs per cubic foot.

This equation takes account of *all* factors which affect fuel inputs in flying operations. The fraction specifies the determinants of fuel consumed in en route flying; the other terms represent the fuel used in ground operations associated with landing and taking off and in miscellaneous ground operations. The numerator of the fraction shows the power, net of exhaust thrust, required to overcome the parasite drag—which is the friction that would exist between the air and the airplane if the latter were supported other than by its dynamic reaction with the air—and the drag (induced) generated in developing the necessary lift. The denominator shows that the fuel required varies inversely with the combustion energy per pound of fuel and the efficiency with which it is used.

Although it would be highly desirable to spell out in quantitative terms the determinants of the independent variables it is impossible to do so. The terms b_1, b_2, s_1 are structural characteristics of the airplane, which, in construction, are influenced by economic considerations within limits set by the state of technology in the aircraft industry. Similarly the thermal and propulsive efficiencies are both subject to limited control within rather complex technical restrictions,[24] while the energy content of the fuel depends upon the state of technology in the petroleum and aircraft-engine industries.

Since most of the fuel consumed in ground operations (that used in engine warm-up, taxiing, take-off, and landing) is directly related to the number of landings in a particular operation, it is taken as a function of

[24] The determinants of efficiency are discussed more fully in 'Empirical Determination of a Multidimensional Marginal Cost Function' referred to above, footnote 22.

the number of landings. The rest of the fuel used in ground operations is taken as a factor to be determined empirically in each case.

In applying equation (11, 9) to an airline operation it is necessary to use weighted averages for the variables since the speed, weight, and altitude vary in flight and the airplane structural characteristics vary between types of aircraft. Alternatively, separate computations could be made for each appropriate phase of the total operation.

The advantages of having a cost statement of this nature lie in its generality and relative quantitative precision. Since the equation includes all factors that can influence fuel consumption in flying operations it is capable of taking account of any change in such consumption, whatever the cause of the change. The *form* of the equation is not affected by the type of equipment employed, the conditions of operation, or institutional factors. These influence only the values of the variables.

Thus in a formal way the equation is independent of technological change. However, technical change in aircraft, at least substantial change, may involve both of two things. Several variables may be changed simultaneously, as in the introduction of a new type of airplane. Also unless a particular change is consciously isolated, for example in experimentation, a change in one variable may involve a change in others, or good engineering practice may require adjustments in others. It is apparently true, however, that in the design of aircraft the variables are changed independently, at least conceptually, and over moderate ranges. Even in the case where several variables might change simultaneously the effect upon fuel consumption would be indicated in equation (11, 9).

The remainder of this section is devoted to the derivation of marginal cost functions associated with changes in the quantity of output, in the quality of product, and in the techniques of production. What is here called marginal cost is only the marginal fuel cost in physical terms.[25]

A. THE MARGINAL COST OF QUANTITATIVE CHANGES

The marginal cost (as defined) of a change in any of the determinants of fuel consumption can be determined, assuming that the variables are mutually independent, merely by taking the appropriate partial derivatives. As was pointed out, in practice several of the variables may be changed simultaneously. However, to the extent that they *can* be changed independently, the proposed procedure is valid. Further in the cases

[25] This marginal-cost concept is not the same as the 'coefficient of costicity' developed by M. Brèquet (reported in Phelps-Brown, E. H., 'Cost Categories and the Total Cost Function,' *Econometrica,* July 1936, pp. 242-63). M. Brèquet's concept is really the elasticity of the total unit cost with respect to its various determinants. Per cent variable, a similar cost concept, is developed by Ford K. Edwards in Interstate Commerce Commission, *Explanation of Rail Cost Finding Procedures and Principles Relating to the Use of Costs,* Washington, 1948, and other writings.

where a change—especially a large change—in one variable requires changes in the others as a practical matter, side relationships could be set up showing the interdependence of the variables. No attempt to set up such side relationships will be made here.

Equation (11, 1) indicates that output in air transportation can be increased by increasing speed, payload capacity, or hours. A change in speed constitutes changing the basic product characteristic in air transportation. Changes in weight and hours, *ceteris paribus*, can be taken as being quantitative changes. To change hours with quality constant requires that speed and, hence, the number of landings *per hour* remain fixed.[26] Therefore in measuring the marginal cost of quantitative changes we must assume that

$$B = k'H \tag{11, 10}$$

where k' is a constant. Then the marginal cost (partial derivatives) of fuel consumed with respect to hours and weight is:

$$\frac{\delta F}{\delta H} = \frac{3600\left(\dfrac{b_1 s_1}{2}\rho V_e^3 + \dfrac{2W^2}{b_2 s_1 \rho V_e} - T\right)}{cee_t} + k'M_b \tag{11, 11}$$

and[27]

$$\frac{\delta F}{\delta W} = \frac{(4)(3600)H}{b_2 s_1 \rho V_e} \cdot \frac{W}{cee_t} \tag{11, 12}$$

It is to be noted that weight in equation (11, 9) and its derivatives is gross weight, while weight in equation (11, 1) is payload weight. The relation between these quantities depends primarily upon the type of aircraft and the fuel load; the latter, in turn, depends upon the length of stage between fueling stops.

Although output can be increased by increasing either weight or hours, management is not always free to increase output by whichever of these means is the less costly; the time distribution of demand may be controlling. An increase in demand for service at a time when aircraft are flying without full payloads allows an increase in weight. A demand for service at other times is a demand for additional flying hours. So, even here, it is a little difficult to draw a firm, fine line between quantitative and qualitative changes in output.

The marginal cost, i.e. the marginal real cost of fuel, is linear in the case of changes in hours; but in the second case it is curvilinear, cost increasing as the square of the weight. Here is an empirically derived marginal-cost curve with the upward concavity of the elementary texts.

[26] Otherwise the length of hop would have to be changed, in which case the service would be rendered to different cities, constituting an entirely different product.

[27] Ignoring the effect of weight on fuel consumed in landing and ground operations, which effect cannot be predicted.

B. THE MARGINAL COST OF QUALITATIVE CHANGES

Changes in speed, altitude, and the number of landings all affect the quality of the airline's product.[28] The marginal fuel cost of changing each of these entities can be obtained by computing the appropriate partial derivative. Since speed is the most important product characteristic in aviation we shall begin with it.

The object is to obtain a statement of the effect of producing *a given output* at different speeds. Speed, weight, and hours all appear in both equations, (11, 1) and (11, 9). From equation (11, 1) hours and speed are interdependent. Therefore it is necessary to eliminate H from equation (11, 9) before differentiating with respect to V_e. It is also necessary to express V_b (equation (11, 1)) which is block speed in miles per hour as a function of V_e (equation (11, 9)) true-air speed in feet per second and to express payload weight in tons (11, 1) as a function of gross weight in pounds (11, 9).

It can be shown that

$$V_b = \frac{aV_e}{1 + ak_oV_e} \tag{11, 13}$$

where

a = factor for converting feet per second to miles per hour, .686,

k_o = the ratio of the time 'lost' at each stop (hours) to the distance between stops (miles); both are constants,

and it can be seen by inspection that

$$W'_q = \frac{W' - W'_t}{2000} \tag{11, 14}$$

where

W' = gross weight per airplane, pounds,

W'_t = tare weight per airplane, pounds, taken as constant.

Then, substituting for H in equation (11, 9) the fraction $\dfrac{2000x(1 + k_eaV_e)}{aV_e(W - W_t)}$ from equation (1), differentiating equation (11, 9) with respect to V_e yields the marginal cost in fuel of airspeed changes; thus:

$$\frac{\delta F}{\delta V_e} = \frac{2000 \cdot 3600x}{cee_t(W - W_t)} \left\{ \frac{b_1s_1}{a}\rho V_e + \frac{3k_o\rho(b_1s_1)V_e^2}{2} - \frac{4W^2}{b_2s_1\rho V_e^3} - \frac{2k_oW^2}{b_2s_1\rho V_e} + \frac{T}{aV_e^2} \right\} \tag{11, 15}$$

[28] The size and type of the airplane can influence the quality of product The effect of changes in aircraft type upon fuel consumption are indicated in equations (11, 12) above and (11, 15)-(11, 24) below.

Changes in speed are of two kinds. Given a type of airplane, it can be operated at various speeds within a specified range. A different, e.g. new, type of airplane can be operated over a different range of speeds. Thus a change in the type of aircraft will involve a change in the value of V_e that one would expect to find in equation (11, 9). Also, many of the other variables would normally differ in value between aircraft types, although they could be held constant at least over some ranges of speed variation if it were desirable to do so, e.g. in experimentation. There is nothing in this which gives rise to any difficulty in the present context.

The first fact, that a given aircraft can be operated over a range of speeds, does cause trouble. b_1, b_2, T, e, and e_t are all functions of speed. In contemporary commercial aircraft with piston engines the variations with speed are sufficiently small to justify the assumption of independence among these variables. However, with jet- or turbine-driven propeller power plants the variation with speed is too large to be ignored. Therefore, for the solution of practical problems involving such aircraft, side relationships specifying the interdependence would be required These relationships are rather complex and no attempt will be made to include them in this paper. For these reasons equation (11, 15) is a usable approximation only for contemporary aircraft.[29]

Changes in number of landings and in altitude may also affect product quality. The appropriate marginal cost functions can be obtained by inspection; they are:

$$\frac{\delta F}{\delta B} = M_b \tag{11, 16}$$

and

$$\frac{\delta F}{\delta \rho} = \frac{3600H}{cee_t}\frac{b_1 s_1}{2} V_e^3 - \frac{2W^2}{b_2 s_1 \rho^2 V_e} \tag{11,17}$$

C. MARGINAL COSTS OF TECHNICAL CHANGE

All the other elements in equation (11, 9) can be taken as technical conditions of production, variations in which need not affect either the quantity or the quality of output.

Changes in airplane structure or design:

[29] The author is indebted to Edward S. Taylor, Professor of Aircraft Engines at Massachusetts Institute of Technology, for comments (included in a letter to Mrs. Elizabeth Gilboy, dated 28 November 1950) which incorporated some of the ideas expressed in the two paragraphs above.

$$\frac{\delta F}{\delta s_1} = \lambda\left(\frac{b_1 \rho V_e^3}{2} - \frac{2W^2}{b_2 \rho V_e s_1^2}\right) \tag{11, 18}$$

$$\frac{\delta F}{\delta b_1} = \lambda\left(\frac{s_1 \rho V_e^3}{2}\right) \tag{11, 19}$$

$$\frac{\delta F}{\delta b_2} = -\lambda\left(\frac{2W^2}{s_1 \rho V_e b_2^2}\right) \tag{11, 20}$$

where

$$\lambda = \frac{3600H}{cee_t}.$$

Equations (11, 18)-(11, 20), like the other partial derivatives, assume that the variables on the right side of equation (11, 9) are independent, or more accurately that all 'independent' variables other than the one in question can be held constant while it is changed. It has already been pointed out that under some conditions there is some interdependence between the velocity, for example, and the b and s factors. However, changes in s_1, b_1, and b_2 can be made with V_c held constant. Similarly in many circumstances one would not change s_1, b_1, or b_2 without changing one or more of the other quantities. In other words, in any given set of circumstances there is an *optimum* relation between these factors and a change in one of them may require a change in the others to attain a new optimum combination. For example, in standard engineering practice it is hard to imagine why the empty weight of the airplane (given its general characteristics) would not increase if an aircraft were modified so as to increase the wing area, s_1. But it is gross weight which appears in equation (11, 9) and this could be kept constant (within broad limits) by decreasing the useful load, as by carrying less fuel. It may often be true, then, that one of these variables might be changed in order to change another variable and thus to reach a combination which is optional under some set of circumstances, but it remains true that changes in b_1, b_2, or s_1 need not cause changes in the other variables. It is the purpose of this part of the present essay to show the effect of isolated changes. Therefore the fact that isolation is possible is all that is required for the validity of equations (11, 18)-(11, 20).

It will be noted that no marginal cost with respect to changes in thrust, T, is presented. This is so because thrust depends upon the value of the rest of the term within brackets in equation (11, 9). For this reason there is some inaccuracy in the derivatives (11, 12) and (11, 15)-(11, 20). In the case of conventional aircraft the error is negligible since T is small.

In the case of jet- or turbine-driven propeller installations the variation would not be negligible.[30]

Changes in engine performance and associated variables:

$$\frac{\delta F}{\delta c} = -\frac{3600HP_r}{c^2 e_t e} \qquad (11, 21)$$

$$\frac{\delta F}{\delta e_t} = -\frac{3600HP_r}{c e_t^2 e} \qquad (11, 22)$$

$$\frac{\delta F}{\delta e} = -\frac{3600HP_r}{c e_t e^2} \qquad (11, 23)$$

$$\frac{\delta F}{\delta M_b} = B \qquad (11, 24)$$

$$\frac{\delta F}{\delta E} = 1 \qquad (11, 25)$$

where

$$P_r = \frac{b_1 s_1}{2}\rho V^3 + \frac{2W^2}{b_2 s_1 V \rho} - T.$$

These equations give the marginal cost of fuel when operating under different technical conditions, i.e. they show the fuel saving or expense of changing the technical conditions of production. Thus with fuel prices applied, they show the value of structural or operational innovations, and hence (given the marginal cost of introducing the innovations) (a) the ones that may be economically introduced under any given technological situation, and (b) the directions in which research might be most profitably pursued to change the technological situation.

Changes in these factors do not necessarily require technological change in the sense of invention or advances in pure science, although they could be brought about as a result of incorporating such changes. Some may be associated with the improvement of managerial practices, for example changes in e_t, e, M_b, or E.[31] The introduction of a new type of aircraft will involve changes in many of the factors simultaneously, but in the designing and developing of aircraft each can be varied independently. Even given the type of aircraft many of them can be varied independently. Of

[30] These facts were also called to the writer's attention by Professor Taylor in the letter referred to above, footnote 29.

[31] In the technical jargon a management may be operating off an isoquant, in that, given the technological situation, more fuel than is required for the amounts of the other factors employed is being expended. A managerial change to economize fuel would constitute an approach to the isoquant.

course the introduction of a new aircraft may also change the values of V and W and perhaps the permissible values of ρ.

It is possible to compute the marginal costs of the other inputs in an exactly analogous fashion. Those for aircraft requirements are all very simple, being of two forms:

$$\frac{\delta A}{\delta x} = \frac{1}{V_b W'_q H'_H} \tag{11, 26}$$

and

$$\frac{\delta A}{\delta H'_H} = \frac{x}{V_b W'_q (H'_H)^2} \tag{11, 27}$$

The marginal costs of money wages of flight personnel are complex and it would serve no useful purpose to reproduce the functions here.

V. Some Tests [32]

So far the coefficients of three inputs have been set up and their determinants explained. There remains the problem of providing some empirical verification of the formal relationships established. Sufficient empirical work has not been done at this time to constitute satisfactory 'proof' of the findings, but enough testing has been completed to indicate that the equations are operational and give reasonably satisfactory quantitative results in at least some cases.

It should be noted that, except in the case of the relation between various types of investment, the equations have not been derived empirically from data on the air transport industry. Hence using such data can provide a significant test of the relations deduced.

Section II analyzed in some detail the determinants of the number of airplanes required for any given operation and provided a means of estimating total capital requirements. Aircraft needs were seen to depend first upon the expected volume of service and some product characteristics, especially range; and second upon the block speed, average utilization, and the payload capacity per aircraft. The remaining investment is highly correlated with the money investment in aircraft, and for some purposes can be taken as a function of it.

With this discussion in mind, let us glance once more at the capital coefficients for the years 1939 and 1947 in Table 1. The physical coefficient in 1947 was only 29 per cent of that in 1939. The difference is consistent with the explanation of the determinants of capital requirements

[32] The empirical work involved in this section was made possible by the Bureau of Population and Economic Research of the University of Virginia under the direction of Dr. Lorin A. Thompson.

developed earlier, and can be understood in terms of that presentation. Equation (11, 4) is restated:

$$A = \frac{x}{W'_q H'_H V_b} \tag{11, 4}$$

where

A = number of airplanes in use,
H'_H = utilization per aircraft, hours per month,
V_b = block speed, miles per hour,
W'_q = payload capacity per airplane, tons,
x = monthly output, ton miles.

The pertinent quantities changed during the interval in question in the manner shown in Table 3.

The following bit of algebra shows that the change that occurred in the capital coefficient is explained satisfactorily by equation (11, 4). Let

$C = \dfrac{A}{x}$, the physical capital coefficient,
subscript 1 stand for 1939,
subscript 2 stand for 1947.

Then

$$C = \frac{1}{W'_q H'_H V_b} \tag{11, 28}$$

and

$$\frac{C_2}{C_1} = \frac{(W'_q H'_H V_b)_1}{(W'_q H'_H V_b)_2} \tag{11, 29}$$

Substituting the pertinent ratios from Table 3 yields:

$$\frac{C_2}{C_1} = \frac{1}{(1.081)(1.330)(2.042)} = .3406 \tag{11, 30}$$

From Table 3 the actual ratio between the capital coefficients is .293. This difference is largely accounted for by the difference in load factor.

$$\frac{C_2}{C_1} \div \frac{f_2}{f_1} = \frac{.3406}{1.158} = .2941, \text{ the `predicted' coefficient} \tag{11, 30}$$

Thus there is a difference of .23 per cent between the 'predicted' and actual physical capital coefficient over a period of eight years in which many institutional and technological changes occurred. This is, of course, so small an error as to be embarrassing in view of the roughness of some of the values employed. Apparently so extremely small an error is largely fortuitous. However, the test does appear to support the formulation of Section II.

TABLE 3

Number of Aircraft and Determinants of Aircraft Requirements, 1939 and 1947

	1939	1947	1947 as Percentage of 1939
	(1)	(2)	(3)
(1) Number of aircraft[1]	252	676	268
(2) Physical coefficient[1] (airplanes per million ton miles)	1.887	0.5537	29.34
(3) Block speed[2] (miles per hour)	155.6	168.2	108.1
(4) Seat-mile capacity[3] (seats per airplane)	14.66	29.93	204.2
(5) Utilization, average (hours and minutes per day)	6:04[4]	8:04[5]	133.0
(6) Load factor[6] (per cent capacity utilized)	56.25	65.12	115.8

[1] Table 1-A

[2] Civil Aeronautics Administration, *Statistical Handbook of Civil Aviation*, 1948, p. 62. Given as 'average speed,' but apparently is block speed since it agrees roughly with some independent estimates made by this writer elsewhere. The figures are actually for 1944 and 1947; since the war prevented the introduction of new equipment, the 1944 figure is probably very close to correct 1939 value, and is used for it.

[3] Ibid. p. 61. There are no data on total payload capacity, but seat and total capacity probably move rather closely together.

[4] Civil Aeronautics Board, *Annual Airline Statistics*, 1938-42, Table VI.

[5] Ibid. 1947, Table VIII.

[6] Passenger load factor in both cases. Ibid. 1938-42, p. 1, and 1947, p. 32.

In the face of rising wage rates the coefficient for flight-crew labor in dollars per ton mile decreased considerably over the period 1939-47. This is, of course, accounted for by the even greater decrease in the physical coefficient.[33] Equation (11, 7) provides an explanation of the observed change in the coefficients. The only 4-engine equipment that had been introduced to any substantial extent in 1947 was the DC-4,[34] on which a flight engineer was not required. Therefore it is possible to take K in equation (11, 7) as being substantially constant over the interval. The following indicates that the predicted value of the physical coefficient of labor is reasonably accurate.

Equation (11, 7) is reproduced:

$$\frac{L'}{x} = \frac{K}{V_b W'_q} \qquad (11, 7)$$

[33] See Table 1-B.

[34] *AALS*, 1947, Table VIII.

Let

$$\bar{L} = \frac{L'}{x} \tag{11, 31}$$

subscript 1 stand for 1939,
subscript 2 stand for 1947.

Then

$$\frac{\bar{L}_2}{\bar{L}_1} = \frac{K_2(V_bW'_g)_1}{K_1(V_bW'_g)_2} \tag{11, 32}$$

Substituting the appropriate ratios from Table 3 yields ($K_1 = K_2$ by assumption):

$$\frac{\bar{L}_2}{\bar{L}_1} = \frac{1}{(1.081)(2.042)} = .453 \tag{11, 33}$$

The ratio between the actual coefficients is .464, an error of 2.43 per cent.

Equation (11, 9) indicates the determinants of fuel consumption in flying operations. The fraction is drawn from engineering sources which are accepted as valid in this context; the remaining terms, BM_b and E, can be defined to account for any fuel consumed other than in actual flight. Hence there is no need of establishing the formal validity of the equation. However, a test was performed to determine whether the equation would prove useful, using data actually available.

The test was carried out with the assistance of Northeast Airlines and, without the full co-operation of that company, would have been impossible. Data for the month of June 1949 were inserted into the right-hand side of the equation and the fuel consumption for the month 'predicted'; the predicted value was compared with the actual consumption. To reduce the danger of obtaining a relatively satisfactory result merely because of off-setting errors two computations were made, one for each of the major types of aircraft used, the DC-3 and the CV-240. Some DC-4's were also flown during the period but they were used so sporadically that the data on them were not considered accurate enough to use.

The Airline's engineering department provided values for the airplane characteristics and the energy content of the fuel, b_1, b_2, s_1, and c; these were taken as constants for each type of aircraft. The first difficulty was encountered in determining an appropriate gross weight to employ. An estimate, simply a considered judgment, of typical gross take-off weights for each of the types of aircraft was obtained. These weights were substantially less than the maximum allowable since the payload carried is often less than the maximum and some airports are inadequate for operation with maximum loads. In order to determine the amount that should be deducted from take-off weight to allow for the burning-off of fuel a brief inquiry was made into the fueling practices of the Airline and it was

decided that a typical burn-off between fueling points for the DC-3 would be about 2400 pounds and for the CV-240 about 3000 pounds. Half of this amount was deducted and the balance was taken as the typical en route gross weight for each airplane. The crudity of this figure is apparent.[35]

The number of hours flown by each type of aircraft was provided from their logs. This figure includes the time spent on the ground between the loading ramp and take-off and between landing and the unloading ramp. It was necessary to deduct the time spent in ground activities from total flying time. The chief pilot provided the information that about 17.6 per cent of pilot 'flying' hours were spent in taxiing; multiplying the total number of hours by this factor provided an estimate of total taxiing time for the month. Estimates of the average time spent per take-off and per landing were also obtained from the chief pilot.[36] The sum of these figures multiplied by the number of landings made during the month provided the total time spent in landing and taking off. The sum of the taxiing time and the landing and take-off time was deducted from total flying time as obtained from aircraft logs to get the total time, H, for use in the equation.

Because average altitude and speed depend upon whether the airplane is climbing, gliding, or cruising it was necessary to obtain estimates of the amounts of time spent in each altitude. Statements of the rate of climb and descent and the cruising altitude used by each type of aircraft were obtained. Dividing the difference between cruising altitude and the altitude at which normal climb and descent were considered to begin and end by the rate of climb or descent, then multiplying each quotient by the number of stops gave the time spent in climbing and gliding, respectively. Deducting the sum of these from the total flying time previously computed gave the hours spent in level cruise.

The computation of the value of ρ to be used is based upon the assumption that the density of the air varies as in the standard atmosphere.[37] The variation in the density with altitude under this assumption is readily obtainable. Taking the typical altitude suggested by the Airline (5000 feet) a value of .002049 was obtained for cruising; with 2500 feet

[35] Since the gross weight squared enters equation (11, 9), deducting one-half the burn-off is not entirely sound, but given the roughness of the computation it seemed satisfactory.

[36] The time spent in these operations was, for the sake of simplifying the arithmetic, defined as all time in the air not spent at normal cruise, normal climb, or normal glide power settings.

[37] A model used in study of the atmosphere which assumes:

'1. The air is a perfect gas with the gas constant . . . $\frac{53.33089 \text{ ft.}}{°\text{F}}$

2. The pressure at sea level is . . . 29.921 in. Hg.
3. The temperature at sea level is . . . 59°F.
4. . . . the temperature gradient (is) . . . 0.003566°F.
(per foot of altitude above sea level) . . .' (von Mises, op. cit. p. 8.)

TABLE 4

Values Used in the Fuel Equation

	DC-3	CV-240
H	1135.2	724.5
$b_1 s_1$	22.05	19.6
b_2	28.70	32.34
s_1	987.	817.
ρ	.00219	.00213
V_e	12,800,000.	35,700,000.
W	22,500.	36,500.
c	14,587,500.	14,587,500.
e	0.85	0.85
e_t	0.29	0.30
B	2,814.	1,305.
M_b	120.	130.
E	12,000.	15,000.
T	-------	90,200.

Note: Only a value for the product, $b_1 s_1$, is available.

as the typical altitude in climb and glide a value of .002209 was obtained for these operations.[38] The final value of ρ used in the computations was an arithmetic average of these, weighted by the number of hours spent in cruising and in climbing and gliding respectively. Estimates of typical speeds for both aircraft in each of the three flight attitudes were obtained from the chief pilot. These were converted to feet per second and an average, weighted as in the case of ρ, was obtained. Since the cube of the velocity appears in the equation the speed used in each flight attitude was cubed and weighted in the same manner to find an average velocity cube for use for each aircraft. Thus all the values needed for the numerator in equation (11, 9) were obtained.

The value of the denominator of the fraction was less difficult to compute. Eighty-five hundredths was taken for propulsive efficiency.[39] Thermal efficiency was computed for several conditions of flight and averaged according to the time spent under each set of conditions.[40] The

[38] Diehl, Walter S., *Engineering Aerodynamics*, New York, 1928, Appendix.

[39] This value was suggested by Professor Edward S. Taylor. Other values suggested by him were employed in computing thermal efficiency.

[40] For more complete description of the procedure in determining the values for the efficiencies see the article referred to in footnote 1.

power required for the CV-240 (but not for the DC-3) is reduced significantly by the fact that the exhaust stacks are so arranged as to provide some forward thrust. The thrust in pounds at the altitudes in question were provided by the engineering department of the Airline; and since the appropriate speeds were known, it was possible to convert the thrust for climbing, gliding, and cruising into power equivalents, which were deducted from the power required of the propeller.

The determination of fuel consumed in landing, taking-off, and other ground operations presented many problems and was not entirely satisfactory. The Airline furnished figures for the DC-3 for fuel consumed per stop in landing, engine run-up prior to take-off, taking-off, and taxiing. Because of the extensive experience with that airplane, these were considered adequate empirical estimates. Multiplying by the number of landings gave BM_b. To this was added a figure for the daily warm-up and engine inspection, the only other ground operation taken into consideration. No figure for this was available from the Airline. However, a value of 50 pounds per aircraft per day has been found by M. G. Beard [41] to be appropriate.[42] This figure multiplied by the number of aircraft in use and the number of days in the month is taken as the total value of the fuel consumed by the DC-3's in the daily check and warm-up, E.

For the CV-240 insufficient experience with the airplane made satisfactory empirical data unobtainable from the Airline. Fuel consumed per minute in taxiing is estimated by R. Dixon Speas [43] at 6 pounds; this, times the number of minutes spent in taxiing (determined as indicated above), was used for the total fuel consumed in taxiing. For fuel spent in actual landing the same rate per minute was employed on the grounds that no direct estimate was available and in the actual landing operation throttles are closed so that at this point fuel is probably used at considerably less than the taxi rate, while during the approach for landing some power is used, probably more than in average taxiing.

Data on the actual rate of fuel consumption for take-off power was available from the Airline, 49 pounds per minute, but no accurate estimate of time at take-off power was available. One and one-half minutes per take-off was taken as representative.

No data at all were available on the fuel consumed in warm-up and daily inspection, so twice the value of the DC-3 per inspection was taken simply because the maximum power of the CV-240 engine is twice that of the DC-3. The results of these computations are indicated in Table 4.

[41] Beard, M. G., 'Airline Fuel Consumption,' *Flying*, May 1943, p. 56.

[42] This value is also within the limits of 30-60 pounds found by R. Dixon Speas, Jere T. Farah, and Sanford Hinton, 'Cruise Control for Flying Efficiency,' *Aviation*, August 1943, p. 207.

[43] Speas, R. Dixon, *Airline Operations*, Washington, 1948, p. 180.

Substituting these figures in equation (11, 4) yields for the DC-3:

$$F = \frac{\left\{(3600)(1135.2)\left[\left(\frac{22.05}{2}\right)(.00219)(12{,}800{,}000) + \frac{2(22{,}500)^2}{(28.70)(987)(.00219)(228)}\right]\right\}}{(14{,}587{,}500)(.85)(.29)} + (2{,}814)(120) + 12{,}000$$

and for the CV-240

$$F = \frac{\left\{(3600)(724.5)\left[\left(\frac{19.6}{2}\right)(.00213)(35{,}700{,}000) + \frac{2(36{,}500)^2}{(32.34)(817)(.00213)(303)} - 90{,}200\right]\right\}}{(14{,}587{,}500)(.85)(.30)} + (1{,}305)(130) + 15{,}000$$

The results of these computations and the comparison of the predicted with the actual total fuel consumption [44] for the month of June are indicated in Table 5.[45] The total error is indicated in Table 6 as 5 per cent. Given the fact that nearly all the data are estimates, some of them very crude estimates, the results appear satisfactory. The errors of 4 per cent and 6 per cent in predicting the fuel consumption of the two types of aircraft probably include some off-setting errors.

Undoubtedly the most dubious figures are the estimates of fuel consumed in landing and associated operations. It was not possible to ascertain how the Airline had computed the figures for the DC-3, but the personnel in the engineering department felt that they were reasonably reliable because of the long experience with the airplane. Nevertheless, since roughly 43 per cent of the fuel used is consumed in these operations, any substantial error in the figures per landing would cause a very marked change in the predicted result. The landing figures for the CV-240 are far less reliable, being little more than reasonable guesses. Until considerably more empirical work is done on fuel consumption in ground operations any formula of the type used here is subject to a substantial range of error. There may be as large a relative error in the figure for daily warm-up and inspection but it is of much less significance in absolute terms.

[44] The data from the Airline were in gallons and were converted to pounds by multiplying by a factor of 6.

[45] Determining the number of significant figures in a complex computation such as this is always difficult The rounding-off used in Tables 4 and 5 is rather arbitrary but appears to be adequate.

TABLE 5

Predicted and Actual Fuel Consumption of the DC-3 and CV-240

	DC-3	CV-240
(1) Fuel consumed in flight	427,000	643,000
(2) Fuel consumed in landing, taking-off, taxiing, and engine run-up	338,000	171,000
(3) Fuel consumed in warm-up and daily inspection	12,000	15,000
(4) Predicted total consumption	780,000	820,000
(5) Actual total consumption	809,322	870,966
(6) Error	29,322	50,966
(7) Per cent error	4	6

TABLE 6

Predicted and Actual Fuel Consumption for Both Airplanes Together

(1) Total predicted	1,600,000
(2) Actual total	1,680,288
(3) Error	80,288
(4) Per cent error	5

The prediction of fuel consumed in flight is relatively reliable but even here there is considerable room for error. The figures for altitude and speed might have been estimated more accurately, although it is not certain, by a laborious analysis of flight logs. What was employed instead was merely the considered judgment of the chief pilot, based on his experience. Since fuel consumption varies as the cube of the velocity, errors in this term could cause serious errors in the final prediction. As explained above the figure used for typical gross weight is also somewhat suspect. It is based upon two guesses: first, as to typical take-off weight, and second, what the typical fuel burn-off en route is. The estimates of propulsive and thermal efficiency are crude too. The rest of the data is probably relatively respectable.

Lastly it should be pointed out that an under-estimation of fuel consumption is what one would expect. The actual fuel consumption figure was obtained from service logs, which merely record the amount of fuel poured into the airplanes. Some of this fuel is wasted in various ways, spilling, for example; some of it evaporates; some of it is used in maintenance checks other than the one included in the calculation. An empirical determination of the amount of fuel dissipated in these ways would be far more laborious than it is worth from the point of view of the economist. Since no attempt was made to estimate these quantities, the 5 per cent error observed does not seem excessive.

PART V

CONSUMPTION AND FINAL DEMAND

Chapter 12

THE ROLE OF DEMAND IN THE ECONOMIC STRUCTURE

James S. Duesenberry
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If we view the economy as a structure of relationships we must consider more than input-output and capital-output relations. The relations governing the behavior of consumers will be equally important, for they play a major role in determining the set of final products produced by the economy.

The ultimate aim of analysis in terms of general interdependence is to achieve a system which is as nearly closed as possible. It is doubtful whether, in the present stage of knowledge, a completely closed system can be achieved. Some elements in the system, e.g. some aspects of investment decisions, must remain arbitrary in the sense that they are not understood and cannot be predicted from knowledge of other economic variables. If the system is closed except for a few elements of the type just mentioned, those elements play the role of *deus ex machina* and control the system. When we analyze the structure of an automobile we take as data the actions of the driver. But given the way in which he manipulates the controls, the actions of all parts of the car at each point of time can be deduced from knowledge of the structure of the car. In the same way the actions of all parts of the economy can be deduced from movements of the arbitrary variables if we know the structure of the economy.

It is possible to stop with a system which includes only technical relations of the input-output, capital-output type, taking the bill of goods as a datum. But since the bill of goods plays such an important role in determining the activity of the economy it is desirable if possible to take account of the relationships governing consumer behavior in the systematic analysis of general interdependence rather than to take the outcome of those relations as data.

It can be seen at once that, even if all the relations governing consumer behavior were known, there would still be a gap before those relations could be connected with the system of technical relations. The size of personal income and its distribution and the prices of consumers' goods are obviously among the variables determining consumer behavior. Knowledge of the latter relations would permit us only to make conditional pre-

dictions about the bill of goods. That is, we could only predict what the bill of goods would be for each set of prices and incomes. On the other hand the bill of goods enters into the determination of the income distribution by influencing the distribution of employment and enters into the determination of prices by determining derived demand for raw materials and other resources. Given the technical relations and a bill of goods plus some other data the prices and income distribution can be determined. The bill of goods depends on the latter two variables, so that the bill of goods, distribution of incomes, and price structure implicit in the technical relations and demand relations are the ones which make all those variables mutually consistent.

Some work on problems of income distribution and price determination has been undertaken but in this chapter we shall confine ourselves to the problem of making conditional predictions of consumer purchases, taking income and prices as data.

As already pointed out, the make-up of the consumption bill of goods is likely to depend on the level and distribution of income and on the relative prices of various types of consumption goods. With a fixed set of prices, the part of the bill of goods which varies with income can be dealt with as though consumption were another industry. If the consumption function for the i^{th} commodity is $C_i = a_i y + b_i$, the a_i can be regarded as an input-output coefficient showing the amount of product i required to produce a unit of household services (or if different types of income are distinguished, labor services, land services, and capital services). Households can be treated as an industry selling services to other industries and buying consumption goods from other industries. The b_i then play the role of final bill of goods. If the consumption functions are non-linear, certain computational difficulties arise, since the consumption coefficients vary with income. That is not a very serious problem, however, and need not concern us here. An approach similar to the one just described was used by Cornfield, Evans, and Hoffenberg in their paper, 'Full Employment Patterns in 1950.' [1]

The task of estimating income-consumption relationships for particular commodities is difficult enough but there is a more fundamental difficulty. When we have to deal with production problems there are valid reasons for regarding the 'fixed coefficients' assumption as a good working hypothesis. Some substitution unquestionably exists but there are strong reasons for supposing the amount of substitution is not great enough to create large errors when it is neglected. In the case of consumption the assumption of fixed coefficients is much less plausible. There is a good deal of general evidence for the view that relative prices do have a sig-

[1] Cornfield, Jerome, Evans, W. Duane, and Hoffenberg, Marvin, 'Full Employment Patterns in 1950,' *Monthly Labor Review*, February 1947.

nificant effect on consumer behavior. If that is so, we have to take them into account in order to make realistic estimates of the consumption part of the bill of goods. The necessity of introducing prices into a system greatly increases the computational problem in dealing with a closed system. But that is a relatively minor problem compared with the task of estimating the price elasticities themselves.

Because of the amount of work which has been done on income-consumption relationships in comparison with the amount which has been done on the problem of price elasticities we thought it desirable to begin by working on the latter problem.

When one is faced with a large-scale problem such as that of estimating a detailed bill of goods for an input-output system, there is some temptation to proceed by the use of mechanical methods on a large scale. We could have plunged into the computation of multiple regressions of the consumption of various commodities on prices and income. But as we shall show, the results of such computations are of dubious value. Consequently we felt that discretion was the better part of valor and proceeded with some caution.

The present paper is confined to the estimation of demand elasticities with respect to relative prices for three broad commodity groups: food, clothing, and housing. We have used some rather unorthodox methods. In particular, we abandoned the use of aggregate time-series data in favor of intertemporal comparisons of budget studies. The reasons for this choice are discussed in detail below. In brief our results on estimates of the elasticities are .8 for food, 1.3 for clothing, and 0 for housing. It will be noted that these are not the ordinary price elasticities but elasticities with respect to relative prices, *real* income being held constant. The confidence limits and the results of some prediction checks are given in Section v below.

I. The Problem of Predicting Consumer Behavior

Theoretical work in this field has been carried on at least since the time of Jevons, Walras, and Menger, and empirical work since the time of Engel and LePlay. But, in spite of the large volume of work which has been done, anyone who approaches the task of making practical predictions of consumer behavior has a certain feeling of futility. The number and variety of phenomena to be explained are so great and the variables entering the problem so numerous that it is difficult even to get a clear idea of the nature of the problem.

The theoretical structure which has been erected is elaborate and elegant but not very helpful for empirical work. It is of limited usefulness because it produces only existence theorems of a peculiarly 'iffy' kind.

The accepted theory assumes that every consuming unit has a well-ordered preference system. That means that with respect to any two combinations of goods and services, X' and X^o, each individual will be able to say that either,

(1) X' is preferred to X^o

(2) X' is indifferent to X^o

(3) X^o is preferred to X'

It can then be shown that the individual's purchases will depend only on his income and the prices with which he is faced. If the concept is extended so that combinations of goods include goods consumed at different times, the relevant variables are extended to include assets, present and expected future interest rates, expected future prices, and non-interest incomes.

What we appear to obtain from the theory then is a list of the variables which determine consumer behavior. That is not much, but if it were true it would significantly reduce the scope of the empirical problem. But it is not really true. The theorem just stated is based on the assumption that preferences are given. Yet preferences obviously change so that the theory really only gives us a classification of variables. It tells us that some variables determine the behavior of consumers when their preferences are given and others determine the character of their preferences. Such a classification can be useful only if preferences change slowly relative to the changes in the other variables. Not much empirical work has been done on that point but it can at least be said that it is not obvious that the economic variables change faster than the preferences themselves.

Because of the weakness of the theoretical results, empirical work in the consumption field has been rather loosely tied to theory. A great many budget studies have been made and estimates of income elasticities have been made from them. But as prediction tools the results are rather unsatisfactory since no account is taken of price changes or changes in preferences over time.

Systematic attempts at prediction of consumer behavior have, for the most part, been based on the analysis of time series.

II. Traditional Empirical Technique

If preferences really were constant the unknown parameters of preference functions or the demand functions derived from them could be estimated by making use of observations showing the response of consumers to changes in the economic data. The traditional empirical technique takes that proposition as its starting point.

However, it is recognized by all writers that certain changes in prefer-

ences do take place in time. It has been usual to assume that all of those changes can be represented in one of two ways. (1) Some changes in taste take place gradually and at fairly steady rates. Those changes are generally treated by introducing a time trend into the relation whose parameters are to be estimated. (2) There are a large number of factors which may have rather small and reversible effects on taste from moment to moment. These variables are conceived of as being drawn at random in accordance with some probability function (not necessarily a normal one). Introduction of these elements makes the problem of demand measurement a statistical one. As a result the observations have to be considered as samples, and estimates of the parameters are made according to rules deduced from sampling theory.

A large number of problems arise out of the existence of stochastic elements in demand relations, e.g. multi-collinearity, identification, the convergence properties of estimates. We shall not concern ourselves with these problems at this point, however, important though they may be. Instead we wish to call attention to the extreme simplicity of the hypotheses used for dealing with changes in taste. All of the multitude of factors which can produce changes in tastes are put into one of two classes: those which are regarded as producing trends and those which are regarded as producing stochastic terms.

III. The Adequacy of the Traditional Approach

Without trying to be exhaustive we can attempt to list some of the factors which are likely to produce changes in taste. They include: (1) introduction of new products; (2) fashion changes, as in dress; (3) irreversible effects of changes in prices or availability of commodities; (4) the consequences of mutual interdependence of consumer preferences; (5) changes in the composition of the population with respect to age, marital status, occupational type, or geographical location; and (6) short-run changes in such factors as weather or expectations.

A little consideration will show that most of these factors are not of a kind which can be easily put into the trend-stochastic-term mold. Factors like those listed under (5) may with some plausibility be regarded as producing trends. Even here there are some difficulties. For example, the rate of urbanization may be dependent on the national income or its rate of change because rural-urban migration depends on the existence of job opportunities in the cities. Factors like those under (6) may be regarded as producers of stochastic terms. But what about the other factors?

The introduction of new products does not produce a simple trend in the consumption of any group of products. When a new product is introduced, changes in consumption not explained by prices and incomes

occur. But there is no reason for this to produce a trend-like movement. The demand for a new product or the group of products of which it is a member will shift upward after it is introduced but will reach a saturation point for any given level of income and prices. The effect of introducing any particular new good may be described by a trend term for a relatively short period of growth, but unless the saturation point is known there is no justification for projecting an observed trend supposed to result from the introduction of new products. It may be said that some new product is always being introduced. But since different new products have different impacts on competing goods the steadiness of the innovation process would still be of no help. It goes without saying that new products do not produce results which can be described by stochastic variables. The effects of new products are irreversible and systematically related to one another in time.

Fashion changes may sometimes be described by stochastic variables but some of them may have effects of long duration which cannot be described in terms of random drawings. Similarly a rise or fall of a price or a change in the availability of goods may cause a more or less permanent change in taste. For example, the shortage of butter during the war caused a shift of consumption toward margarine which was not reversed when larger butter supplies became available.

The mutual interdependence of consumer preferences is a factor of a different order from the ones mentioned so far. If consumer tastes are shifted by mutual interaction it is possible to account for the movements of consumption by introducing new variables into the explanation. We can, for instance, make each individual's consumption depend on the lagged aggregate consumption. But if we fail to do so our statistics will show movements of consumption which appear to be accounted for by trend factors.

If we admit that non-random shifts in taste, new products, and irreversible responses to price changes (or availabilities) are important influences on consumption we cannot content ourselves with the kinds of methods which have hitherto been used in the statistical analysis of demand. We must seek some methods which enable us to deal effectively with those factors.

In addition we have to find some methods which will permit us to escape the difficulties of multi-collinearity, serial correlation, and the interdependence of supply and demand.

IV. Determination of Price Elasticities

Let us return for a moment to the logic of the basic consumer behavior theory. The theory asserts that at any one moment each individual has

an ordered preference system or acts as though he had. It is then shown that his action at that moment is determined by the values of the economic variables such as prices, incomes, price expectations, and so on. A functional relation between the amounts of commodities he buys and the economic variables is supposed to exist. What is the nature of that relationship? It is what we might call a potential action function. It states that if one set of conditions existed at the moment each consumer would act in one way but if a different set of conditions existed he would act differently. Now, in the nature of the case, we can never verify that proposition. For at any one moment only one set of conditions can exist. We can observe how an individual reacts to those conditions but we can never observe how he would have reacted at that moment to a different set of conditions. We believe that he would act differently because we often observe that apparently similar individuals act differently when placed in different circumstances. (This idea could be refined by introducing statistical concepts, but that hardly seems necessary here.)

That belief is inherent in the logic not only of all demand theory but of most other branches of economic theory.

If we try to discover the character of the 'potential' relation between consumption of some commodities and some economic variables by using observations on the quantities purchased and the values of the economic variables, we have to assume that, to some extent anyway, the 'potential' relation is the same throughout the period of observation. By introducing random terms we concede that some of its parameters (usually the constant) change from moment to moment, but that on the average those parameters are the same. Similarly when we introduce a trend we assert that one of the parameters changes through time in a smooth and systematic way. But as we have already shown, there are other kinds of shifts which are not included under those headings.

In the face of the difficulties presented by those shifts we would not be very pessimistic if we argued that the potential function is stable for such short periods that there is no hope of determining it. That position is hardly tenable if we are allowed to expand the range of variables included in our analysis. The basic psychology of members of our society changes only very slowly. It might well be that consumption is so powerfully influenced by non-economic variables that there is no stable relation between consumption and the economic variables. But in principle at least, the solution to that problem would be to take the non-economic variables into account in empirical studies.

But if that is the only defense the pessimists are probably right. Many of the non-economic variables have a 'rare events' kind of distribution. They are constant for many years and then exert their influence for a short period only to become dormant again. The influence they exercise

may be permanent but we cannot put them into empirical terms unless we can experiment so that we get repeated observations on their influence. Other non-economic variables may change more often but may be very difficult to measure or even to observe at all.

When long time series are available it may be possible to test the stability of a relationship by splitting the data. If the results are satisfactory we need not worry. But at present long time series are seldom available. If we split a 20- to 30-year series into subseries of 10 or 15 observations and then fit relationships involving 3 or 4 parameters there are not many degrees of freedom left for comparing the coefficients obtained from the two halves. Moreover the effective number of degrees of freedom is reduced by serial correlation, and we still face the complications arising from multi-collinearity and the interdependence of supply and demand. We cannot expect very much from splitting the data into subseries.

A method is needed which permits us to eliminate systematically as much as possible of the effect of changes in parameters over time. That can be done if we can find observations which can be regarded as being more or less equally affected by those shifts, while undergoing different changes in the independent variables in which we are interested. To do that we have made use of intertemporal comparisons of budget studies. Those data obviously have their own inadequacies but we shall first discuss our method before we discuss the numerous problems which arose in applying it.

We are interested in finding price elasticities. The income elasticity problem will be discussed later. Our method of estimation is as follows. We assume the basic relation governing the consumption of a commodity to be of the form

$$y_{ij} = k\epsilon_i\delta_j x_{ij}^a z_{ij}^b F_{ij}^c S_{ij}\lambda_{ij} \tag{12, 1}$$

where

- y_{ij} = consumption of the commodity in question in the year i and city j,
- x_{ij} = the relative price of the commodity,
- z_{ij} = the real income of a certain group of families,
- F_{ij} = the number of persons in the families in question,
- S_{ij} = a measure of their social class,
- δ_j = a term measuring the effect of all variables peculiar to city j (these are supposed to be constant for some years),
- ϵ_i = the effect of all other variables which affect all cities alike, e.g. expectational variables.

The value of ϵ_i changes from year to year but is the same for all cities in a given year. Finally λ_{ij} includes the effect of those variables which are specific to year i and city j. It may be taken to include the effect of de-

viations from the mean of ϵ_i and δ_j. We assume that the parameters, a, b, and c, are approximately constant in various cities at one time and over fairly long periods. (The magnitude of errors resulting from shifts in elasticities is shown below to be small.)

Now consider a pair of budget studies made in the same city in two different years and covering families in the same social class. We can choose observations from those studies in such a way as to obtain pairs of observations (one from each year) covering families having the same real income and family size. Applying equation (12, 1) we have:

$$y_{ij} = k\epsilon_i\delta_j x_{ij}{}^a z_{ij}{}^b F_{ij}{}^c S_{ij}\lambda_{ij} \tag{12, 1}$$

for the first observation and

$$y_{i+t,j} = k\epsilon_{i+t}\delta_j x_{i+t,j}{}^a z_{i+t,j}{}^b F_{i+t,j}{}^c \lambda_{i+t,j} \tag{12, 2}$$

for the second.

All the terms of the two equations are the same except for the y's, x's and ϵ's. Let us divide equation (12, 2) by equation (12, 1). We obtain:

$$\frac{y_{i+t,j}}{y_{ij}} = \frac{\epsilon_{i+t}}{\epsilon_i}\left(\frac{x_{i+t,j}}{x_{ij}}\right)^a \frac{\lambda_{i+t,j}}{\lambda_{ij}} \tag{12, 3}$$

We thus eliminate the common terms and obtain a relation between the ratio of the quantities in the two years and the ratio of the prices. This relation is, however, affected by the ϵ's and λ's. Suppose, however, that we have a number of pairs of observations of the type just discussed and that those observations involve the same pair of years but different cities. For various reasons which we shall discuss later different changes in relative prices occur in different cities. We now have a series of observations of the form $Y_j = EX_j{}^a L_j$, where the capital letters correspond to the ratios in (12, 3). Let us take the regression of log Y_j on log X_j. We get a result in the form, log Y_j = log $\breve{E} + \breve{a}$ log X_j + log L_j. $\breve{a}$ is an estimate of the elasticity of demand with respect to relative prices. (To obtain the ordinary elasticity of demand we have to add something to take account of the effect of price changes on real income.) $\breve{a}$ will be an unbiased estimate of a provided that the term $\dfrac{\lambda_{i+t,j}}{\lambda_{ij}}$ is uncorrelated with $\dfrac{x_{i+t,j}}{x_{ij}}$.

Before discussing our results let us consider the nature of the λ's. They will include first of all the observational error in the x's, which is likely to be fairly substantial in budget studies even when fairly large samples are used. It should be noted, however, that any errors such as systematic over- or under-estimates of either income or purchases will cancel out in the ratio computation. Since the rather large samples used

will eliminate most of the purely random observational error it is probable that the ratio of the λ's is not much affected by observational error in the sense of error due to inaccurate reporting. The type of observational error most likely to show up is that due to lack of comparability between the samples. The definitions of family types acceptable for interviewing differ from one study to another. If those differences are systematic in the sense that they are the same in all pairs of cities they will influence the ϵ's but not the λ's. However, it is likely that considerable error due to lack of comparability will show up in the λ terms. On the whole there seems to be no very good reason why this type of error should be correlated with price changes.

The other type of error appearing in the λ's results from all those factors which are peculiar to a particular city and year. One city may be growing faster than another and therefore have a change in the age distribution of population. If it were merely larger or if it always had a younger population the influence of its size would be eliminated in taking the ratio. But the differential change in its age distribution will not be eliminated. Note, however, that it is only the difference in change in age distribution which is important. If all cities have the same change in age distribution only the constant in the regression is affected. The same situation applies to all variables. The λ ratio will be affected if a relevant variable in one city changes more or less than the average for other cities. The λ ratio will be correlated with the price ratio if there is some connection between price changes and the variable undergoing the differential price change. It should be noted that these remarks do not apply to income size and distribution, family size, or social class distributions, because of the controls used in selecting observations.

We cannot give a categorical answer to the question whether there is a tendency toward correlation between the price ratios and the λ ratios. However, we can get some insight into the problem if we consider what factors are likely to be of most importance in causing differential changes in relative prices. In general these will be of two types, factors leading to differential shifts in demand and factors causing changes in costs or supplies.

Under the first heading the most important items will be changes in income and population. We have controlled the income factor so that no correlations between errors and income changes for the city as a whole need disturb us. However, we have not been able to control the age distribution of the population. As we have already pointed out, age distribution is likely to be related to rate of growth of population, while the latter will influence price changes. We have studied the data used with those considerations in mind but there does not appear to be much relation between price changes and rates of growth of population among

the cities studied. Such correlation as exists has the wrong sign. The southern cities which had the highest rates of population growth showed downward movements in the price of food by comparison with the average for all cities. That seems to indicate that the dominant influences on relative prices were on the supply side. Of these factors the most important are (1) the increase in diversified farming in the South, and (2) improvements in transportation which reduced food costs in non-food-producing areas relative to costs in food-producing areas, e.g. the Midwest.

Since supply factors are not likely to be correlated with shifts in demand not controlled by our methods, there does not seem to be any reason to fear a bias in the results.

One other type of bias remains to be discussed. Errors of observation in the independent variable tend to produce a downward bias in the regression coefficient. The question arises, therefore, whether there are likely to be significant errors in the relative price data. The data used were Bureau of Labor Statistics consumer price indices. The errors in these indices are of three kinds. On the one hand there is a pure observational error arising from the fact that periodic checking of prices may not give an accurate estimate of average prices. It is reasonable to suppose that this error is rather small, since a large number of observations is made for each commodity in each city. The second type of error arises from the fact that all prices are not observed. The prices of some items are estimated on the assumption that they move proportionately to those of items whose prices are observed. Some error undoubtedly arises from this source. The third type of error results from the fact that an index is used. Since weights may change either because of substitution or because of changes in taste, the prices used are not a perfect measure of the *relevant* prices.

As in the case of errors in the dependent variable the critical question is whether the errors are systematic or not. If the deviations between the movements of the commodities whose prices are observed and those which are estimated are proportional for all cities, the errors will increase the constant term, E, which shows the effect of all the systematic errors. The same effect will be produced by the systematic element in the errors in the weights. Consequently the observational error bias is only that which results from the non-systematic part of the error. There is no way to estimate it but it seems probable that the bias is rather small except in the case of housing.

We now come to the more serious limitations of our study. These are (1) the smallness of the range of variation in the sample; (2) the increase in the variance of random errors due to the use of the ratio method; and (3) the small number of observations available.

In the nature of the case it is difficult to get wide variations in relative prices. Even in the raw data relative prices of large commodity groups vary only within a limited range. Changes in relative prices of over 25 per cent seldom occur even over fairly long periods involving wars and depressions When we make use of regressions involving price and quantity changes in different cities during the same period the deviations from the mean are even smaller. Only the deviations give any information, and since prices in all cities move under the same forces, only a limited variation is possible. A range of 10 percentage points on either side of the mean percentage change is about as large as can be obtained. In our attempt to control variations not due to price changes we find it necessary to eliminate a large part of the variations in the price changes themselves. (It should be noted that this is true to a large extent of time-series correlation methods. The correlation between relative prices and incomes is likely to be very high so that the amount of *independent* variation in relative prices is much smaller than the total variation shown by the time series for prices alone.) The variance of a regression coefficient depends in part on the variance of the independent variable so that our estimates are weakened by inability to observe wide variation in the independent variable.

The difficulty is made the greater by the increase in the variance of the errors which results from using ratios. If we consider the variance of the log of the errors, then we know that if the errors are uncorrelated with one another the variance of differences will be twice the variance of the original errors. In addition the basic errors of observation in budget-study data may be greater than those in aggregate data. (This is not necessarily the case and if we consider the accuracy of aggregate data for years earlier than 1929 it may well be that the budget-study data are more accurate than the aggregates.) The increased variance of the errors works to further increase the sampling variance of our estimates.

Neither of these difficulties would be particularly serious if a large number of observations was available. Unfortunately only a limited number of budget studies has ever been made and of those studies only a few meet the requirements of completeness and comparability which we have had to set up. We have been able to obtain data for 5 pairs of cities for the period 1918-29; 6 for 1929-1934/5; 21 for the period 1918-1934/5; and 6 for the period 1934/6-1947/8. Fitting regressions for the separate periods is obviously very unsatisfactory when the correlation coefficients are low and the number of observations is small. Such regressions have been fitted in order to give some idea of the stability of the elasticities over the whole period. But to estimate the elasticities with any degree of accuracy it was necessary to find a device for pooling the observations from the separate periods.

A certain difficulty arises here. There is every reason to believe that economic relationships are unstable over time. The parameters of the 'potential relationship' of which we spoke earlier change from time to time. If we consider relations of the logarithmic type either the constant or the elasticity may change. We have already shown how to eliminate the effects of shifts in the constant when it can be assumed that the elasticity is approximately constant for some years. As a matter of fact it can be shown that when elasticities are calculated for the separate periods, the method discussed above gives a good estimate of the elasticity of one of the two years. (Which one depends on which year is used as a base in calculating the relative price.) The estimate will have an error equal to the change in the elasticity multiplied by the regression of the deviations of the base prices in an individual city from the geometric mean of those prices on the corresponding deviation of percentage change in price. Some experiments show that that regression is small. (The proof of the proposition just stated is given in the Mathematical Note, p. 479.)

Consequently we need not be disturbed by changes in elasticities during one of our time periods. Now, however, we wish to calculate an average elasticity over a long period of time. We shall first consider the case in which the elasticity is stable over time and show how to eliminate the effect of shifts in the constant from period to period.

Let us return to the relation described by equation (12, 1):

$$y_{ij} = k\epsilon_i\delta_j x_{ij}^{a} z_{ij}^{b} F_{ij}^{c} S_{ij}\lambda_{ij}$$

It was shown in Section IV that by taking the ratio of $y_{i+t,j}$ to y_{ij} we could obtain a relation

$$\frac{y_{i+t,j}}{y_{ij}} = \frac{\epsilon_{i+t,j}}{\epsilon_{ij}}\left(\frac{x_{i+t,j}}{x_{ij}}\right)^{a}\frac{\lambda_{i+t,j}}{\lambda_{ij}}$$

For any one pair of years we can compute a set of such ratios, one for each city. We can, as was suggested earlier, take a logarithmic regression of the ratio of the y's on the ratio of the x's. The log of the ratio of the ϵ's will appear as a constant in the relation while the slope of the regression will be an estimate of a. Now suppose we have computed such regressions for several periods and plotted the regression lines and the scatter of points. They might appear like those in Figure 1A or like those in Figure 1B. (To keep the diagrams clear all the points are shown as falling exactly in the regression.) If we were to combine all the observations to get a single regression, the lines of best fit would be AB for the data in Figure 1A and $A'B'$ for those in Figure 1B. In Figure 1A the regression from the combined observation is much steeper than the true regression while in Figure 1B it slopes in the wrong direction. The reason

is obvious. If there is a positive correlation between the mean price ratios for the periods and the constants of the regressions (which equal the ratio of the ϵ's), for the corresponding period it will appear that high prices are associated with high consumption. Conversely a negative correlation between the mean price ratio and the ratio of the ϵ's produces the appearance of a very large elasticity of demand.

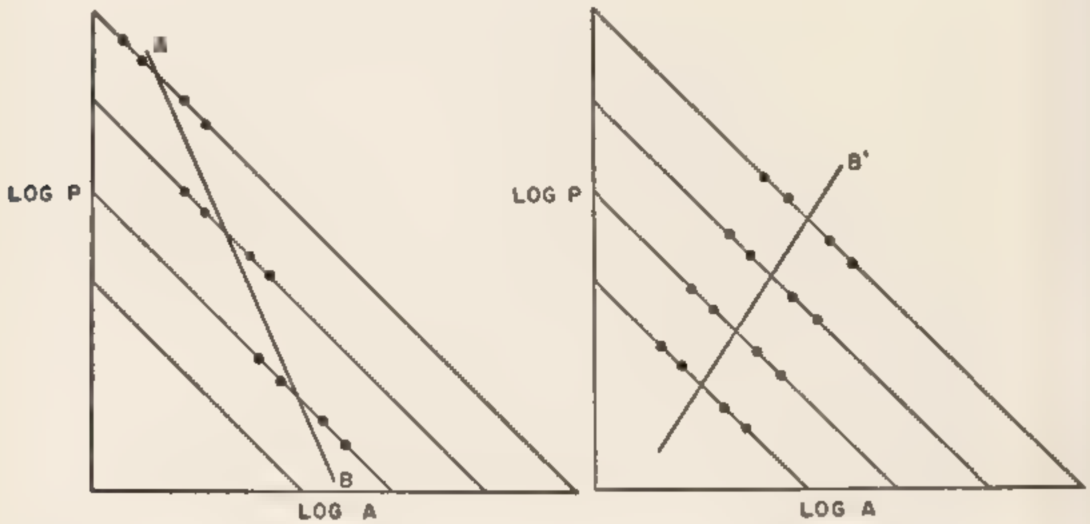

FIGURE 1A FIGURE 1B

Now it is very likely that the ϵ ratios will be correlated with the intertemporal movements of prices. Any shift in taste in favor of or against a commodity will tend to cause its price to rise or fall. Of greater importance in the present connection is the effect of the depression. When a large drop in income takes place people will not adjust their consumption in the manner which would be expected from considering comparisons between individuals with different incomes at the same point in time. Our method involves comparing individuals with the same income at different points of time. A family with a given income in a depression year probably had a higher income during the preceding prosperity. Their consumption will not be the same as that of families with the same income in a prosperous period. We have tried to allow for part of the difference by comparing families with the same expenditure rather than with the same income. That allows for the tendency for the savings propensity to fall in depressions. But the fall in income affects the distribution as well as the total of expenditures at a given level. Purchases of clothing and durables are likely to be reduced much more than those of food, because stocks of the durables are large.

As we shall show there is a marked positive correlation between the errors and the price movements so that the regression coefficients are too low when estimated from the combined observations. To cope with that

difficulty we have used an extension of the method used in obtaining the single period regressions.

Consider the relation (12, 3):

$$\frac{y_{i+t,j}}{y_{ij}} = \frac{\epsilon_{i+t,j}}{\epsilon_{ij}}\left(\frac{x_{i+t,j}}{x_{ij}}\right)^{a}$$

For a given pair of years there is one such set of observations for each city. Let us take logs of both sides. We have:

$$\log\frac{y_{i+t,j}}{y_{ij}} = \log\frac{\epsilon_{i+t,j}}{\epsilon_{ij}} + a\log\frac{x_{i+t,j}}{x_{ij}}$$

Now let us take the mean of each side (the antilogs of these means are the geometric means of the two sides) and subtract it from each observation. We have:

$$\log\frac{y_{i+t,j}}{y_{ij}} - \overline{\log\frac{y_{i+t,j}}{y_{ij}}} = \log\frac{\epsilon_{i+t,j}}{\epsilon_{ij}} - \overline{\log\frac{\epsilon_{i+t,j}}{\epsilon_{ij}}} + a\left(\log\frac{x_{i+t,j}}{x_{ij}} - \overline{\log\frac{x_{i+t,j}}{x_{ij}}}\right)$$

Since the ϵ's are the same for all cities in a given period they will drop out, and we have:

$$\log\frac{y_{i+t,j}}{y_{ij}} - \overline{\log\frac{y_{i+t,j}}{y_{ij}}} = a\left(\log\frac{x_{i+t,j}}{x_{ij}} - \overline{\log\frac{x_{i+t,j}}{x_{ij}}}\right)$$

We can treat the observations for each period in that manner. The adjusted observations in each period will now be independent of the ϵ's. If we now pool the adjusted observations for all of the periods and take the regression of $\log\frac{y_{i+t,j}}{y_{ij}} - \overline{\log\frac{y_{i+t,j}}{y_{ij}}}$ on $\log\frac{x_{i+t,j}}{x_{ij}} - \overline{\log\frac{x_{i+t,j}}{x_{ij}}}$ we shall obtain an estimate of a which is independent of any correlation which may exist between the ϵ's and the prices.

The result will be subject to random errors because of the λ's which were neglected in the calculations above but will not be biased unless the λ's are correlated with the prices. We concluded above that such correlations are not important in the present case so that the regression should yield an unbiased estimate of $\underline{a}$. As a result we are able to get 38 observations so that a reasonably accurate estimate of $\underline{a}$ can be made.

However, all of the argument above is based on the assumption that the elasticity is constant. We have to consider how the results are affected if $\underline{a}$ changes from period to period. Some calculations which are given in the Note (p. 482) yield the following results. Let the elasticity in the earliest period be $\underline{a}$ and let it be $\underline{a} + \delta_1$, $\underline{a} + \delta_2$, $\underline{a} + \delta_3$ in the following periods. The difference between the regression coefficient which is sup-

posed to measure elasticity and g will equal (1) the sum of δ_1, δ_2, and δ_3 each weighted by the ratio of the variance of log X_{ij} for its own period to the variance of log X_{ij} for all periods combined; (2) the sum of δ_1, $\delta_2 - \delta_1$, and $\delta_3 - \delta_2$ each weighted by the logarithmic regression of absolute price on change in price for its own period. The second term is likely to be small in magnitude.

If the elasticity is shifting erratically in time about a mean of g the signs of the δ's will vary and will largely cancel out each other so that the estimated elasticity will not be far from g unless the δ's have a very large variance relative to g, which does not seem likely. If, however, the elasticity rises or falls steadily from one period to another the estimated elasticity will be higher or lower than the elasticity of the last period. If, for example, the true elasticities were -1, $-.9$, $-.8$, $-.7$ in successive periods and the variances of the percentage changes in price were of the same magnitude, the estimated elasticity would be $(-1.0 - .1 - .2 - .3) = -.4$, which would be a serious under-estimate even for the final elasticity.

This looks like an unfortunate situation but there is one possible escape. We have separate estimates for the elasticity in each period. As we have shown they are relatively unbiased. Now if the elasticity is changing appreciably and in the same direction over time there will be a marked difference between the average of the regressions computed for the separate periods and the regression based on all the data. The sampling variation of the separate period regressions will be large because of the small number of observations. Nevertheless, if there is any very strong trend in the elasticity it should be revealed by the difference between the average of the single period regressions and the total regression.

V. Numerical Results

We are, at last, prepared to discuss the numerical results of our computations. Our estimates of the demand elasticities with respect to relative prices for food, clothing, and housing are given in Table 1 together with their standard errors and confidence limits. These estimates were obtained by pooling all the observations from the four periods studied, in the manner described in the last section. The data used covered only wage earners and clerical workers but as we shall show there seems to be no reason why they should not be applied to all groups.

The following points should be noted. The confidence limits for food are rather high. Nevertheless the lower confidence limit is substantially above the estimate for the elasticity of demand for food obtained by Gershick and Haavelmo [2] by the 'reduced form' method. In spite of the

[2] Girshick, M. A., and Haavelmo, Trygve, Statistical Analysis of the Demand for Food Examples of Simultaneous Estimation of Structural Equations,' *Econometrica*, April 1947.

TABLE 1

Estimates of Relative Price Elasticities

Item	a = Estimated Elasticity	σ_a	Correlation Coefficient	N	Lower Confidence Limit $P = .7$	Upper Confidence Limit $P = .7$
	(1)	(2)	(3)	(4)	(5)	(6)
(1) Food	- .811	.221	.53	38	- .581	-1.041
(2) Clothing	-1.333	.214	.73	38	-1.111	-1.555
(3) Housing	- .078	.209	-.06	38	0[1]	- .17[1]

[1] On *a priori* grounds it can be assumed that the elasticity is not positive, the lower limit is therefore set at 0 and the upper limit is calculated by using only the upper tail of the probability distribution.

width of the confidence limit fairly accurate estimates of demand for food can be made in most circumstances simply because changes in relative prices are for the most part held within fairly narrow limits.

After finding that the housing elasticity was quite low we recomputed the relative prices for the other commodities, taking housing prices out of the denominator of the relative price. New estimates of the elasticities for food and clothing were made. That was done on the ground that if there is no substitution between housing and other things the reverse is true. Consequently the price of housing is an irrelevant variable and its removal from the relative price computation should improve the fits for the other commodities. That is what happened.

The elasticities given are based on the second computation, so that the relative prices to which the elasticities apply are the price of food (or clothing) relative to a weighted average of prices of goods other than housing. The details of the price computation are discussed in Appendix 3.

Having computed the elasticities we wished to test them by applying them to aggregate data. To do that we had to take into account the effects of changes in income. One way to do that is to take a regression of consumption of food, clothing, or housing on aggregate income. However, in view of what was said above about the possibility of shifts in demand, that method had to be rejected. If shifts in demand have occurred they would influence the regression of consumption on income unless they were random with respect to income, which would be very unlikely.

The alternative was to obtain an income elasticity from budget-study data. We used the data from *Spending and Saving in Wartime*.[3] Regressions of consumption on expenditure and family size were calculated for food, clothing, and housing. The family-size variable was introduced to

[3] Department of Labor, Bureau of Labor Statistics, Bulletins #723 and #724, *Spending and Saving in Wartime*.

eliminate the effects of the correlation between family size and family income. A linear regression was used but it does not appear that very different results would be obtained by a logarithmic regression. Our regressions took the form

$$C_i = AE_i + bn_i + C_i$$

where

C_i = consumption of the commodity in question by a given family,
E_i = family expenditure,
n_i = family size.

Summing over all families we obtain:

$$\Sigma C_i = A \sum_{i=1}^{1=N} E_i + b\Sigma n_i + NC_i$$

where

N = the number of families in the country,
ΣE = aggregate income,
Σn_i = the population.

Dividing through by the population size we have:

$$\Sigma \frac{C_i}{p} = A\Sigma \frac{E}{p} + b + \frac{N}{p} C_i$$

This is a linear relation between per capita consumption of the commodity in question, per capita expenditure, and average family size. In further calculations we assumed that average family size is approximately constant. Values of the regression coefficients are given in Table 2.

We then estimated per capita consumption of food, clothing, and housing by calculating

$$\Sigma \left(\frac{C_i}{p}\right)_j = \left\{\Sigma \left(\frac{C_i}{p}\right)_{1941} + A \left[\Sigma \left(\frac{E_i}{p}\right)_j - \Sigma \left(\frac{E_i}{p}\right)_{1941}\right]\right\} \{1 + aP_j\}$$

where

A = the elasticity of demand for the commodity in question,
P_j = the percentage change in the relative price from the 1941 relative price.

The first part of the expression measures the effect of change in income from 1941 while the second part measures the effect of changes in relative prices.

In this calculation the year 1941 is taken as a base from which shifts in demand are to be measured. That does not mean that 1941 is any more

TABLE 2

Expenditure Income Regressions

Item	Regression of Item on Income	Regression of Item on Family Size	Intercept	Multiple Correlation Coefficient
	(1)	(2)	(3)	(4)
(1) Food	.23	36.31	22.99	.98
(2) Housing	.15	-10.20	173.97	.98
(3) Clothing	.13	11.75	-62.52	.97

normal than any other year. If we are to speak of shifts in demand we must have some base from which to measure them and for that purpose 1941 is as good as any other year.

Before considering the results of our calculations we have to ask ourselves what we should expect. If there were no shifts in demand we should obtain a close relation between our estimates of consumption and actual consumption. How good an estimate we should expect is not quite clear. Aside from any errors in our calculation of income and price elasticities there are errors in the basic data on aggregate consumption as a whole and for particular commodities. Even if our estimates were perfect we should not get a perfect estimate of consumption as reported by the Department of Commerce. Judging from the statistical discrepancy the estimates of aggregate consumption expenditure are fairly accurate from 1929 on. But estimates for subgroups of commodities are presumably less accurate, since the total figure gains in accuracy by cancellation of errors in the subgroups. Also some additional error is introduced by deflation. For the period before 1929 the errors are probably higher. However, if our estimates were perfect and there were no shifts in demand, we should expect that the deviations of our estimates from the reported data would be fairly random. Some errors in the Commerce data would persist over several years but most of them would be random. On the other hand, errors in our estimates or shifts in demand would produce a systematic pattern of errors.

What can we expect to find in regard to shifts in demand? We have already mentioned two kinds of shifts. There should be a cyclical shift in demand due to the fact that consumption of different commodities will respond differently to a fall in income. Consumption of those commodities whose purchase can be postponed should fall more than would be expected from calculations based on a budget study in a prosperous period. On the basis of our method of calculation we should expect that clothing purchases in the depression would be lower than our estimate, while housing purchases would be substantially higher.

The second kind of demand shift which we anticipated is an upward shift in demand for food during World War II. This shift results from the fact that, in spite of rationing, food consumption by relatively low income groups rose very sharply during the war. In view of the fact that an index of per capita food consumption remained almost constant for 30 years from 1909 to 1939 the wartime rise in food consumption is surprising. The explanation seems to lie in the fact that income rose sharply during the war while many products on which that increased income would normally have been spent were unavailable. Expenditure was thus diverted toward food, which was available in greater aggregate volume than ever before. High and middle income families were forced to curtail their consumption but low income families greatly increased their consumption, as is shown by the rise in aggregate food consumption during the war.

When more goods of other types became available, prewar patterns were not re-established, so that food consumption after the war was higher than would be expected in view of postwar prices and income. (Properly we should say that prices stayed higher than would be expected in view of supplies available.)

The actual and estimated per capita consumption of food, clothing, and housing in terms of 1935-9 prices are given in Table 3. The errors are far from random. In the case of housing there is a concentration of negative errors during the depression years. That is in accord with the view that real housing expenditures resist the decline in income. It can hardly be explained by changing the income elasticity because it would require too low an income elasticity. Raising the price elasticity would not eliminate the errors because the errors during the depression are not well correlated with the movements of housing prices. Rents continued to fall after 1933 while the error grew smaller. The estimates of housing consumption for the 1920's are reasonably satisfactory so that it appears that our method gives fairly accurate results.

The deviations for food are an artifact of the arithmetic of the situation. If families with a given total expenditure spend more on housing in a depression than in prosperity (because their housing expenditure is based on a higher expenditure in other years) they must spend less on something else. Our estimates for clothing during the depression years do not show great errors, so that food and miscellaneous must have carried the adjustment.

The situation is somewhat as follows. When income falls people try to maintain their standards of living as well as possible. Reduction in saving is one way of achieving that aim but it provides only a limited cushion. Once that cushion is used up, reductions in expenditure have to be made. The amount of reduction per dollar of loss of income will differ with dif-

TABLE 3A

Estimated and Actual Food Consumption

Year	Average per Capita Income in 1941 Prices	Income (Col. 1) less 1941 Average per Capita Income	Col. 2 x a[1] (a = +.23)	Col. 3 + 1941 Average per Capita Consumption (148)	Percentage Change in Relative Prices in 1941	Col. 5 x ϵ_p[2] (ϵ_p = −.81)	Estimated Consumption (Col. 6 + 1 x Col. 4)	Actual Consumption	Difference
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1) 1914	$480	$−107	$−24.61	$123.39	+14.7	−11.91	$109	$115	$− 6
(2) 1919	489	− 98	−22.54	125.46	+ 6.9	− 5.59	118	124	− 6
(3) 1921	434	−153	−35.19	112.81	+ 0.8	− 0.65	112	110	+ 2
(4) 1923	517	− 70	−16.10	131.10	+ 4.0	− 3.24	128	125	+ 3
(5) 1925	513	− 74	−17.02	130.98	+11.7	− 9.48	118	111	+ 7
(6) 1927	527	− 60	−13.80	134.20	+10.3	− 8.34	123	112	+11
(7) 1929	578	− 9	− 2.07	145.93	+ 9.5	− 7.70	135	126	+ 9
(8) 1930	526	− 61	−14.03	133.97	+ 6.5	− 5.26	127	124	+ 3
(9) 1931	492	− 95	−21.85	126.15	− 2.3	+ 1.86	128	123	+ 5
(10) 1932	431	−156	−35.88	112.12	− 8.4	+ 6.80	120	112	+ 8
(11) 1933	409	−178	−40.94	107.06	− 6.5	+ 5.26	113	105	+ 8
(12) 1934	430	−157	−36.11	111.89	− 2.2	+ 1.78	114	106	+ 8
(13) 1935	452	−135	−31.05	116.95	+ 0.5	− 0.40	116	111	+ 5
(14) 1936	491	− 96	−72.08	125.92	+ 1.4	− 1.13	124	121	+ 3
(15) 1937	504	− 83	−19.09	128.91	+ 1.3	− 1.05	128	125	+ 3
(16) 1938	491	− 96	−22.08	125.92	− 2.4	+ 1.94	128	127	+ 1
(17) 1939	515	− 72	−16.56	131.44	− 3.3	+ 2.67	135	131	+ 4
(18) 1940	541	− 46	−10.58	137.42	− 2.9	+ 2.35	141	138	+ 3
(19) 1941	587	0	0	148.00	0	0	148	148	0
(20) 1946	745	+158	+36.34	184.34	+ 7.3	− 5.91	173	194	−21
(21) 1947	735	+148	+34.04	182.04	+11.9	− 9.64	164	182	−18
(22) 1948	728	+141	+32.43	180.43	+12.7	−10.29	162	178	−16

[1] a = regression coefficient of food on income. See Table 2.

[2] ϵ_p = relative price elasticity. See Table 1.

TABLE 3B

Estimated and Actual Clothing Consumption

	Year	Average per Capita Income In 1941 Prices	Income (Col. 1) less 1941 Average per Capita Income	Col. 2 x a[1] (a = +.13)	Col. 3 + 1941 Average per Capita Consumption (77)	Percentage Change in Relative Prices from 1941	Col. 5 x ϵp[2] (ϵ = –1.33)	Estimated Consumption (Col. 6 + 1 x Col. 4)	Actual Consumption	Difference
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1)	1914	$480	$–107	$–13.91	$63.09	– 5.1	+ 6.78	$67	$69	$– 2
(2)	1919	489	– 98	–12.74	64.26	+23.2	–30.86	44	52	– 8
(3)	1921	434	–153	–19.89	57.11	+15.3	–20.35	45	60	–15
(4)	1923	517	– 70	– 9.10	67.90	+ 5.0	– 6.65	63	79	–16
(5)	1925	513	– 74	– 9.62	67.38	– 0.5	+ 0.66	68	78	–10
(6)	1927	527	– 60	– 7.80	69.20	– 4.6	+ 6.12	73	84	–11
(7)	1929	578	– 9	– 1.17	75.83	– 5.8	+ 7.71	82	82	0
(8)	1930	526	– 61	– 7.93	69.07	– 3.5	+ 4.66	72	72	0
(9)	1931	492	– 95	–12.35	64.65	– 2.4	+ 3.19	67	68	– 1
(10)	1932	431	–156	–20.28	56.72	– 3.9	+ 5.19	60	56	+ 4
(11)	1933	409	–178	–23.14	53.86	– 2.4	+ 3.19	56	49	+ 7
(12)	1934	430	–157	–20.41	56.59	– 0.6	+ 9.80	57	55	+ 2
(13)	1935	452	–135	–17.55	59.45	– 3.1	+ 4.12	62	59	+ 3
(14)	1936	491	– 96	–12.48	64.52	– 3.5	+ 4.66	68	65	+ 3
(15)	1937	504	– 83	–10.79	66.21	– 1.4	+ 1.86	67	62	+ 5
(16)	1938	491	– 96	–12.48	64.52	+ 0.8	– 1.06	64	62	+ 2
(17)	1939	515	– 72	– 9.36	67.64	+ 1.3	– 1.73	66	66	0
(18)	1940	541	– 46	– 5.98	71.02	+ 1.5	– 2.00	70	68	+ 2
(19)	1941	587	0	0	77.00	0	0	77	77	0
(20)	1946	745	+158	+20.54	97.54	+ 6.9	– 9.12	89	103	–14
(21)	1947	735	+148	+19.24	96.24	+ 6.5	– 8.64	88	90	– 2
(22)	1948	728	+141	+18.33	95.33	+ 5.4	– 7.12	88	86	+ 2

[1] a = regression coefficient of clothing on income. See Table 2.

[2] ϵp = relative price elasticity. See Table 1

TABLE 3C

Estimated and Actual per Capita Consumption of Housing, Fuel, and Light

Year	Average per Capita Income in 1941 Prices	Income (Col. 1) less 1941 Average per Capita Income	Col. 2 x a[1] (a = +.15)	Estimated Consumption (Col. 3 + 1941 Average per Capita Consumption) (101)	Actual Consumption	Difference
	(1)	(2)	(3)	(4)	(5)	(6)
(1) 1914	$480	$-107	$-16.05	$ 84.95	$ 99	$-14
(2) 1919	489	- 98	-14.70	86.30	100	-14
(3) 1921	434	-153	-22.95	78.05	89	-11
(4) 1923	517	- 70	-10.50	90.50	93	- 3
(5) 1925	513	- 74	-11.10	89.90	89	+ 1
(6) 1927	527	- 60	- 9.00	92.00	91	+ 1
(7) 1929	578	- 9	- 1.35	99.65	93	+ 7
(8) 1930	526	- 61	- 9.15	91.85	92	0
(9) 1931	492	- 95	-14.25	86.75	90	- 3
(10) 1932	431	-156	-23.40	77.60	88	-10
(11) 1933	409	-178	-26.70	74.30	89	-15
(12) 1934	430	-157	-23.55	77.45	89	-12
(13) 1935	452	-135	-20.25	80.75	89	- 8
(14) 1936	491	- 96	-14.40	86.60	91	- 4
(15) 1937	504	- 83	-12.45	88.55	92	- 4
(16) 1938	491	- 96	-14.40	86.60	92	- 5
(17) 1939	515	- 72	-10.80	90.20	94	- 4
(18) 1940	541	- 46	- 6.90	94.10	97	- 3
(19) 1941	587	0	0	101.00	101	0
(20) 1946	745	+158	+23.70	124.70	124	+ 1
(21) 1947	735	+148	+22.20	123.20	130	- 7
(22) 1948	728	+141	+21.15	122.15	132	-10

[1] a = regression coefficient of housing on income. See Table 2.

ferent kinds of goods. Moreover there is no reason why the reduction for a particular good should be the same as would be expected from a static comparison of consumption at different levels of expenditure. But we do know that, to the extent that the cut for any one item is less than would be indicated by a static comparison, it must be greater for some other item.

The errors for food and housing during the depression years are fairly well explained by the cyclical factors. The postwar errors for food have already been explained in terms of the upward shift in demand for food during the war.

The situation with respect to clothing is different. We expected to obtain an over-estimate of clothing expenditure during the depression but to reach fairly accurate estimates for the 1920's. Instead we obtain fairly accurate estimates for the 1930's and for the post-World War II period, but expenditures during the 1920's were greatly under-estimated. An error distribution of that type cannot be explained by purely cyclical considerations. In the case of clothing there is either something radically wrong with our theory or we are faced with a real shift in demand.

After wasting a good deal of effort in a search for causes of a downward shift in demand for clothing we came to the conclusion that the apparent shift is the result of our methods of dealing with income effects. Farmers and industrial wage earners spend considerably less on clothing at any income level than clerical, professional, and business people. At the same time the latter groups have substantially higher incomes than farmers and wage earners. There is, therefore, a correlation between occupational status and income. When a regression is fitted to budget-study data for income and expenditures the regression coefficient is increased because of those correlations.

Suppose there are two groups in the economy with different expenditure patterns. Suppose the relations between clothing consumption and income are

$$C_1 = ay_1 + b_1 \text{ for one group and,}$$

$$C_2 = ay_2 + b_2 \text{ for the other.}$$

The average consumption at any income will be

$$C = ay + b_1P_1(y) + b_2[1 - P_1(y)]$$

where

P_1 = the proportion of group 1 families at income y.

Let

$$P_1 = \alpha_y + \beta$$

Then the budget study data will fit the relation

$$C = a + \alpha(b_1 - b_2)y + \beta(b_1 - b_2) + b_2$$

If a regression from a budget is used to estimate the effect of changes in income the estimate of the 'marginal propensity to consume clothing' will be $a + \alpha(b_1 - b_2)$. If the group with the higher clothing consumption is a relatively high income group α will be positive and b_1 will be greater than b_2. The slope of the income-clothing consumption relation will be over-estimated. Similar results will be obtained if it is assumed that the slopes instead of the constants differ between groups.

In 1941 real income per capita was the highest on record up to that time. When we estimated the consumption at other levels of income we applied the slopes of the regression obtained from the 1941 budget study to the difference in income between 1941 and the year in question. When we made estimates for the 1920's when incomes were smaller than in 1941 we under-estimated clothing consumption because we used too high a regression coefficient. Moreover, since income was rising during the 1920's, the over-estimate was greater the farther back we went.

On the other hand our estimates for the 1930's were reasonably accurate. We had expected them to be inaccurate because we thought that the fall in clothing consumption in the depression would be greater than a budget study would indicate. In other words, we expected that the marginal propensity to consume for clothing would be higher for cyclical changes in income than for 'secular' changes. But since we over-estimated the secular marginal propensity we came out with small errors in our estimates.

The results obtained from our estimates of aggregate consumption do not contradict our estimates of elasticities but neither do they support them. However, since other estimates made by the same method worked out fairly well, there seems to be no reason to discount our estimate for clothing.

The systematic deviations from our predictions have been explained by postulating four shifts in demand during the period under examination: cyclical shifts for food and housing; the postwar upward shift in demand for food; and the downward shift in demand for clothing due to the connection between clothing consumption and occupational status.

An additional test of our results can be made by comparing the errors in our estimates of the aggregates with the estimates of the shifts in demand obtained as a by-product of our method of estimating price elasticities. In making our estimates of price elasticities we fitted a regression to the data for each pair of years. Our demand function was of the form $C_1 = k\epsilon_1 p_1{}^a$ for the first year of each pair. For the second year we have $C_2 = k\epsilon_2 p_2{}^a$. We calculated a by taking the ratio

$$\frac{C_2}{C_1} = \frac{\epsilon_2}{\epsilon_1}\left(\frac{p_2}{p_1}\right)^a$$

TABLE 4

Estimates of Values of $\log \frac{\epsilon_2}{\epsilon_1}$ by Periods

Item and Period	$\log \frac{\epsilon_2}{\epsilon_1}$	σ	N	Lower Confidence Limit	Upper Confidence Limit	Antilogs $\frac{\epsilon_2}{\epsilon_1}$	Antilogs Lower Confidence Limit	Antilogs Upper Confidence Limit
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) Food								
1918-1933/6	+.0102	.0066	21	+.0033	+.0171	1.02	1.00	1.04
1918-1925/9	+.0207	.0124	6	+.0078	+.0336	1.05	1.02	1.08
1925/9-1933/6	+.0239	.0124	6	+.0110	+.0368	1.06	1.03	1.09
1933/6-1946/7	+.0058	.0136	5	−.0083	+.0199	1.00	.98	1.05
(2) Clothing								
1918-1933/6	−.2130	.0096	21	−.2230	−.2030	.61	.60	.63
1918-1925/9	−.0133	.0134	6	−.0272	+.0006	.97	.94	1.00
1925/9-1933/6	−.0625	.0134	6	−.0764	−.0486	.86	.84	.89
1933/6-1946/7	+.0336	.0197	5	+.0131	+.0541	1.08	1.03	1.13
(3) Housing								
1918-1933/6	+.0615	.0107	21	+.0504	+.0726	1.15	1.12	1.18
1918-1925/9	+.0421	.0202	6	+.0210	+.0630	1.10	1.05	1.16
1925/9-1933/6	+.0137	.0202	6	−.0073	+.0347	1.03	.98	1.08

and taking the logarithmic regression of $\frac{C_2}{C_1}$ on $\frac{p_2}{p_1}$. In such a regression $\log \frac{\epsilon_2}{\epsilon_1}$ appears as a constant term. When the constant term is positive an upward shift in demand from the second year to the first year is indicated. When it is negative a downward shift is indicated. In all cases a shift in demand means a shift in the price-quantity demand schedule with income held constant. The estimated values of $\log \frac{\epsilon_2}{\epsilon_1}$ and their confidence limits for each period are given in Table 4.

The term $\frac{\epsilon_2}{\epsilon_1}$ reflects two sorts of influences. On the one hand there are in any particular year a number of factors which may affect consumption in that particular year. These factors include short-run expectation factors, variables connected with the stocks of durable goods in the hands of consumers, and the asset position of consumers. In years like 1918 there was a certain amount of informal rationing of food and housing, while expenditures on clothing and miscellaneous items were probably unusually high. The factors just mentioned are more or less ephemeral in the sense that they operate in one particular year and are changed in the next year. The ratio $\frac{\epsilon_2}{\epsilon_1}$ reflects the effect of the differences in the influence of those factors as between the two years compared with one another. The ratio is also affected by any systematic lack of comparability between the samples. The value of $\frac{\epsilon_2}{\epsilon_1}$ also reflects the effect

of what are called trend factors, e.g. sustained changes in taste and in the effects of cyclical factors. The second type of factor will show up fairly well in aggregate data since it produces a non-random pattern in the errors. The first kind will not be so well reflected in the aggregate since only one year at a time is affected.

Let us turn now to Table 4. Column 6 shows the estimated value of $\frac{\epsilon_2}{\epsilon_1}$ while Columns 7 and 8 show its lower and upper confidence limits for a fiducial probability of .7.

The results for clothing show a very satisfactory agreement with the results obtained from comparing our estimates with aggregate consumption and with our explanation of the errors in that comparison. $\frac{\epsilon_2}{\epsilon_1}$ is definitely less than 1 for 1918-1935/6 and for 1925/7-1935/6. The major part of the decline is probably accounted for by the cyclical factor which should affect clothing rather strongly. In addition, since the income groups used were fairly high (for clerical and wage earners) there were probably higher proportions of wage earners included in the later periods. That, as was explained above, would produce an apparent downward shift in clothing demand.

In the case of housing the upward shift from 1918 to 1925/7 probably reflects the wartime housing shortage in 1918. The further shift in favor of housing indicated by values of $\frac{\epsilon_2}{\epsilon_1}$ above 1 for 1918-1935/6 and 1925/7-1935/6 reflects the cyclical factor.

For food the agreement is less satisfactory. The upward shift from 1918-1925/7 probably reflects the effects of wartime rationing. The upward shift from 1925/6-1935/6, however, contradicts the results of our aggregate comparisons. The figure for 1935/6-1947/8 does not show the rather strong upward shift indicated by the aggregates, although it does not contradict it. We can only suppose that some of the other factors affecting $\frac{\epsilon_2}{\epsilon_1}$ have accounted for these discrepancies.

In spite of those discrepancies we feel that comparison of the ϵ ratios with the other data tends to support the view that our method gives correct results.

It may be said that what we have done is to find the errors and then explain them away by postulating shifts in demand. That is not entirely true, however. The method under consideration was chosen just because we believed that important shifts in demand do occur. Moreover the constants in our regression coefficients indicated the occurrence of demand shifts as the logic of the method would lead us to expect. It has been shown elsewhere that cyclical shifts in the allocation of income between consumption and saving do take place. If they do, should not housing be one of the commodities most re-

sistant to change? With regard to the wartime shift in demand for food we can only say that it is reasonable psychologically. The explanation of the change in clothing demand when only income and price are taken into account is consistent with the facts so that there seems to be no reason to discount it.

On the whole it seems to the writers that we have done something more than 'explain away' the errors from our predictions. Indeed, it seems that the character of the errors is such as to justify our search for a method which takes shifts in demand explicitly into account.

VI. Conclusion

The numerical results have a certain usefulness in themselves. Of course elasticities of demand for such broad groups of commodities as food, clothing, and housing have to be applied with care. Nevertheless, our study shows that the elasticity of demand for food is a good deal higher than has been commonly supposed. That is obviously important when such problems as that of estimating the probable cost of the Brannan plan must be considered. (The elasticity given here applies, of course, to food at retail. Allowance for retail mark-ups has to be made before it can be applied to estimates of the demand for raw food.) Similarly the estimate for housing demand should be useful in estimating the magnitude of the housing shortage and the probable length of the housing boom.

However, it is obviously desirable to try to obtain estimates of demand elasticities for much finer groups of commodities than those used here.

Because of lack of data we may find it necessary to fall back on time series in further developments of our work. However, since we have elasticities for large groups it will be possible to eliminate some of the special difficulties of working with time series by using the elasticity for the group as a 'control total.'

Aside from the estimates of elasticities themselves the present study has some methodological significance. Our results tend to confirm the view that responses to cyclical changes in income are radically different from responses to secular changes. Consequently studies of demand parameters must be made in such a way as to avoid the use of data from depression periods when parameters of secular relations are the object of the study.

Second, it appears that more attention has to be given to the role of occupational and social class differences in consumption. In our study these factors appeared important only in the case of clothing. However, they are likely to take on greater importance for particular subgroups of the food group and for some of the items in the 'miscellaneous' category not dealt with here, for example medical care.

The 'shifts' in demand due to cyclical movements of income and to

occupational differences in taste are, in a sense, not genuine shifts. They merely result from our over-simplification of the problem. The same thing might be said of the wartime rise in food consumption. The wartime shift in demand reflects the influence of past consumption in the same way as do the cyclical adjustments. The difference is that in a depression consumers have to reduce consumption of everything, while in the postwar period they could keep up food consumption while still increasing consumption of other things.

Our position is thus a fairly optimistic one. We have not found any shifts in demand which could not be accounted for by including past consumption as a variable or by taking account of class differences in taste. Of course when we deal with individual commodities we shall have to introduce new products into the picture. But they are an observable factor. The hopeful thing is that we have not found any shifts in demand which have to be accounted for by introducing mysterious and arbitrary changes in taste.

MATHEMATICAL NOTE

The Effect of Shifts in Elasticities on Estimates of Elasticities

In this note we shall give the proof of the two propositions about errors introduced by shifts in elasticities mentioned in the text.

Let us consider first the question of the single period regressions. Suppose that in year one the true demand relationship is $y_{1j} = kp_{1j}^{a}$ and that in year two the true relationship is $y_{2j} = kp_{2j}^{a+\delta}$. (The error terms are neglected here since the ϵ's will merely introduce a constant into the relationship and the λ's will merely cause an additive error whose character has been discussed in the text.)

Our procedure is to take the regression of the ratio of the y's for the two periods on the corresponding ratio of the p's. We have:

$$\frac{y_{2j}}{y_{1j}} = \left(\frac{p_{2j}}{p_{1j}}\right)^{a} (p_{2j})^{\delta}$$

The ratio of the y's is thus equal to the ratio which would have resulted from the price change on the basis of the old elasticity times the adjustment required when we apply the increase in elasticity to the new price.

If we take the logarithmic regression of the ratio of the y's on the ratio of the p's we have:

$$R = \frac{\frac{1}{N}\Sigma\left(a\log\frac{p_{2j}}{p_{1j}} + \delta\log p_{2j}\right)\log\frac{p_{2j}}{p_{1j}} - \left[a\,\overline{\log\frac{p_{2j}}{p_{1j}}} + \delta\,\overline{\log p_{2j}}\right]\overline{\log p_{2j}}}{\frac{1}{N}\Sigma\log\frac{p_{2j}}{p_{1j}}^2 - \overline{\log\frac{p_{2j}}{p_{1j}}}^2}$$

$$= \frac{a + \delta\left[\frac{1}{N}\Sigma\left(\log p_{2j}\log\frac{p_{2j}}{p_{1j}}\right) - \overline{\log p_{2j}}\cdot\overline{\log\frac{p_{2j}}{p_{1j}}}\right]}{\frac{1}{N}\Sigma\log\frac{p_{2j}}{p_{1j}}^2 - \overline{\log\frac{p_{2j}}{p_{1j}}}^2}$$

The second term of the regression is, as was stated in the text, the logarithmic regression of the price in the second year on the percentage change in price between the two years multiplied by δ. If the initial prices are unrelated to the changes in prices then the regression in question will be 1. In that case our method gives an estimate of the elasticity in the second year.

Let us now turn to the problem of combining the observations from different periods. Suppose that the demand functions in successive periods are:

$$\begin{array}{rl} \text{Period (0)} & y_{oj} = kp_{oj}^{a} \\ (1) & y_{1j} = kp_{1j}^{a+\delta_1} \\ (2) & y_{2j} = kp_{2j}^{a+\delta_2} \\ (3) & y_{3j} = kp_{3j}^{a+\delta_3} \end{array}$$

Proceeding as before, our computations give us three sets of ratios:

$$\frac{y_{1j}}{y_{oj}} = \left(\frac{p_{1j}}{p_{oj}}\right)^a (p_{1j})^{\delta_1}$$

$$\frac{y_{2j}}{y_{1j}} = \left(\frac{p_{2j}}{p_{1j}}\right)^a \frac{p_{2j}^{\delta_2}}{p_{1j}^{\delta_1}}$$

$$\frac{y_{3j}}{y_{2j}} = \left(\frac{p_{3j}}{p_{1j}}\right)^a \frac{p_{3j}^{\delta_3}}{p_{2j}^{\delta_2}}$$

Let $\dfrac{y_{ij}}{y_{i+1j}} = R_{ij}$ and $\dfrac{p_{ij}}{p_{i+1j}} = \pi_{ij}$. Taking logarithms we have

$$\log R_{1j} = a \log \pi_{1j} + \delta_1 \log p_{1j}$$

$$\log R_{2j} = a \log \pi_{2j} + \delta_2 \log p_{2j} - \delta_1 \log p_{1j}$$

$$\log R_{3j} = a \log \pi_{3j} + \delta_3 \log p_{3j} - \delta_2 \log p_{2j}$$

Next we put the logs of both sides in terms of deviations from means. That is equivalent to dividing each R_{ij} by the geometric mean of the R's for period i.

Then

$$\log R'_{1j} = a \log \pi_{1j} + \delta_1 \log p_{1j} - [a \overline{\log \pi_{1j}} + \delta_1 \overline{\log p_{2j}}]$$

$$\log R'_{2j} = a \log \pi_{2j} + \delta_2 \log p_{2j} - \delta_1 \log p_{1j}$$
$$- [a \overline{\log \pi_{2j}} + \delta_2 \overline{\log p_{2j}} - \delta_1 \overline{\log p_{1j}}]$$

$$\log R'_{3j} = a \log \pi_{3j} + \delta_3 \log p_{3j} - \delta_2 \log p_{2j}$$
$$- [a \overline{\log \pi_{3j}} + \delta_3 \overline{\log p_{3j}} - \delta_2 \overline{\log p_{2j}}]$$

We now take the regression of the $\log R'_{ij}$ on $\log \pi_{ij} - \overline{\log \pi_{ij}}$ (the means for the π's being calculated separately for each period). This is equal to

$$\frac{\left\{\begin{array}{l} a + 1/N[\delta_1 \Sigma \log p'_{1j} \log \pi'_{1j} + \delta_2 \log p'_{2j} \log \pi'_{2j} - \delta_1 \log p'_{1j} \log \pi'_{2j} \\ \quad + \delta_3 \log p'_{3j} \log \pi'_{3j} - \delta_2 \log p'_{2j} \log \pi'_{3j}] \end{array}\right\}}{1/N\Sigma[\log \pi'_{1j}]^2 + \Sigma[\log \pi'_{2j}]^2 + \Sigma[\log \pi'_{3j}]^2}$$

By definition,

$$p'_{1j} = \frac{p'_{2j}}{\pi'_{2j}} \quad \text{and} \quad p'_{2j} = \frac{p'_{3j}}{\pi'_{3j}}$$

Thus the term:

$$\delta_2 \Sigma \log p'_{2j} \log \pi'_{2j} - \delta_1 \log p'_{1j} \log \pi'_{2j}$$

can be rewritten as:

$$(\delta_2 - \delta_1)\Sigma \log p'_{2j} \log \pi'_{2j} + \delta_1 \Sigma[\log \pi'_{2j}]^2$$

Similarly the term,

$$\delta_3 \log p'_{3j} \log \pi_{3j} - \delta_2 \log p_{2j} \log \pi'_{3j} =$$
$$(\delta_3 - \delta_2)\Sigma \log p'_{3j} \log \pi'_{3j} + \delta_2 \Sigma \log \pi'_{3j}$$

Thus we have

$$R = \frac{\left\{\begin{array}{l} a + \delta_1 \Sigma \log p'_{1j} \log \pi'_{1j} + (\delta_2 - \delta_1) \Sigma \log p'_{2j} \log \pi'_{2j} \\ \quad + (\delta_3 - \delta_2) \Sigma \log p'_{3j} \log \pi'_{3j} + \delta_1 \Sigma [\log \pi'_{2j}]^2 \\ \quad + \delta_2 \Sigma [\log \pi'_{3j}]^2 \end{array}\right\}}{1/N \Sigma [\log \pi'_{1j}]^2 + [\log \pi'_{2j}]^2 + [\log \pi'_{3j}]^2}$$

All the terms in the denominator except the last two involve cross products of changes in price on base year price and they are likely to be quite small. The last two terms are variances. It can be seen that if the variances of the price changes for all periods are of the same order of magnitude, the last two terms will raise the elasticity above a by $\delta_1 + \delta_2$.

APPENDICES

Appendix 1

SUMMARY OF 1939 CAPITAL STRUCTURE OF AMERICAN INDUSTRIES
(In Millions of 1939 Dollars)

	INDUSTRIES	OUTPUT (1)	CAPACITY (2)	FIXED CAPITAL STOCK: Undepreciated (3)	FIXED CAPITAL STOCK: Replacement cost (4)	FIXED CAPITAL STOCK: Depreciated (5)
1-8	Agriculture	9815.4	12250.0	16585.3		9959.3
9	Fishing	113.8	130.9	98.0		64.2
10	Flour and grist mill products	1250.3	1987.8	386.0		197.4
11	Canning and preserving	838.1	1178.8	352.6		176.3
12	Bread and bakery products	1458.0	2582.6	782.0		478.3
13	Sugar refining	557.8	934.4	312.9		177.1
14	Starch and glucose products	119.4	171.1	140.3		74.8
15	Alcoholic beverages	722.6	795.9	355.8		241.6
16	Nonalcoholic beverages	367.8	367.8	126.9		95.6
17	Tobacco manufactures	1322.2	1342.8	158.2		80.0
18	Slaughtering and meat packing	3003.0	3273.3	752.4		384.3
19	Manufactured dairy products	2093.8	2773.2	761.0		406.0
20	Edible fats and oils, n.e.c.	220.4	255.6		72.9	
21	Other food products	1218.1	2114.0		470.8	
22	Iron mining	150.9	309.5	489.6		300.4
23	Blast furnaces	550.8	898.5		1376.0	
24	Steel works and rolling mills	2790.4	4326.2		7782.0	
25	Iron and steel foundry products	463.7	995.1		863.8	
26	Shipbuilding	437.6	834.0	192.2		112.6
27	Firearms	17.7	21.6		21.4	
28	Munitions	100.1	174.5		348.9	
29	Agricultural machinery	422.6	626.9	292.2		144.9
30	Engines and turbines	135.1	135.1	66.7		30.9
31	Automobiles	4047.9	5419.6	1924.5		967.9

SUMMARY OF 1939 CAPITAL STRUCTURE OF AMERICAN INDUSTRIES
(Continued)

DOLLARS OF FIXED CAPITAL PER DOLLAR OF CAPACITY			REPLACEMENT REQUIREMENTS	TOTAL INVENTORIES HELD BY AND FOR THE INDUSTRY	DOLLARS OF INVENTORIES REQUIRED PER DOLLAR OF ANNUAL OUTPUT
Undepre-ciated (6)	Replace-ment cost (7)	Depre-ciated (8)	(9)	(10)	(11)
1.354		.813	1731.5	674.2	.069
.754		.491	10.9	17.0	.150
.194		.099	13.0	871.8	.697
.299		.150	16.2	118.7	.141
.303		.185	31.0	104.9	.072
.334		.190	9.5	165.1	.294
.820		.437	6.9	72.2	.605
.447		.303	13.1	158.6	.220
.345		.260	5.4	41.6	.113
.119		.060	6.9	502.0	.308
.200		.117	23.0	117.4	.039
.275		.147	49.6	54.4	.026
	.286		3.2	40.5	.184
	.222		22.1	126.7	.104
1.582		.970	23.7	0.3	.002
	1.531		47.6	161.5	.293
	1.798		272.5	535.9	.192
	.867		30.2	54.5	.118
.231		.135	6.6	58.7	.134
	.991		1.0	8.3	.470
	2.000.		14.9	16.5	.164
.466		.231	11.0	95.5	.226
.493		.228	2.8	33.3	.247
.355		.179	86.8	563.4	.139

SUMMARY OF 1939 CAPITAL STRUCTURE OF AMERICAN INDUSTRIES
(Continued)

		(1)	(2)	(3)	(4)
32	Aircraft	279.5	614.8	103.4	
33	Transportation equipment, n.e.c.	265.1	1732.9	486.8	
34	Industrial and household equipment, n.e.c.	2243.4	3560.0	1416.2	
35	Machine tools and metal-working equipment	451.7	645.3	323.6	
36	Merchandising and service machines	327.5	519.2	206.9	
37	Electrical equipment, n.e.c.	2130.6	2843.3	630.6	
38	Iron and steel, n.e.c.	2228.4	3909.5	1574.8	
39	Nonferrous metal mining	372.2	435.2		465.1
40	Smelting and refining of nonferrous metals	1127.4	1651.3		989.0
41	Aluminum products	206.9	206.9		123.6
42	Nonferrous metal manufactures and alloys	810.5	1526.3	478.6	
43	Nonmetallic mineral mining	394.4	604.0	660.9	
44	Nonmetallic mineral manufactures	1580.7	2100.6	2031.7	
45	Petroleum and natural gas	1679.0	1679.0	7233.3	
46	Petroleum refining	2461.1	3001.4	1978.8	
47	Anthracite coal	189.6	310.4	439.5	
48	Bituminous coal	733.5	1146.4	1570.9	
49	Coke and manufactured solid fuel	309.1	441.6		552.0
50	Manufactured gas	391.9	583.3	1468.1	
51	Communications	1523.7	1546.9	5513.6	
52	Electric public utilities	2445.5	3801.6	10154.1	
53	Chemicals	3745.0	5067.7	1879.6	
54	Lumber and timber	1435.9	2279.1	1226.0	
55	Furniture and other manufactures of wood	1173.4	1599.5	542.2	
56	Wood pulp, paper and paper products	2019.6	2531.9	1532.6	
57	Printing and publishing	2646.0	3400.5	1451.7	
58	Cotton yarn and cloth	1396.9	1942.0		1601.0
59	Silk and rayon products	619.0	827.9		579.8
60	Woolen and worsted manufactures	896.2	1956.3	805.0	

SUMMARY OF 1939 CAPITAL STRUCTURE OF AMERICAN INDUSTRIES
(Continued)

(5)	(6)	(7)	(8)	(9)	(10)	(11)
80.2	.168		.130	3.7	99.3	.356
285.6	.281		.165	17.7	75.4	.284
688.9	.398		.194	51.0	415.1	.185
158.5	.501		.246	12.0	75.3	.167
100.6	.398		.194	8.2	70.4	.215
332.7	.222		.117	23.4	386.6	.181
744.4	.403		.190	73.1	385.7	.173
		1.069		20.9	1.4	.004
		.599		35.7	188.5	.167
		.597		5.5	40.5	.196
282.7	.314		.185	17.0	135.1	.167
435.7	1.093		.721	26.4	1.2	.003
1001.1	.967		.477	81.3	172.1	.109
3458.8	4.308		2.060	362.2	18.3	.011
962.2	.659		.321	56.5	286.0	.079
236.6	1.416		.762	18.3	1.0	.005
878.8	1.370		.767	71.4	2.5	.004
		1.250		21.9	46.2	.099
1087.6	2.517		1.865	39.9	31.2	.080
3913.5	3.564		2.530	209.0	0.5	.003
8852.0	2.671		2.329	319.8	180.6	.070
1256.3	.371		.248	85.9	733.5	.196
764.0	.538		.335	39.4	193.5	.135
268.7	.339		.168	17.6	210.4	.179
832.5	.605		.329	55.7	278.7	.138
684.5	.427		.201	58.5	223.8	.085
		.825		50.7	541.6	.388
		.700		19.9	80.1	.129
392.4	.412		.201	27.9	195.8	.219

SUMMARY OF 1939 CAPITAL STRUCTURE OF AMERICAN INDUSTRIES
(Continued)

		(1)	(2)	(3)	(4)
61	Clothing	3824.5	4153.1	291.1	
62	Other textile products	707.2	810.0	400.9	
63	Leather	346.4	475.9		96.6
64	Leather shoes	864.1	1120.7	199.0	
65	Leather products, n.e.c.	177.0	227.4	24.9	
66	Rubber products	903.3	1273.5	472.6	
67	Industries, n.e.c.	1749.1	1912.4	2873.1	
68 ,69	Construction	10091.1	19002.3		1437.7
70	Transportation, n.e.c.	1105.0	1615.1	4851.3	
71	Coastwise and inland water transportation	254.0	274.0	428.0	
72	Transoceanic water transportation	205.7	248.9		641.5
73	Steam railroad transportation	4449.0	7981.3	25750.0	
74	Trade	17121.0	19877.0	10602.4	
76	Banking	3747.6	7495.0	2630.7	
77	Insurance	3370.7	3910.0	391.0	
78	Other business services	328.1	397.1	77.5	
79	Advertising	1871.8	2264.8	441.9	
80	Services allied to transportation	172.1	243.4	73.0	
81	Automobile repair	432.3	432.3	204.6	
82	Other repair	301.8	301.8	142.7	
83	Rental agencies	64.4	#	#	#
85	Home renting	9914.0	9914.0		70577.8†
86	Hotels, etc.	527.6	850.0	1253.8	
87	Laundry, etc.	837.6	879.5	632.1	
88	Personal services	847.3	953.2	613.9	
89, 90 91	Professional entertainment, motion picture theaters, amusement places	992.8	1157.6	760.5	

#No data available †Market value

SUMMARY OF 1939 CAPITAL STRUCTURE OF AMERICAN INDUSTRIES
(Continued)

(5)	(6)	(7)	(8)	(9)	(10)	(11)
137.9	.070		.033	13.9	469.2	.123
192.1	.497		.237	13.0	138.3	.195
		.203		2.2	83.2	.240
90.0	.179		.081	8.0	115.2	.133
11.9	.110		.052	1.0	24.1	.136
227.1	.371		.179	21.2	101.4	.112
1453.3	1.502		.760	114.7	230.1	.131
		.076		164.9	869.9	.086
3410.5	3.004		2.112	181.6	51.9	.047
218.7	1.562		.798	10.7	7.0	.028
		2.577		19.5	2.0	.010
	3.226			762.4	378.1	.084
5764.3	.534		.290	464.8	118.9	.007
2345.9	.351		.313	62.4	0.5	*
369.1	.100		.094	9.3	0.6	*
40.9	.195		.103	0.1	1.2	.004
233.3	.195		.103	9.0	127.6	.068
38.9	.300		.160	1.6	0.2	.001
130.9	.473		.303	8.9	2.8	.006
91.4	.473		.303	6.0	6.6	.022
#	#	#	#		0.1	.001
		7.119†		1764.4	0.8	*
834.9	1.475		.982	37.0	20.1	.038
302.4	.719		.344	37.8	47.5	.057
308.7	.644		.324	54.0	40.2	.047
417.9	.657		.361	56.4	85.4	.086

*Less than .0005 #No data available

SUMMARY OF 1939 CAPITAL STRUCTURE OF AMERICAN INDUSTRIES (Continued)

MAJOR CATEGORIES

	(1)	(2)	(3)	(4)
Agriculture and fishing (1-9)	9929.2	12380.9	16683.3	
Manufacturing (10-21, 23-38, 40-42, 44, 46, 49, 53-67)	59062.9	83539.8	27533.8	14877.8
Mining (22, 39, 43, 47, 48, 45)	3519.6	4484.5	10394.2	465.1
Construction (68, 69)	10091.1	19002.3		1434.7
Public utilities (50-52, 70-73)	10374.8	16051.1	48165.1	641.5
Total Services	40529.1	48675.7	17824.1	70577.8†
Trade (74)	17121.0	19877.0	10602.4	
Home Renting (85)	9914.0	9914.0		70577.8†
Other Services (76-83, 86-91)	13494.1	18884.7	7221.7	
TOTAL	133506.7	184134.3	120600.5	87996.9

† Market value

SUMMARY OF 1939 CAPITAL STRUCTURE OF AMERICAN INDUSTRIES
(Continued)

(5)	(6)	(7)	(8)	(9)	(10)	(11)
10023.5	1.348		.810	1742.4	691.2	.070
14515.3	.330	.178	.174	1639.9	9896.6	.168
5310.3	2.318	.104	1.184	522.9	24.7	.007
		.076		164.9	869.9	.086
17482.3	3.001	.040	1.089	1542.9	660.3	.064
10788.6	.366	1.450	.222	2511.7	452.4	.011
5764.3	.533		.290	464.8	118.9	.007
		7.119†		1764.4	0.8	*
5114.3	.382		.271	282.5	332.7	.025
58120.0	.655	.478	.316	8124.7	12595.1	.094

†Market value *Less than .0005

Appendix 2

Appendix 2

COMPARISONS OF INPUT-OUTPUT CLASSIFICATIONS

I

THE FOLD-IN TABLE 1 compares eight classifications which have been used in input-output analysis. Other classifications that have been used [1] can be derived straightforwardly from the classifications given. The differences in classifications which arise because industries in one classification are aggregates of industries in another classification will be apparent to anyone examining the table.[2] However, there are conditions affecting the comparability of classifications which are not so readily apparent. The first of these is the distinction between industries whose inputs are the elements of final demand—the bill-of-goods industries—and all other industries, the inputs of which will be included in the coefficient matrix. Classifications containing identical industries may differ in the assignment of industries to the bill of goods. Bill-of-goods industries are those for which no structural relation between inputs is assumed either for theoretical reasons or simply because no adequate data to determine the input structure for a particular industry are available. The first six classifications were defined in the light of data available since 1939, the seventh and eighth in the light of 1929 data. The sixth classification directly derived from the fifth was defined to be as nearly comparable as possible to the seventh, which is an aggregation of the eighth. The bill of goods for the sixth and seventh classifications are:

SIXTH	SEVENTH
1. Rubber	1. Rubber
2. Industries, n.e.c.	2. Industries, n.e.c.
3. Construction	3. Construction
4. Foreign trade	4. Foreign trade
5. Undistributed	5. Undistributed, including government
6. Households	6. Households
7. Government	
8. Investment (net)	

[1] For instance, the classification used in Leontief, W. W., 'Exports, Imports, Domestic Output, and Employment,' *Quarterly Journal of Economics*, February 1946, is a straightforward aggregation of the 96-industry classification. The derivation of the classification used in the article from the 96-industry classification is given on p. 192, ibid.

[2] Classifications five and six are aggregations of three Seven is an aggregation of eight.

In the case of the sixth classification, rubber, industries, n.e.c., construction, and undistributed have been put in the bill of goods simply for comparability with the seventh classification. Enough is known about the input structures of the first three to include the industries in the coefficient matrix.[3] This was not the case when the seventh classification was set up.

There are two other differences in the bill of goods: (1) in the seventh classification government demand is included in undistributed, implying that sufficient data were not available for the itemization of government purchases to separate government demand from other demands which had to be computed as a residual; and (2) in the sixth classification there is an industry for investment (net). If a classification includes no industry for net investment, it means that all technical coefficients are gross in the sense that they reflect not only the use of inputs consumed in current production, but inputs used for increasing capital stock as well. Establishment of an industry for net investment makes it possible to distribute inputs for increasing capital stock[4] separately from inputs used in current production; these latter include, of course, inputs for maintenance and replacement of capital stock.[5]

In the earliest input-output analysis for which the seventh and eighth classifications were used, undistributed was entered in the bill of goods, while in the later analysis undistributed has been included in the coefficient matrix. Undistributed is a residual obtained by subtracting from the total output of each industry the sum of the known distributions of the output of the industry to other industries. The better the empirical data, the smaller the undistributed items become. Meanwhile, the existence of undistributed detracts from the results that can be secured with input-output analysis. If, as was formerly done undistributed is included as a bill-of-goods item, the bill of goods, the data on the basis of which outputs are predicted, loses precision. If undistributed is included in the coefficient matrix, the assumption is made that the undistributed items originating in each industry bear a constant proportion to each other. Though this assumption is not justified theoretically, to include undistributed in the coefficient matrix is the better alternative, since it is impossible to make a meaningful estimate of undistributed to be included in a bill of goods used for predicting outputs.

[3] The alternatives for handling undistributed will be discussed below.

[4] Capital stock may be defined in such a way as to include inventories, and either both inventory and plant investment can be included in the same investment industry, or separate industries defined for each.

[5] For dynamic analysis, in which capital growth is taken into account, another set of technical coefficients (or coefficient matrix) is required. The coefficient matrix used in static analysis relates consumption of inputs, including capital flows to current output. The coefficients of the second matrix required for dynamic analysis relate changes in the stocks of capital (inventories, plant, and equipment) to changes in the levels of outputs.

Classifications differ in the treatment of foreign countries. Although exports are in all classifications included in the bill of goods, imports are handled differently in the seventh and eighth classifications than in the first six. In the first six classifications an imported commodity not domestically produced (tea, coffee, tin, silk, etc.) is entered in the column of the consuming industry (i.e. the industry where it first enters domestic production). An imported commodity with a domestically produced counterpart is included, however, as part of the output of the industry producing the domestic counterpart. The former enters only as a material cost and not as an addition to output value, while the latter increases the output value of the industry producing the domestic counterpart.

In the earliest analyses (seventh and eighth classifications) all imports were treated as though they were noncompetitive. Hence the output predictions excluded the value of imports. In later analyses output predictions include the value of competitive imports.

Transportation and trade are treated in input-output analysis as process service industries. This means that instead of the products of other industries flowing through the transportation and trade industries, all other industries are thought of as purchasing transport and distribution services. The total outputs of the transportation and trade industries are equal to their operating costs plus net revenues. The service transportation, for example, is distributed to *commodities* simply by assigning to each commodity the freight revenue charged for the transporting of that commodity. The total of such charges is equal to the operating costs and net revenues of the transportation industries (neglecting passenger service).

However, it is necessary to decide whether the industry producing a particular commodity or industry consuming it should have the transportation (and trade) charge for the commodity incorporated in its input structure. In the earlier analyses these charges were incorporated in the input structure of the producing industry. In the BLS classification they are included in the input structure of the purchasing industry.

When the distribution costs are charged to the producing industry they become part of the value of output of that industry, and all the input ratios for the industry will be based on an output value including these charges. If distribution costs are variable from year to year, input ratios from a purchasers' value matrix will be unstable. If, on the other hand, distribution costs are charged to the consuming industry, input ratios from the resulting producers' value matrix will be free from this source of instability.

Note that these alternatives for treating distribution costs are formally the same as the alternatives for treating imports.

The treatment of secondary products is another source of lack of comparability between classifications. A secondary product exists whenever a commodity is produced in more than one industry. The commodity is a secondary product to all industries producing it except for the industry in which it is classified. The problem arises because basic data are collected on an establishment basis. Establishments in a great many instances produce an array of products not all of which fall in the same industry as defined by the classification scheme. The first alternative for handling secondary products is to aggregate industries in such a way as to eliminate secondary products. This is an out-of-the-frying-pan-into-the-fire method, however, because the price of eliminating secondary products in this way is increasing the product-mix of the industries. In the input-output classifications identifying 50 or fewer industries, the level of aggregation is sufficiently great so that secondary products are not significant.

In both the 96-industry and 192-industry classifications several methods have been used in dealing with secondary products. To the extent that the alternative methods have not been used uniformly in both classifications, the classifications are not immediately comparable.

The correct solution to the secondary products problem from the point of view of the mathematical criteria for industry aggregation is to abandon the establishment as the basic unit in favor of the commodity. This solution is not a practical one at this time because the establishment is the basic data-collecting unit from which input-output statistics are obtained in the United States.

The practical alternatives (in addition to grosser aggregation of industries, discussed above) are charging products directly to the user from the industry in which they are produced, ignoring the distinction between primary and secondary products, and charging products from the producing industry to the industry where they are primary and then distributing to the using industry. Neither of these alternatives is really satisfactory. If the first alternative is used, input ratios lose precision because of product-mix. If the second is used, the value of secondary products is counted twice, and the input ratios of the primary industry are based on an output value which includes secondary products.

Finally, there is a residuum of incomparability in classifications based on data of different years because of the changes which take place in the economy over time and the reflection of these changes in the data. In addition, as the use of economic data increases the data become more plentiful and of better quality. The extent of these changes will be apparent to anyone who examines the *Census of Manufactures* volumes for 1919, 1929, 1939, and 1947.

Table 2, which compares the BLS 50- and 192-industry classification for 1947, is an example of the most detailed information now available.

II

In this volume, various classifications have been used. In Chapter 2, the classification used is number seven in Table 1.

For Chapter 6, dealing with capital structure, a simple consolidation of the first classification is used. Industries 1 through 8 are combined in a single industry. Industries 76 through 82, and 86 through 91 are combined in a single industry; and the bill-of-goods industries, 75, 92, and 94, are not included since no coefficients are calculated for them.

The classification used in Chapter 5 is classification three with one change. Households have been included in the coefficient matrix rather than in the bill of goods.[6]

Table 1 is a fold-in table in the pocket in the back of the book. Table 2 follows on p. 502.

[6] Thus, households is Industry 39 in Chapter 5.

TABLE 2

COMPARISON OF CLASSIFICATION FOR THE BUREAU OF LABOR STATISTICS 1947 INPUT-OUTPUT STUDY

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification (with detailed listing of industries and SIC code number[1])
1	Agriculture and Fisheries	1	Meat Animals and Products[2] meat animals, other livestock, and products
		2	Poultry and Eggs
		3	Dairy Products
		4	Food Grains and Field Crops food grains, field crops
		5	Cotton
		6	Tobacco
		7	Oil Bearing Crops
		8	Vegetables and Fruit vegetables, fruit

[1] The listing of industries for the 192-industry classification includes in small print the industries of the detailed classification of industries for the 1947 BLS study. An industry title followed by a number or series of numbers in parentheses indicates an industry in the detailed classification. The relation of a detailed industry to the SIC classification is indicated by the numbers in parentheses. If this is a single four-digit number, it means that the detailed classification industry is equivalent to a four-digit SIC industry. These, it will be remembered, are the most detailed SIC industries. If the parentheses enclose a three-digit number or more than one four-digit number, it indicates that the industry of the BLS detailed classification is an aggregate of SIC four-digit industries. The separate SIC four-digit industries included can be found by referring to the Standard Industrial Classification.

[2] Industries one through nine are not individually comparable to SIC industries. The SIC code numbers for the group of industries are 01, 071, 0723, 0729, 08.

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
	plus offices of veternarians and animal hospitals (0722)	9	All Other Agricultural tree nuts, legumes and grass seeds, sugar and sirup crops, miscellaneous crops, forest products, greenhouse and nursery products, agricultural services
	minus hunting and trapping and game propagation (0741)	10	Fisheries, Hunting, and Trapping fisheries, hunting, and trapping
2	Food and Kindred Products plus synthetic sausage casings (part 2014)	21	Meat Packing and Wholesale Poultry meat packing (2011-13, part 2014), wholesale poultry (2015)
		22	Dairy Products dairy products (2021-25)
		23	Canning, Preserving, and Freezing canned seafood (2031), cured fish (2032), canning and preserving food (2033), dehydrated fruits and vegetables (2034), pickles and sauces (2035), frozen food (2037)
		24	Grain Mill Products flour and meal (2041), prepared animal feeds (2042), cereal preparations (2043), rice cleaning and polishing (2044), blended and prepared flour (2045)
	minus retail bakeries (5462)	25	Bakery Products bread and other bakery products (2051), biscuits, crackers, and pretzels (2052), retail bakeries (5462)
		26	Miscellaneous Food Products confectionery products (2071) chocolate and cocoa products (2072), chewing gum (2073), bottled soft drinks (2081), liquid, frozen, and dried eggs (part 2099),

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
			leavening compounds (2091), shortening and cooking oils (2092), oleomargarine (2093), corn products (2094), flavorings (2095), vinegar and cider (2096), manufactured ice (2097), macaroni and spaghetti (2098), food preparations, n.e.c. (part 2099)
		27	Sugar raw cane sugar (2061), cane sugar refining (2062), beet sugar (2063)
		28	Alcoholic Beverages malt liquors (2082), malt (2083), wines and brandy (2084), distilled liquors (2085)
3	Tobacco Manufactures	29	Tobacco Manufactures cigarettes (2111), cigars (2121), chewing and smoking tobacco (2131), tobacco stemming and redrying (2141)
4	Textile Mill Products	30	Spinning, Weaving, and Dyeing[3] woolen and worsted manufactures, (2211, 2212, 2213, 2216), cotton and rayon textiles (2222-25, 2233-34, 2241, 2261)
	plus hard-surface floor coverings (2274), coated fabrics (2295)	31	Special Textile Products wool carpets and rugs (2271), carpets and rugs, n.e.c. (2273), felt goods, n.e.c. (2291), lace goods (2292), paddings and upholstery filling (2293), processed textile waste (2294), textile goods, n.e.c. (2299)

[3] The detailed BLS classification is based on census industries which are for the most part identical with SIC industries. The differences in classification are so great, however, for the spinning, weaving, and dyeing industries that the census classification is used in the body of the table for the sake of clarity and the respective SIC industries indicated in the footnote.

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
		32	Jute, Linen, Cordage, and Twine jute, linen, cordage, and twine (2296-98)
5	Apparel	33	Canvas Products canvas products (2394)
	plus hunting and trapping and game propagation (0741), minus furs dressed and dyed (3992)	34	Apparel apparel (225, 228, 231-38, 2396-98, 2395)
	minus hard-surface floor coverings (2274), coated fabrics (2295)	35	House Furnishings and Other Non-apparel hard-surface floor coverings (2274), coated fabrics (2295), curtains and draperies (2391), house furnishings, n.e.c. (2392), textile bags (2393), fabricated textile products, n.e.c. (2399)

3

SIC Code	Title (SIC and Census)		Census Code
2211	Scouring and combing plants		2211
part 2221	Yarn mills wool, except carpet		2212
2232	Woolen and worsted fabrics		2213
2262	Finishing wool textiles		2216
	(SIC)	(Census)	
part 2221	Yarn mills	Yarn mills, cotton system	2224
		Yarn mills, silk system	2225
		Yarn mills, wool system except carpet	2212
2231	Broad-woven fabric mills (cotton, silk, and synthetic fiber)	Cotton broad-woven fabrics*	2233
		Rayon and related broad-woven fabrics	2234

* Including tire cord, not woven, part SIC 2221

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
6	Lumber and Wood Products	36	Logging logging (2411)
		37	Sawmills, Planing, and Veneer Mills sawmills and planing mills (2421), veneer mills (2422), shingle mills (2423), cooperage stock mills (2424), excelsior mills (2425), wood preserving (2491)
		38	Plywood plywood plants (2432)
		39	Fabricated Wood Products millwork plants (2431), prefabricated wood products (2433), lasts and related products (2492), mirror and picture frames (2493), wood products, n.e.c. (2499)
		40	Wooden Containers and Cooperage wooden containers (2441-44), cooperage (2445)
7	Furniture and Fixtures	41	Wood Furniture wood household furniture (2511-13, 2519), wood office furniture (2521)
		42	Metal Furniture metal house furniture (2514), mattresses and bedsprings (2515), metal office furniture (2522), public building furniture (2531), professional furniture (2532), restaurant furniture (2591)
		43	Partitions, Screens, Shades, etc. partitions and fixtures, n.e.c. (2541, 2599), window and door screens (2561), window shades (2562), venetian blinds (2563)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
8	Paper and Allied Products	44	Pulp Mills pulp mills (2611)
		45	Paper and Paperboard Mills paper and paperboard (2612-13)
		46	Converted Paper Products paper products (264-69)
9	Printing and Publishing	47	Printing, Publishing, and Allied Products printing and publishing (271-79, 7331-32, 7351)
10	Chemicals	48	Industrial Inorganic Chemicals alkalies and chlorine (2812), inorganic chemicals (2811, 2819)
		49	Industrial Organic Chemicals organic chemicals (2822, 2829)
		50	Plastics Materials plastics materials (2823)
		51	Synthetic Rubber synthetic rubber (2824)
	minus synthetic sausage casings (part 2014)	52	Synthetic Fiber synthetic fiber (2825, part 2014)
	minus fireworks and pyrotechnics (3985)	53	Explosives and Fireworks explosives (2826), fireworks and pyrotechnics (3985)
		54	Drugs and Medicines drugs and medicines (2831-34)
		55	Soap and Related Products soap and related products (2841-43)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
		56	Paints and Allied Products paints, varnish, etc. (2851, 2853), inorganic color pigments (2852)
		57	Gum and Wood Chemicals hardwood distillation (2861), softwood distillation and gum naval stores (2862, 2863), natural dyeing and tanning materials (2864, 2865)
		58	Fertilizers fertilizers (2871-72)
		59	Vegetable Oils cottonseed oil mills (2881), linseed oil mills (2882), soybean oil mills (2883), vegetable oil mills, n.e.c. (2884)
		60	Animal Oils marine animal oils (2885), grease and tallow (2886), fatty acids (2887), animal oils, n.e.c. (2889)
		61	Miscellaneous Chemical Industries printing ink (2891), essential oils (2892), toilet preparations (2893, glue and gelatin (2894), carbon black (2895), compressed and liquified gases (2896), insecticides (2897), salt (2898), chemical products, n.e.c. (2899)
11	Products of Petroleum and Coal	17	Crude Petroleum and Natural Gas crude petroleum and natural gas (1312-15)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
		62	Petroleum Products petroleum refining (2911, 2992, 2999)
		63	Coke and Products coke and by-products (2821, 2931-32), fuel briquets (2991)
		64	Paving and Roofing Materials paving mixtures (2951), roofing felts and coatings (2952)
12	Rubber Products	65	Tires and Inner Tubes tires and inner tubes (3011)
		66	Miscellaneous Rubber Products rubber footwear (3021), reclaimed rubber (3031), rubber industries, n.e.c. (3099)
13	Leather Products	67	Leather Tanning and Finishing leather tanning and finishing (3111)
		68	Other Leather Products industrial leather belting (3121), leather gloves (3150), luggage (3161), handbags and purses (3171), small leather goods (3172), saddlery, harnesses, and whips (3192), leather goods, n.e.c. (3199)
		69	Footwear (Except Rubber) footwear cut stock (3131), footwear (3141), houseslippers (3142)
14	Stone, Clay, and Glass Products	18	Stone, Sand, Clay, and Abrasives dimension stone (141), crushed and broken stone (1423-29), crushed and broken limestone (1422), sand and gravel (1441), clay, ceramic, and refractory materials (145), natural abrasives (1462, 1469), cut-stone and stone products (3281)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
		19	Sulfur sulfur (1477)
		20	Other Nonmetallic Minerals chemical and other fertilizer mineral mining (1472, 1474, 1479), fluorspar (1473), phosphate rock (1475), rock salt (1476), miscellaneous nonmetallic minerals (149)
		70	Glass flat glass and glass products (3211, 3231), glass containers (3221), pressed and blown glassware (3229)
		71	Cement cement, hydraulic (3241)
		72	Structural Clay Products structural clay products (3251, 3253-55, 3259)
		73	Pottery and Related Products pottery and related products (3261-65, 3269)
		74	Concrete and Plaster Products concrete products (3271), gypsum products (3272), lime (3274), mineral wool (3275)
		75	Abrasive Products abrasive products (3291)
		76	Asbestos Products asbestos products (3292), gaskets and insulations (3293)
		77	Other Miscellaneous Nonmetallic Minerals graphite and statuary goods (3294, 3296, 3298), minerals, ground or treated (3295), nonclay refactories (3297)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
15	Iron and Steel	11	Iron Ore Mining iron ore mining (1011)
		78	Blast Furnaces blast furnaces (3311, 3313)
		79	Steel Works and Rolling Mills steel works and rolling mills (3312, 3393, part 3392, 3399)
		80	Iron Foundries iron foundries (3321-22)
		81	Steel Foundries steel foundries (3323)
		92	Iron and Steel Forgings iron and steel forgings (3391)
16	Nonferrous Metals	12	Copper Mining copper mining (1021)
		13	Lead and Zinc Mining lead and zinc mining (1032-34)
		14	Bauxite Mining bauxite mining (1051)
	minus metal mining contract services (1081)	15	Other Metal Mining other metal mining (1042-44, 1062-64, 1069, 1081, 1092-94, 1099)
		82	Primary Copper primary copper (3331)
		83	Copper Rolling and Drawing copper rolling and drawing (3351, part 3392)
		84	Primary Lead primary lead (3332)
		85	Primary Zinc primary zinc (3333)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
		86	Primary Metals, n.e.c. primary metals, n.e.c. (3335, 3339)
		87	Nonferrous Metal Rolling, n.e.c. nonferrous metal rolling, n.e.c. (3359)
		88	Primary Aluminum primary aluminum (3334)
		89	Aluminum Rolling and Drawing aluminum rolling and drawing (3352)
		90	Secondary Nonferrous Metals secondary nonferrous metals (3341)
		91	Nonferrous Foundries nonferrous foundries (3361)
17	Plumbing and Heating Supplies minus vitreous -enameled products (3461)	97	Metal Plumbing and Vitreous Fixtures metal plumbing fixtures (3431), vitreous-enameled products (3461)
		98	Heating Equipment oil burners (3432), heating and cooking apparatus (3439)
18	Fabricated Structural Metal Products	99	Structural Metal Products structural metal products (3441), metal doors, sash, etc., (3442), sheet metal work (3444)
	minus fabricated pipe (3592)	100	Boiler Shop Products and Pipe Bending boiler shop poducts (3443), fabricated pipe (3592)
19	Other Fabricated Metal Products	93	Tin Cans and Other Tinware tin cans and other tinware (3411)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
		94	Cutlery cutlery (3421)
		95	Tools and General Hardware edge tools (3422), hand tools, n.e.c. (3423), files (3424), hand saws and blades (3425)
		96	Hardware, n.e.c. hardware, n.e.c. (3429)
		101	Metal Stampings metal stampings (3462-63)
	plus vitreous-enameled products (3461)	102	Metal Coating and Engraving enameling and lacquering (3465), galvanizing (3466), engraving on metal (3467), plating and polishing (3468)
		103	Lighting Fixtures lighting fixtures (3471)
		104	Fabricated Wire Products nails and spikes (3481), wirework, n.e.c. (3489)
		105	Metal Barrels, Drums, etc. metal barrels, drums, etc. (3491)
		106	Tubes and Foils collapsible tubes (3496), metal foil (3497)
		107	Miscellaneous Fabricated Metal Products safes and vaults (3492), fabricated metal products (3499)
		108	Steel Springs steel springs (3493)
		109	Nuts, Bolts, and Screw Machine Products bolts, nuts, washers, etc. (3494) screw machine products (3495)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
20	Agricultural, Mining, and Construction Machinery minus industrial trucks (3565)	112	Farm and Industrial Trucks tractors (3521), industrial trucks (3565)
		113	Farm Equipment farm machinery (3522)
		114	Construction and Mining Machinery construction and mining machinery (3531)
		115	Oil Field Machinery and Tools oil field machinery and tools (3532)
21	Metal Working Machinery	116	Machine Tools and Metal Working Machinery machine tools (3541), metal woking machinery (3542)
		117	Cutting Tools, Jigs, and Fixtures cutting tools, jigs, and fixtures (3543)
22	Other Machinery (Except Electric)	110	Steam Engines and Turbines steam engines and turbines (3511)
		111	Internal Combustion Engines internal combustion engines (3519)
		118	Special Industrial Machinery food-products machinery (3551), textile machinery (3552), woodworking machinery (3553), paper-industries machinery (3554), printing-trades machinery (3555), special industry machinery (3559)
		119	Pumps and Compressors pumps and compressors (3561)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	192-Industry Classification
		120	Elevators and Conveyors elevators and escalators (3562), conveyors (3563)
		121	Blowers and Fans blowers and fans (3564)
		122	Power Transmission Equipment power transmission equipment (3566)
	plus industrial trucks (3565)	123	Industrial Machinery, n.e.c. industrial furnaces and ovens (3567), mechanical stokers (3568), general industrial machinery (3569)
	minus beauty and barber shop equipment (3991), soda fountain and bar equipment (3997)	124	Commercial Machines and Equipment laundry machinery (3582), measuring and dispensing pumps (3586), computing machines and cash registers (3571), typewriters (3572), scales and balances (3576), office and store machines (3579), beauty and barber shop equipment (3991), soda fountain and bar equipment (3997), vending, amusement, and other coin-operated machines (3575)
		125	Refrigeration Equipment refrigeration equipment (3585)
	plus fabricated pipe and fittings (3592)	126	Valves and Fittings valves and fittings (3591)
		127	Ball and Roller Bearings ball and roller bearings (3593)
		128	Machine Shops machine shops (3599)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
	minus electrical appliances (3621)	135	Electrical Appliances domestic laundry equipment (3581), sewing machines (3583), vacuum cleaners (3584), service and household machines (3589), electrical appliances (3621)
23	Motors and Generators	131	Motors and Generators motors and generators (3614)
24	Radios	139	Radio and Related Products radio and related products (3661)
25	Other Electrical Machinery	129	Wiring Devices and Graphite Products wiring devices (3611), carbon and graphite products (3612)
		130	Electrical Measuring Instruments electrical measuring instruments (3613)
		132	Transformers transformers (3615)
		133	Electrical Control Apparatus electrical control apparatus (3616)
		134	Electrical Welding Apparatus electrical welding apparatus (3617), electrical industrial apparatus (3619)
		136	Insulated Wire and Cable insulated wire and cable (3631, part 3392)
		137	Engine Electrical Equipment engine electrical equipment (3641)
	plus electrical appliances (3621)	138	Electric Lamps electric lamps (3651), electric products, n.e.c. (3699)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
	plus phonograph records (3663)	140	Tubes tubes (3662)
		141	Communication Equipment telephone and telegraph equipment (3664), communications equipment (3669)
		142	Storage Batteries storage batteries (3691)
		143	Primary Batteries primary batteries (3692)
		144	X-Ray Apparatus x-ray apparatus (3693)
26	Motor Vehicles	145	Motor Vehicles motor vehicles (3711-14)
		146	Truck Trailers truck trailers (3715)
		147	Automobile Trailers automobile trailers (3716)
27	Other Transportation Equipment	148	Aircraft and Parts aircraft (3721), aircraft engines (3722), aircraft propellers (3723), aircraft equipment, n.e.c. (3729)
		149	Ships and Boats shipbuilding and repairing (3731), boat building and repairing (3732)
		150	Locomotives locomotives (3741)
		151	Railroad Equipment railroad equipment (3742)
		152	Motorcycles and Bicycles motorcycles and bicycles (3751), transportation equipment, n.e.c. (3799)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
28	Professional and Scientific Equipment	153	Instruments, etc. scientific instruments (3811), mechanical measuring instruments (3821)
		154	Optical, Ophthalmic, and Photo Equipment optical instruments and lenses (3831), ophthalmic goods (3851), photographic equipment (3861)
		155	Medical and Dental Instruments and Supplies surgical and medical instruments (3841), surgical appliances (3842), dental equipment and supplies (3843)
		156	Watches and Clocks watches and clocks (3871), watch cases (3872)
29	Miscellaneous Manufacturing	157	Jewelry and Silverware jewelry, precious metals (3911), jewelers' findings (3912), lapidary work (3913), silverware and plated ware (3914), costume jewelry (3961)
		158	Musical Instruments and Parts pianos (3931), organs (3932), piano and organ parts (3933), musical instruments (3939)
		159	Toys and Sporting Goods games and toys (3941), dolls (3942), children's vehicles (3943), sporting and athletic goods (3949)
		160	Office Supplies pens and mechanical pencils (3951), lead pencils and crayons (3952), hand stamp and stencils (3953), artists materials (3954), carbon paper and inked ribbons (3955)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
		161	Plastic Products plastic products (3971)
		162	Cork Products cork products (3982)
		163	Motion Picture Production motion picture production (7811), 7821)
	minus phonograph records (3663), plus beauty shop and barber shop equipment (3991), furs, dressed and dyed (3992), soda fountain and beer dispensing equipment (3997), fireworks and pyrotechnics (3985), small arms (1951), ammunition (1961)	164	Miscellaneous Manufactured Products phonograph records (3663), artificial flowers (3962), buttons (3963), needles, pins and fasteners (3964), brooms and brushes (3981), matches (3983), candles (3984), jewelry and instrument cases (3986), lamp shades (3987), morticians goods (3988), signs and advertising displays (3993), hair work (3994), umbrellas, parasols, and canes (3995), tobacco pipes (3996)
30	Coal, Gas, and Electric Power	16	Coal Mining anthracite coal (111) bituminous coal (121)
		167	Electric Light and Power electric light and power (4911, 4931, 496)
		168	Natural, Manufactured, and Mixed Gas natural gas transmission and distribution (4922-24), manufactured gas production and distribution (4925-26)
31	Railroad Transportation plus forwarding and arrangement of transportation (part 471)	169	Railroads railroads (4011, 4013, 4021, 4041, 4742-43)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
32	Ocean Transportation	172	Overseas Transportation Ocean transportation (441, part 4453, part 4454, part 4462, part 4463, part 4469, 4421)
33	Other Transportation	170	Trucking trucking (421)
		173	Other Water Transportation other water transportation (4422-23, 4431, 4441, 4452, part 4453, part 4454, part 4462, part 4463, part 4469)
		174	Air Transportation air transportation (45)
		175	Pipeline Transportation pipeline transportation (46)
		178	Local and Highway Transportation local transit (4012, 4111, 4121, 4131, 4141, 4151), highway transportation, n.e.c. (4311, 4321, 4331, 4381, 4399)
	minus forwarding and arrangement of transportation (part 471), miscellaneous services incidental to transportation (478), not covered explicitly	171	Warehousing and Storage warehousing and storage (4221, 4232-33, 4241, 4281, 4291, 4782-84, 4789), forwarding and arrangement of transportation (4712-13, 4721), stockyards (4731)
34	Trade	176	Wholesale Trade wholesale trade (50 minus 5093, part of 511, 512, part of 513, 514)
	plus retail bakeries (5462)	177	Retail Trade retail trade (52, 53, 54 minus 5462, 55, 56 minus part 5671, part 5681, 57, 59, plus merchandise receipts of service industries)
35	Communications plus radio and television (77)	179	Telephone and Telegraph telephone (4811), telegraph (4821, 4899)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
36	Finance and Insurance	181	Banking, Finance, and Insurance banking and finance (60, 61, 62, 67), nonlife insurance (6332-33, 6339, 6342-43, 6349, 6351-52, 6361, 6399) life insurance (6312-13, 6319, 6322-24, 6329), insurance agents and brokers and services (6411, part of 6611)
37	Rental	183	Real Estate and Rentals nonfarm residential rents (6513-14), nonfarm nonresidential rents (6512, 6515-19), real estate agencies (6531, 6541, 6551, part 6561), farm dwelling rents,[4] farm nonresidential rents[4]
38	Business Services minus radio broadcasting (77)	186	Advertising, including Radio and Television advertising (731), radio broadcasting (77), plus advertising revenues of other industries
		187	Business Services wholesale sales offices and agents (part of 511, part of 513), credit and collection agencies (7321), building maintenance services (7341-42, 7349), business services, n.e.c. (7361-7399)
39	Personal and Repair Services	182	Hotels hotels (70 except 7031 and part 7021 excludes eating and drinking receipts), auto courts and tourist camps (7031)
		184	Laundries and Dry Cleaning laundries (7211-14), dry cleaning (7221-22)

[4]Not classifiable on an SIC basis.

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
		185	Other Personal Services photographic studios (7231-32), barber and beauty shops (7241-43), shoe repair and hat cleaning (7251), funeral services (7262-63), pressing, alteration, and garment repair (7271-72), miscellaneous personal services (7291, 7299)
		188	Automobile Repair Services and Garages automobile repair services (75), plus service receipts of trade
		189	Other Repair Services electrical repair shops (7621), watch, clock, and jewelry repair (7631), armature rewinding shops (7694), miscellaneous repair services (7611, 7641, 7692-93, 7695-96, 7699), plus service receipts of trade
	minus medical and other health services (80), offices of veternarians and animal hospitals (0722)	191	Medical, Dental, and Other Professional Services medical and health services (8011, 8021, 8031, 8041, 8071-72, 8092, 8099), hospitals (8061), miscellaneous professional services (7371, 8111, 8911, 8999, part 6611, 0722)
40	Medical, Education, and Non-Profit Organizations plus medical and other health services (80)	192	Non-Profit Institutions education (821-823), schools and educational services, n.e.c. (8241-42, 8299), non-profit membership organizations (86, 8921)
41	Amusements	190	Motion Pictures and Other Amusements motion picture theaters (7831), motion picture distribution (7812), other amusements (794, 795, 799, 7911, 7921, 7931, 7961)

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
42	Scrap and Miscellaneous Industries	165	Waste Products, Metal waste materials (part of 5093)
		166	Waste Products, Nonmetal waste materials (part of 5093)
43	Undistributed	193	Undistributed
44	Eating and Drinking Places	180	Eating and Drinking Places eating and drinking places (58)
45	New Construction and Mainenance	210	Construction[5] new nonfarm residential dwelling unit construction; new nonfarm nonhousekeeping residential construction; nonfarm residential additions and alteration construction; new industrial building construction; new commercial building construction; new hospital, educational, and institutional building construction; miscellaneous new nonfarm nonresidential building construction; nonfarm residential building maintenance and repair construction; nonfarm tenant residential maintenance and repair construction; nonfarm nonresidential building maintenance and repair construction; detached private residential garage construction; new farm

[5] The BLS detailed classification of construction industries differs considerably from the SIC classification of these industries. The included SIC industries are drilling oil and gas wells (part 1331), building, repairing, and dismantling rigs and derricks (part 1332), oil and gas field contractors (1339), general building contractors (1511), highway and street construction (1611), heavy construction, except highway and street (1621), plumbing, heating, and air conditioning (1711), painting, paper hanging, and decorating (1721), electrical work (1731), masonry, stone setting, and other stone work (1741), plastering and lathing (1742), terrazzo, tile, marble, and mosaic work (1743), carpentering (1751), floor laying and other floor work, n.e.c. (1752), roofing and sheet metal work (1761), concrete work (1771), water well drilling (1781), miscellaneous special trade contractors (179), subdividers and developers (part 6551), operative builders (6561), plus other construction expenditures not classifiable on an SIC basis.

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
			operator dwelling construction; new farm tenant dwelling construction; new farm service building construction; new farm land betterment construction; maintenance and repair of farm operators' dwellings; mainteance and repair of farm tenants' dwellings; maintenance and repair of farm service buildings; new railroad construction; railroad maintenance and repair construction; new electric light and power construction; electric light and power maintenance and repair construction; new gas utility construction; gas utility maintenance and repair construction; new telephone and telegraph construction; telephone and telegraph maintenance and repair construction; new sewer construction; new water construction; sewer and water maintenance and repair construction; new petroleum pipeline construction; new local transit construction; other new utilities construction; petroleum pipeline maintenance and repair construction; local transit maintenance and repair construction; new highway, road, and street construction; highway, road, and street maintenance and repair construction; new conservation and development construction; conservation and development maintenance and repair construction; oil and gas well drilling; new military and naval construction; park, playground, and miscellaneous construction
46	Inventory Change	230	Inventory Change
47	Foreign Countries	222	Foreign Trade, Competitive

TABLE 2 (Continued)

Ind. No.	BLS 50-Industry Classification	Ind. No.	BLS 192-Industry Classification
		225	Foreign Trade, Noncompetitive
48	Government	215	Federal Government
		220	State and Local Government
49	Gross Private Capital Formation	205	Producer Durables
50	Households	200	Households

Appendix 3

Appendix 3

ESTIMATION OF PRICE ELASTICITIES FROM BUDGET STUDIES

THE TEXT of Chapter 12 discusses the purpose and logic of the methods used to estimate price elasticities from intertemporal comparisons of pairs of budget studies. This section will describe the sources and reliability of the basic data and the specific steps followed in the computations.

I. Sources of Expenditure Data

Our method required the selection of pairs of budget studies, made in the same city in two different years, as comparable as possible with respect to real income, family size, occupational and social status, and other characteristics of the families covered. Studies were selected in the first place for adequacy of coverage and method, to insure reliability of the data. The majority were conducted by the Bureau of Labor Statistics, using trained interviewers to complete the schedules and covering large groups of families.

The surveys were paired by city, period, and general characteristics of the families included. Expenditures were compared by matching families of comparable size, real income, and total expenditures from each of two studies for the same city.

A large number of available budget studies were eliminated because of our special requirements. In some instances investigations were not used because samples were too small, or methods and editing procedures seemed inaccurate. A few surveys could not be used because no comparable samples were available for the same cities in different years. This was particularly true of investigations of higher income groups. Some were discarded to avoid overlapping of time periods for the same cities, since observations for each of the selected periods were pooled in the final computation of elasticities.

Table 1 shows the pairs of studies which were finally used and their sources, grouped by periods.

II. Reliability of the Data

Since the budget studies were not designed for our purposes and differ in coverage, eligibility requirements, and method of presentation, the

TABLE 1

SOURCES OF BUDGET STUDIES USED TO COMPUTE PRICE ELASTICITIES

Period and City	First Year	Source	Second Year	Source
1918-29			1918-29	
San Francisco, California	1918	'Cost of Living in the United States,' Bureau of Labor Statistics, Bulletin #357.	1924/5	Spending Ways of a Semi-Skilled Group, University of California Press, 1931; Huntington, Emily, and Luck, Mary, Living on a Moderate Income, 1937.
Detroit, Michigan	1918	"	1929	'Standard of Living of Employees of Ford Motor Company in Detroit,' Monthly Labor Review, June 1930.
Baltimore, Maryland	1918	"	1927/8	'Cost of Living of Federal Employees in Five Cities,' Monthly Labor Review, August 1929.
Boston, Massachusetts	1918	"	1927/8	"
Chicago, Illinois	1918	"	1927/8	"
New York City, N. Y.	1918	"	1927/8	"
1924/5 - 1934/5			1924/5 -	1934/5
San Francisco, California	1924/5	Spending Ways of a Semi-Skilled Group, University of California Press, 1931; Huntington, Emily, and Luck, Mary, Living on a Moderate Income, 1937.	1934/5	'Money Disbursements of Wage Earners and Clerical Workers in the Pacific Region,' Bureau of Labor Statistics, Bulletin #639.
Detroit, Michigan	1929	Standard of Living of Employees of Ford Motor Company in Detroit,' Monthly Labor Review, June 1930.	1934/6	Money Disbursements of Wage Earners and Clerical Workers in Eight Cities in the East North Central Region,' Bureau of Labor Statistics, Bulletin #636.

Baltimore, Maryland	1927/8	'Cost of Living of Federal Employees in Five Cities,' <u>Monthly Labor Review</u>, August 1929.	1934/6	'Money Disbursements of Wage Earners and Clerical Workers in Twelve Cities of the South, Bureau of Labor Statistics, Bulletin #640.
Boston, Massachusetts	1927/8	"	1934/6	'Money Disbursements of Wage Earners and Clerical Workers in the North Atlantic Region,' Bureau of Labor Statistics, Bulletin #637, Vol. II.
New York City, N. Y.	1927/8	"	1934/6	'Money Disbursements of Wage Earners and Clerical Workers in New York City,' Bureau of Labor Statistics, Bulletin #637, Vol. I.
Chicago, Illinois	1927/8	"	1935/6	Study of Consumer Purchases, Urban Series, 'Family Expenditures in Chicago,' Bureau of Labor Statistics, Bulletin #642, Vol. II.
1918 - 1933/6			1918 - 1933/6	
Buffalo, New York	1918	'Cost of Living in the United States,' Bureau of Labor Statistics, Bulletin #357.	1935/6	'Money Disbursements of Wage Earners and Clerical Workers in the North Atlantic Region,' Bureau of Labor Statistics, Bulletin #637, Vol.II.
Philadelphia, Pennsylvania	1918	'Cost of Living in the United States,' Bureau of Labor Statistics, Bulletin #357	1934/5	'Money Disbursements of Wage Earners and Clerical Workers in the North Atlantic Region,' Bureau of Labor Statistics, Bulletin #637, Vol. II.
Pittsburgh, Pennsylvania	1918	"	1934/5	"
Portland, Maine	1918	"	1935/6	"
Scranton, Pennsylvania	1918	"	1933/4	"
Birmingham, Alabama	1918	"	1933/4	'Money Disbursements of Employed Wage Earners and Clerical Workers in Twelve Cities of the South, Bureau of Labor Statistics, Bulletin #640.

TABLE 1 (Continued)

SOURCES OF BUDGET STUDIES USED TO COMPUTE PRICE ELASTICITIES

Period and City	First Year	Source	Second Year	Source
Houston, Texas	1918	"	1934/6	"
Jacksonville, Florida	1918	"	1934/5	"
Memphis, Tennessee	1918	"	1933/4	"
Mobile, Alabama	1918	"	1934/5	"
Norfolk, Virginia	1918	"	1933/6	"
Richmond, Virginia	1918	"	1933/4	"
Cincinnati, Ohio	1918	"	1933/4	'Money Disbursements of Wage Earners and Clerical Workers in Eight Cities in the East North Central Region,' Bureau of Labor Statistics, Bulletin #636.
Cleveland, Ohio	1918	"	1933/4	"
Indianapolis, Indiana	1918	"	1933/4	"
Kansas City, Kansas	1918	"	1933/4	'Money Disbursements of Wage Earners and Clerical Workers in Five Cities in the West North Central Mountain Region,' Bureau of Labor Statistics, Bulletin #641.
Minneapolis, Minnesota	1918	"	1935/6	"
St. Louis, Missouri	1918	"	1935/6	"
Los Angeles, California	1918	"	1934/5	'Money Disbursements of Wage Earners and Clerical Workers in the Pacific Region,' Bureau of Labor Statistics, Bulletin #639.
Seattle, Washington	1918	"	1934/5	"

1932/3 - 1947			1932/3 - 1947	
Milwaukee, Wisconsin	1935/6	'Money Disbursements of Wage Earners and Clerical Workers in Eight Cities in the East North Central Region,' Bureau of Labor Statistics, Bulletin #636.	1946	Department of Labor, Bureau of Labor Statistics, Mimeographed Release.
Scranton, Pennsylvania	1933/4	'Money Disbursements of Wage Earners and Clerical Workers in the North Atlantic Region,' Bureau of Labor Statistics, Bulletin #637, Vol. II.	1946	Department of Labor, Bureau of Labor Statistics, Mimeographed Release.
Richmond, Virginia	1933/4	'Money Disbursements of Wage Earners and Clerical Workers in Twelve Cities of the South,' Bureau of Labor Statistics, Bulletin #640.	1947	Family Income and Expenditures in 1947,' <u>Monthly Labor Review</u>, April 1949.
Manchester, New Hampshire	1933/4	'Money Disbursements of Wage Earners and Clerical Workers in the North Atlantic Region,' Bureau of Labor Statisics, Bulletin #637, Vol. II,	1947	"
Washington, D.C.	1932/3	'Changes in Cost of Living of Federal Employees in the District of Columbia,' <u>Monthly Labor Review</u>, July 1934.	1947	"

samples are not perfectly comparable for any of the cities. Our object was to match studies and draw samples from within each survey in such a way as to minimize the effects of differences in occupational and social status, family composition, income, and other factors not relevant to the estimates of price elasticities. We consider that all of the matched samples finally used are reasonably comparable in the most important respects. Some of the more important discrepancies which remain are described here.

A. OCCUPATIONAL AND SOCIAL STATUS

With the exception of the 1933 survey of professional government employees in the District of Columbia and the 1946 and 1947 surveys of family expenditures in five cities, all of the studies covered families of wage earners or wage earners and clerks. However, differences in buying habits certainly exist between industrial workers of different skill groups and between factory and clerical workers. Many of the surveys included more than one type of worker and did not present expenditure data separately for different occupational groups.

The 1918 Bureau of Labor Statistics survey, which covered many cities and accounts for a large proportion of our sample, included families of all types of wage earners as well as some clerical workers' families. For Detroit, this survey has been matched with a group of predominantly unskilled wage earners employed in the Ford factory in 1929. Errors in the opposite direction are involved when the 1918 sample is compared with the survey of federal workers in 1927–8, probably representing a larger proportion of technical and clerical workers. Again, the study of streetcar men in 1925, also matched with the 1918 sample for San Francisco, differed markedly from the earlier period in its coverage of a very homogeneous group of semi-skilled workers.

The 1934–6 survey of money disbursements is the other major nationwide survey which has been compared both with the cities included in 1918 and with various separate studies. In contrast to 1918, this survey gives some information on the occupational status of the chief earners which points up some of the discrepancies in our comparisons. For example, in San Francisco only slightly over one-fourth of the families in 1934–6 were semi-skilled workers comparable to the streetcar men surveyed in 1925; of 446 families, chief earners in 164 were clerks, 77 were skilled, and 75 unskilled workers. In Detroit, only 44 of 598 families had unskilled workers as heads as against most of the families in 1929. In both cases, the occupational distribution of the smaller sample selected for comparison is not known; the discrepancy is undoubtedly less when similar income groups are matched.

Incomparability in social status is probably greatest when the 1946

and 1947 surveys of family expenditures are compared with other budget studies. These surveys differed from the others used in attempting to obtain a representative sample of all occupational groups in the community. Since data are not presented by occupation, we had to rely entirely on matching income groups.

In addition to the differences in occupation of the principal earner, and aside from the cyclical influences discussed below, the studies vary with regard to employment status, national origin, and other characteristics of the particular sample. This is particularly true when a few of the special-purpose studies are compared with the more general survey, i.e. the regularly employed and homogeneous samples of Ford workers and streetcar men discussed above.

The extent of home ownership is another factor which differs from one sample to another and may affect the budget comparisons. It has been found that on the average home owners spend somewhat more for housing than renters, and as a result have less leeway for other expenses. Federal workers, for instance, included a higher proportion of home owners in several of the cities in comparison with both the 1918 and the 1934–6 surveys. The survey of streetcar men in San Francisco also included a relatively high proportion of families owning their own homes.

The comparison of 1918 with 1934–6 involves both differences in definition and in the extent of home ownership. In the later period, the rental expenditure entered for home owners includes all actual current expenditures other than payments on principal or mortgage, down payments, and permanent improvements. In 1918, the rent item for home owners was determined by assessing a rental value. The difference in computing rent may result in a relative underestimate for 1934–6. For example, in New York City home owners averaged $348 in current housing expenditures in 1936 while the estimated rental value was $493. The balance was treated as part of annual net income.

The extent of home ownership varied widely between cities in both periods, ranging from under 5 to 50 per cent in 1918 and from 12 to 51 per cent in 1934–6. In most of the cities, a larger proportion of the sample were home owners in the later period. The information on ownership in 1934–6, however, refers to the entire sample for each city and is not available by income groups or for the family types comparable with the 1918 study. The discrepancy may be less in the particular income group matched with the 1918 samples.

B. FAMILY COMPOSITION

The various budget studies differ with respect to family size and composition, largely because of varying eligibility conditions. For example,

the Ford survey covered only families with not less than two or more than three children living at home; in 1918 families with one or more children were included, while other surveys included all family types.

In most of the comparisons, it was possible to match income and expenditure groups for similar family types and by family size within very narrow limits. In only a few cases was the discrepancy large enough to affect the results materially. When 1918 samples were compared with 1934–6, it was found that the average family was somewhat larger in 1918, particularly in the higher income classes. The difference in most of the comparisons ranged from about .1 to .7 persons; the largest discrepancy in this group was 1.7 persons in Scranton, Pennsylvania. The families covered in 1946–7 were somewhat smaller than those with which they were matched in 1934–6, apparently due to the selection of the sample rather than specific eligibility requirements. The difference in comparable expenditure groups varied from .6 to 1.9 persons, and in most cases was large enough to be a factor in comparative expenditure patterns.

C. EFFECT OF CYCLICAL CHANGES ON COMPARISON OF INCOME AND EXPENDITURES

We have attempted in comparing budget studies to match the price-adjusted expenditures as closely as possible. The periods covered by the surveys range from prosperity to depression, and include the effects of war and postwar conditions in 1918 and in 1946–7. Because of income shifts and changes in savings patterns, the comparison of similar dollar expenditures may conceal considerable differences in the economic status of the families when pre-depression studies are compared with 1934–6, and when the postwar periods are matched with other surveys.

Families in 1918 tended to have positive savings, while the reverse was true of the streetcar men in 1925, the federal workers in 1928, the Ford workers in 1929, and most of the families in 1934–6. In these and other similar comparisons, expenditures have been matched rather than incomes.

The average real income and expenditure of Ford workers in 1929 is only slightly lower than the comparable group of families in 1935. It is likely, however, that the same families surveyed in 1929 would be earning and spending considerably less than those families in the $900-1200 income bracket in 1935 with which they have been matched for comparison of expenditures.

The federal workers in 1928 had higher real incomes as a group than did the most comparable families in 1934–6. Price-adjusted expenditures were also higher for federal workers' families; the difference varied from 10 per cent in New York to 28 per cent in New Orleans. Again, it is prob-

able that the families covered in the later period actually were of higher income status than the federal workers in the years prior to the depression.

The comparison of families in 1946 and 1947 with earlier years is complicated both by the higher levels of postwar income and by the use of broader income classes in the later study. In 1934/6, $300 class intervals were used as against $1000 brackets in 1946/7.

In the postwar period, a new high level of income was reached; many families possessed liquid assets and demand for many goods exceeded supply. The picture in 1934/6 was, of course, just the reverse. Even when expenditures are adjusted to the increased prices and families with similar total expenditures are compared, these families do not have the same relative position in the income scale and may also differ in occupations, family composition, and expenditure patterns.

D. REPRESENTATION OF INCOME GROUPS IN SAMPLES

Most of the surveys, as previously described, covered the families of wage earners and low-salaried clerical workers. All of the studies except those in 1946/7 included only regularly employed workers and set definite maximum and minimum income limitations as part of the eligibility requirements. The inclusion of some marginal workers in the postwar period is not significant because of the generally high employment and income levels in those years.

The specific income limitations in the surveys do not necessarily result in comparable income distributions between periods, however. For example, the 1934/6 survey, which excluded families receiving relief, showed higher real incomes than families surveyed in 1918.

The distribution of all families covered in each survey by income brackets, in the table below, shows a higher proportion of families in the later study with incomes under $900 and over $1800:

	INCOME CLASS	PER CENT OF ALL FAMILIES (35 CITIES) 1918	1934/6[1]
		(1)	(2)
(1)	Under $900	2.7	5.4
(2)	$900-1200	20.0	19.9
(3)	$1200-1500	32.7	25.8
(4)	$1500-1800	22.6	22.6
(5)	$1800-2100	13.2	17.0
(6)	$2100-2500	5.8	6.2
(7)	$2500 and over	2.9	3.1

[1] The average income for all families of types comparable with the 1918 survey was about $200 higher in 1934/6 than in 1918.

Since expenditures were higher in 1934-36 than in 1918 for comparable incomes, in most cases the 1918 income class was compared with the one a step lower in 1934-36. The families thus matched are probably more comparable with respect to income status to the extent that the loss of income and of secondary employment shifted families down the income scale.

Similar difficulties arose in matching a number of the studies because of differences in the average incomes of the families represented in the sample. The federal workers, for example, were a group of relatively high income families in comparison with both 1918 and 1934-36. Some of the families covered in 1946 and 1947 could not be matched with any part of the earlier surveys because of higher incomes, even after deflating for price increases. Because of these incomparabilities in income distribution, it was often possible to use data for only a limited group from the entire sample covered by a particular survey.

III. Sources and Adjustment of Price Index

The Bureau of Labor Statistics Cost-of-Living index was used to adjust expenditures in the first year to those in the second year, thereby arriving at estimates of changes in the quantities purchased. In most cases, the six Bureau of Labor Statistics index subdivisions of food, clothing, housing, fuel and light, house furnishings and furniture, and miscellaneous expenditures matched the expenditure categories used in the budget studies, and no adjustment was necessary. Where minor discrepancies existed in the arrangement of expenditure items, adjustments were made to obtain comparability with the Bureau of Labor Statistics index and between the two years.

Some of the surveys, including the Bureau of Labor Statistics Studies of Money Disbursements in 1934-36, and Family Expenditures in 1946 and 1947, combined fuel and light with housing expenditures. The cost-of-living index for these categories was reweighted and combined; both the housing and fuel and light indices for the first and second year were weighted by the average family expenditure for these items in the particular city and period, and a combined index number for each year was used.

In Chapter 12 we discussed the estimation of new elasticities for food and clothing purchases after eliminating housing from the relative price. This was accomplished by recomputing the cost-of-living index to obtain a weighted average of prices of all commodities other than housing. Weights for the different periods were derived as follows:

1. 1918-24: Average family expenditures for the particular city as given in the Bureau of Labor Statistics survey of the cost-of-living, 1918.

2. 1930-47: Average family expenditures for each city used in recomputing the Bureau of Labor Statistics index weights on the basis of the 1934 6 survey of family expenditures. These weights are given in Bureau of Labor Statistics, Bulletin #699.

3. 1925-9: The average of weights (1) and (2) for each city.

IV. Method of Analysis

Space does not permit the reproduction here of all the computations which were performed with each of the pairs of budget studies shown in Table 1, to arrive at estimates of the change in quantities of goods purchased. Since the method used was the same in each case, the following example of the comparison between expenditures of families of wage earners and clerks in Cleveland, Ohio, in 1918 and 1935/6 will illustrate the steps in the procedures:

1. Matching the samples for comparability. Both surveys covered families in the same general social class; therefore the entire sample could be used subject to the conditions of adequate size of sample, and comparable family size and real income. Each of the two surveys presented expenditure data by income groups. The particular families to be compared were selected by matching income classes for comparable family types as follows:

	Year	Income Class	Number of Families	Average Number of Persons per Family	Average Real Income [1]	Average Price-Adjusted Expenditures
			(1)	(2)	(3)	(4)
(1)	1918	$ 900-1200	25	4.1	$1072	$1036
	1934/5	900-1200	42	3.5	1069	1165
(2)	1918	1500-1800	61	4.7	1573	1500
	1934/5	1200-1500	73	4.1	1343	1435
(3)	1918	1800-2100	47	5.1	1835	1727
	1934/5	1500-1800	67	4.2	1635	1624
(4)	1918	2100-2500	22	5.6	2174	2047
	1935/6	2100-2500	15	5.3	2226	2191

[1] Income deflated according to price index.

In the final computations, families with incomes ranging from $1200-1800 in 1918 were used. The subsequent steps in the computations are shown for the $1500-1800 income group.

2. Use of price index to adjust expenditures. Expenditures in the base

year, by major commodity groups, were adjusted to expenditures in the second year:

ITEM	ACTUAL EXPENDITURES, 1918	PERCENTAGE PRICE CHANGE, 1918-1935/6	1918 EXPENDITURES ADJUSTED TO 1935/6 PRICES
	(1)	(2)	(3)
(1) Food	$ 558	−27.5	$ 405
(2) Clothing	246	−19.2	199
(3) Housing, fuel and light combined	311	− 0.1	311
(4) Furniture and furnishings	92	− 6.8	86
(5) Miscellaneous	354	+41.0	499
Total	1561	− 4.0	1500

3. Interpolation to match total expenditures. Changes in the quantities consumed in 1918 and 1935/6 were then compared by interpolating to obtain the same expenditure total and computing the percentage change in the price-adjusted expenditures:

ITEM	ACTUAL EXPENDITURES, 1918, IN 1935/6 PRICES	ACTUAL EXPENDITURES, 1935/6	INTERPOLATION OF 1918 EXPENDITURES FOR 1935/6 TOTAL	PERCENTAGE CHANGE, 1918-1935/6
	(1)	(2)	(3)	(4)
(1) Food	$ 405	$ 509	$ 387	+31.5
(2) Clothing	199	146	190	−23.2
(3) Housing, fuel and light	311	343	298	+13.1
(4) Furniture and furnishings	86	67	82	−18.3
(5) Miscellaneous	499	370	477	−22.4
All items	1500	1435	1435	

In this instance the actual expenditure in each category was reduced by 4.3 per cent, the amount of the difference between the 1918 and 1935/6 totals. In no case did the interpolation exceed a 10 per cent adjustment, and usually it was closer to 5 per cent. It was assumed that within these narrow limits no real error would be introduced by postulating a proportionate distribution of the total difference.

4. Calculation of relative price change. Expenditure change, the x in the correlation, is now taken from the last column of the table given above. The final correlation, as described in the text, required the computation of changes in relative prices. This involved two steps: (a) reweighting and combining the price index to eliminate housing, and (b)

calculating the change in relative price for each pair of observations used.

(a) Price index recomputed to eliminate housing:

ITEM	1935 6 AVERAGE	1918 AVERAGE
	(1)	(2)
(1) Food	98.9	136.4
Weight	31.6	35.6
Index × weight	3125.24	4855.84
(2) Clothing	96.9	120.0
Weight	11.0	16.0
Index × weight	1065.90	1920.00
(3) Miscellaneous	98.4	69.8
Weight	28.6	21.8
Index × weight	2814.24	1521 64
(4) Combined weights	71.2	73.4
(5) Combined index	98.4	113.0

(b) Change in relative prices for food and clothing:

ITEM	1 + CHANGE IN PRICE OF PARTICULAR ITEM	1 + CHANGE IN COMBINED PRICES	COL. 1 ÷ COL. 2	COL. 3 — 1
	(1)	(2)	(3)	(4)
(1) Food	72.5	87.1	83.2	−16.8
(2) Clothing	80.8	87.1	92.8	− 7.2

INDICES

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